Clarkson University

AEROSOL PARTICLE MOTION

Equation of Motion

Consider an aerosol particle in fluid flow as shown in Figure 1. The equation of motion of a spherical aerosol particle of mass m and diameter d is given as

$$m\frac{d\mathbf{u}^p}{dt} = \frac{3\pi\mu d}{C_c}(\mathbf{u}^f - \mathbf{u}^p) + m\mathbf{g} \tag{1}$$

Here $\mathbf{u}^p$ is the particle velocity, $\mathbf{u}^f$ is the fluid velocity, $\mathbf{g}$ is the acceleration of gravity and the buoyancy effect in air is neglected. Here it is assume that the particle is away from walls and the Stokes drag is assumed.

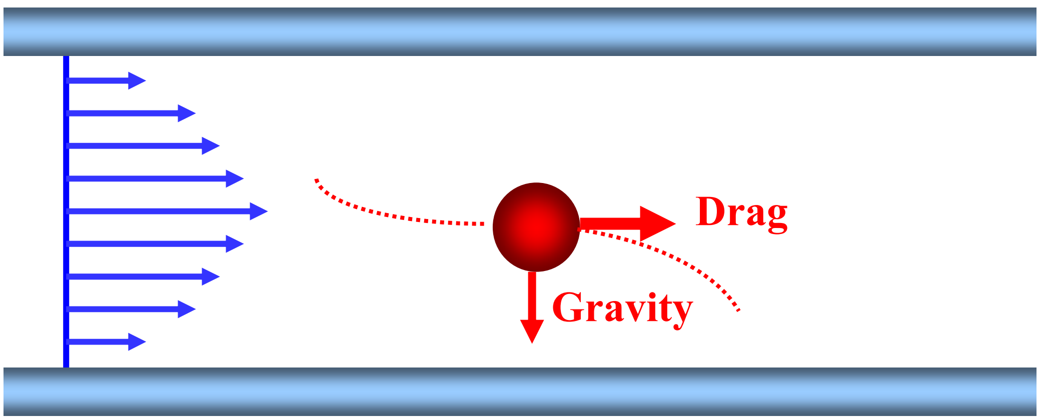

Figure 1. Schematics of an aerosol motion in a gas flow.

Dividing Equation (1) by $\frac{3\pi\mu d}{C_c}$ and rearranging, we find

$$\tau\frac{d\mathbf{u}^p}{dt} = (\mathbf{u}^f - \mathbf{u}^p) + \tau\mathbf{g} \tag{2}$$

where the particle response (relaxation) time is defined as

$$\tau = \frac{mC_c}{3\pi\mu d} = \frac{d^2\rho^p C_c}{18\mu} = \frac{Sd^2 C_c}{18\nu}, \tag{3}$$

where $m = \frac{\pi d^3 \rho^p}{6}$, ν is the kinematic viscosity of the fluid and $S = \rho^p/\rho^f$ is the density ratio. In practice, for non-Brownian particles, $C_c \approx 1$ and

$$\tau \approx \frac{d^2 \rho^p}{18\mu} \tag{4}$$

Terminal Velocity

For a particle starting from rest, the solution to (2) is given as

$$\mathbf{u}^p = (\mathbf{u}^f + \tau \mathbf{g})(1 - e^{-t/\tau}) \tag{5}$$

where $\mathbf{u}^f$ is assumed to be a constant vector. For $\mathbf{u}^f = 0$ and large t, the terminal velocity of particle u^t is given by

$$u^t = \tau g = \frac{\rho^p d^2 g C_c}{18\mu} \tag{6}$$

Table 7 – Relaxation time τ for a unit density particle in air (p = 1 atm, T = 20 ^{o} C).

Diameter, μm	$u^t = \tau g$	τ sec	Stop Distance u_o = 1 m/s	Stop Distance u_o = 10 m/s
0.05	0.39 μm/s	4×10^{-8}	0.04 μm	4×10^{-4} mm
0.1	0.93 μm/s	9.15×10^{-8}	0.092 μm	9.15×10^{-4} mm
0.5	10.1 μm/s	1.03×10^{-6}	1.03 μm	0.0103 mm
1	35 μm/s	3.57×10^{-6}	3.6 μm	0.0357 mm
5	0.77 mm/s	7.86×10^{-5}	78.6 μm	0.786 mm
10	3.03 mm/s	3.09×10^{-4}	309 μm	3.09 mm
50	7.47 cm/s	7.62×10^{-3}	7.62 mm	76.2 mm

Stopping Distance

In the absence of gravity and fluid flow, for a particle with an initial velocity of $\mathbf{u}_o^p$, the solution to (2) is given by

$$\mathbf{x}^p = \mathbf{u}_o^p \tau (1 - e^{-t/\tau}) \tag{7}$$

$$\mathbf{u}^p = \mathbf{u}_o e^{-t/\tau} \tag{8}$$

where $\mathbf{x}^p$ is the position of the particle. As $t \to \infty$, $u^p \to 0$ and

$$x^p = u_o^p \tau \tag{9}$$

is known as the stopping distance of the particle. For an initial velocity of 1000 cm/s, the stop distance for various particles are listed in table 7.

Particle Path

For constant fluid velocity, integrating Equation (5), the position of the particle is given by

$$\mathbf{x}^p = \mathbf{x}^p_o + \mathbf{u}^p_o \tau(1 - e^{-t/\tau}) + (\mathbf{u}^f + \tau \mathbf{g})[t - \tau(1 - e^{-t/\tau})] \tag{11}$$

Here $\mathbf{x}^p_o$ is the initial position of the particle. For a particle starting from rest, when the fluid velocity is in x-direction and gravity is in the negative y-direction, Equation (10) reduces to

$$x^p / u^f \tau = [t/\tau - (1 - e^{-t/\tau})] \tag{12}$$

$$y^p / u^f \tau = -\alpha[t/\tau - (1 - e^{-t/\tau})] \tag{13}$$

where the ratio of the terminal velocity to the fluid velocity α is given by

$$\alpha = \frac{\tau g}{u^f \tau} \tag{14}$$

Figure 2 shows the variation of vertical position of the particle with time.

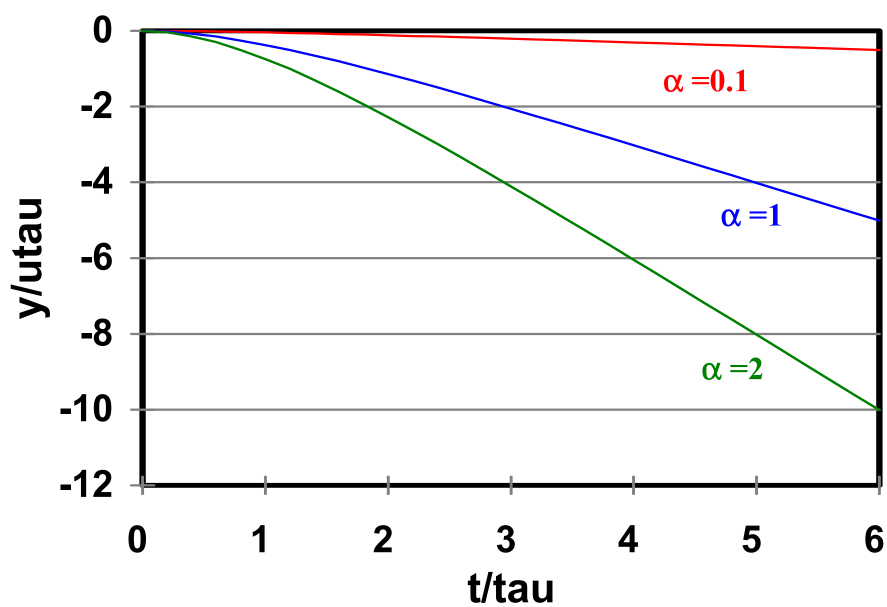

Figure 2. Variations of the particle vertical position with time.

From Equations (12) and (13), it follows that

$$y^p = -\alpha x^p \tag{15}$$

That is the particle paths are straight lines. Figure 3 shows sample particle trajectories.

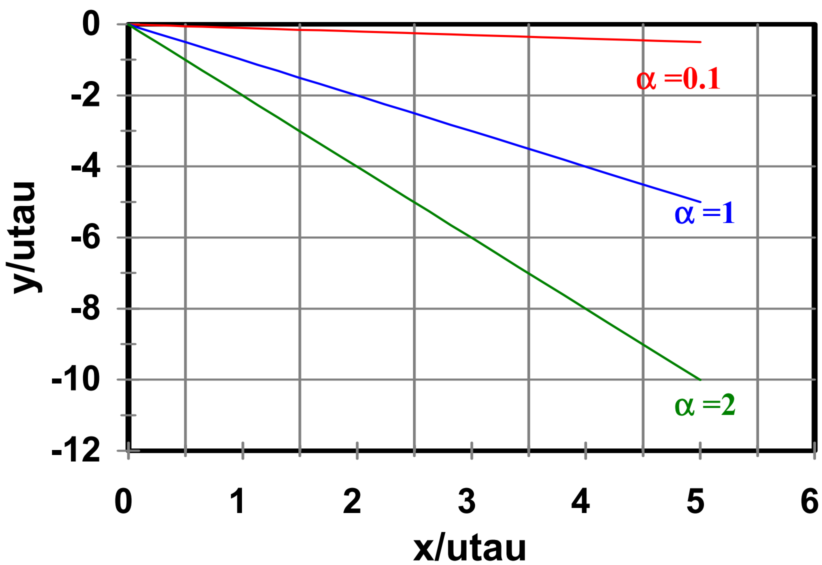

Figure 3. Sample particle trajectories.

Buoyancy Effects

For small particles in liquids, the buoyancy effect must be included. Thus, Equation (1) is replaced by

$$(m + m^a)\frac{d\mathbf{u}^p}{dt} = \frac{3\pi\mu d}{C_c}(\mathbf{u}^f - \mathbf{u}^p) + (m - m^f)\mathbf{g} \tag{16}$$

where m^f is the mass of the equivalent volume fluid given as

$$m^f = \frac{\pi d^3 \rho^f}{6} \tag{17}$$

and m^a is the apparent mass with ρ^f being the fluid density. For spherical particles, $m^a = \frac{1}{2} m^f$.

Keeping the same definition for particle relaxation time as given by (3), Equation (2) may be restated as

$$(1+\frac{1}{2S})\tau\frac{d\mathbf{u}^{\mathbf{p}}}{dt}=(\mathbf{u}^{\mathbf{f}}-\mathbf{u}^{\mathbf{p}})+\tau\mathbf{g}(1-\frac{1}{S}) \tag{18}$$

The expression for the terminal velocity then becomes

$$u^{t}=\tau g(1-\frac{1}{S})=\frac{\rho^{p}d^{2}gC_{c}}{18\mu}(1-\frac{\rho^{f}}{\rho^{p}}) \tag{18}$$

Note that the Basset force and the memory effects are neglected in this analysis.