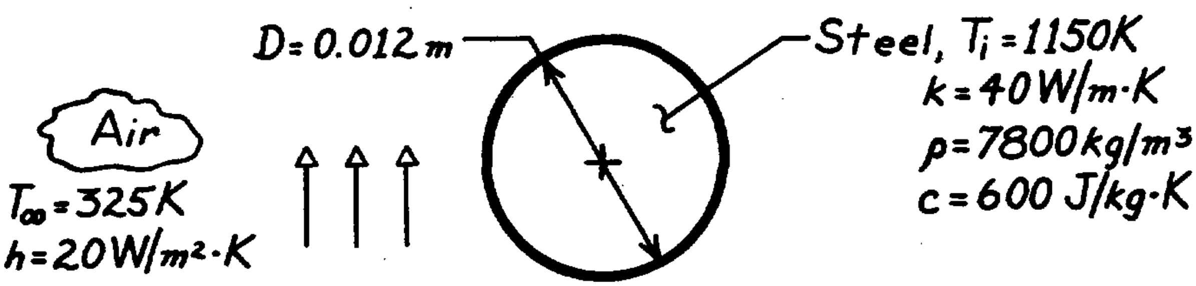
PROBLEM 5.5

KNOWN: Diameter and initial temperature of steel balls cooling in air.

FIND: Time required to cool to a prescribed temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible radiation effects, (2) Constant properties.

ANALYSIS: Applying Eq. 5.10 to a sphere ($L_c = r_o/3$),

$$Bi = \frac{hL_c}{k} = \frac{h(r_o/3)}{k} = \frac{20\ W/m^2 \cdot K\ (0.002m)}{40\ W/m \cdot K} = 0.001.$$

Hence, the temperature of the steel remains approximately uniform during the cooling process, and the lumped capacitance method may be used. From Eqs. 5.4 and 5.5,

$$t = \frac{r V c_p}{hA_s} \ln \frac{T_i - T_\infty}{T - T_\infty} = \frac{r\left(p D^3/6\right)c_p}{hp\ D^2} \ln \frac{T_i - T_\infty}{T - T_\infty}$$

$$t = \frac{7800 kg/m^3 (0.012m) 600 J/kg \cdot K}{6 \times 20\ W/m^2 \cdot K} \ln \frac{1150 - 325}{400 - 325}$$

$$t = 1122\ s\ = 0.312 h$$ <

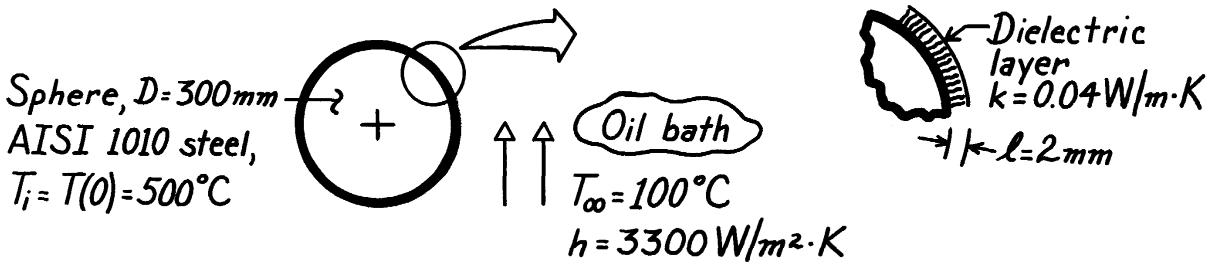
COMMENTS: Due to the large value of T_i, radiation effects are likely to be significant during the early portion of the transient. The effect is to shorten the cooling time.

PROBLEM 5.7

KNOWN: Solid steel sphere (AISI 1010), coated with dielectric layer of prescribed thickness and thermal conductivity. Coated sphere, initially at uniform temperature, is suddenly quenched in an oil bath.

FIND: Time required for sphere to reach 140°C.

SCHEMATIC:



PROPERTIES: *Table A-1*, AISI 1010 Steel $\left(\bar{T} = [500+140]^\circ C/2 = 320^\circ C \approx 600K\right)$:

$\rho = 7832\ kg/m^3,\ c = 559\ J/kg \cdot K,\ k = 48.8\ W/m \cdot K.$

ASSUMPTIONS: (1) Steel sphere is space-wise isothermal, (2) Dielectric layer has negligible thermal capacitance compared to steel sphere, (3) Layer is thin compared to radius of sphere, (4) Constant properties.

ANALYSIS: The thermal resistance to heat transfer from the sphere is due to the dielectric layer and the convection coefficient. That is,

$$R'' = \frac{\ell}{k} + \frac{1}{h} = \frac{0.002m}{0.04\ W/m \cdot K} + \frac{1}{3300\ W/m^2 \cdot K} = (0.050 + 0.0003) = 0.0503 \frac{m^2 \cdot K}{W},$$

or in terms of an overall coefficient, $U = 1/R'' = 19.88\ W/m^2 \cdot K$. The effective Biot number is

$$Bi_e = \frac{UL_c}{k} = \frac{U(r_o/3)}{k} = \frac{19.88\ W/m^2 \cdot K \times (0.300/6)m}{48.8\ W/m \cdot K} = 0.0204$$

where the characteristic length is $L_c = r_o/3$ for the sphere. Since $Bi_e < 0.1$, the lumped capacitance approach is applicable. Hence, Eq. 5.5 is appropriate with h replaced by U,

$$t = \frac{\rho c}{U}\left[\frac{V}{A_s}\right] \ln\frac{\theta_i}{\theta_o} = \frac{\rho c}{U}\left[\frac{V}{A_s}\right] \ln\frac{T(0) - T_\infty}{T(t) - T_\infty}.$$

Substituting numerical values with $(V/A_s) = r_o/3 = D/6$,

$$t = \frac{7832\ kg/m^3 \times 559\ J/kg \cdot K}{19.88\ W/m^2 \cdot K}\left[\frac{0.300m}{6}\right] \ln\frac{(500-100)^\circ C}{(140-100)^\circ C}$$

$$t = 25{,}358s = 7.04h. \qquad <$$

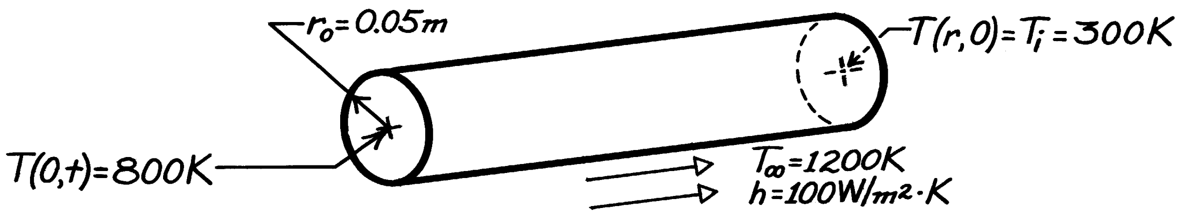
COMMENTS: (1) Note from calculation of R'' that the resistance of the dielectric layer dominates and therefore nearly all the temperature drop occurs across the layer.

PROBLEM 5.9

KNOWN: Diameter and radial temperature of AISI 1010 carbon steel shaft. Convection coefficient and temperature of furnace gases.

FIND: Time required for shaft centerline to reach a prescribed temperature.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, radial conduction, (2) Constant properties.

PROPERTIES: AISI 1010 carbon steel, *Table A.1* $\left(\bar{T} = 550\ K\right)$: $r = 7832\ kg/m^3$, k = 51.2 W/m·K, c = 541 J/kg·K, $\alpha = 1.21 \times 10^{-5}\ m^2/s$.

ANALYSIS: The Biot number is

$$Bi = \frac{hr_o/2}{k} = \frac{100\ W/m^2 \cdot K\left(0.05\ m/2\right)}{51.2\ W/m \cdot K} = 0.0488.$$

Hence, the lumped capacitance method can be applied. From Equation 5.6,

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{hAs}{r\,Vc}\right)t\right] = \exp\left[-\frac{4h}{r\,cD}t\right]$$

$$\ln\left(\frac{800 - 1200}{300 - 1200}\right) = -0.811 = -\frac{4 \times 100\ W/m^2 \cdot K}{7832\ kg/m^3\left(541\ J/kg \cdot K\right)0.1\ m}t$$

$$t = 859\ s. \qquad <$$

COMMENTS: To check the validity of the foregoing result, use the one-term approximation to the series solution. From Equation 5.49c,

$$\frac{T_o - T_\infty}{T_i - T_\infty} = \frac{-400}{-900} = 0.444 = C_1\ \exp\left(-V_1^2 Fo\right)$$

For $Bi = hr_o/k = 0.0976$, Table 5.1 yields $\varsigma_1 = 0.436$ and $C_1 = 1.024$. Hence

$$\frac{-\left(0.436\right)^2\left(1.2 \times 10^{-5}\ m^2/s\right)}{\left(0.05\ m\right)^2}t = \ln\left(0.434\right) = -0.835$$

$$t = 915\ s.$$

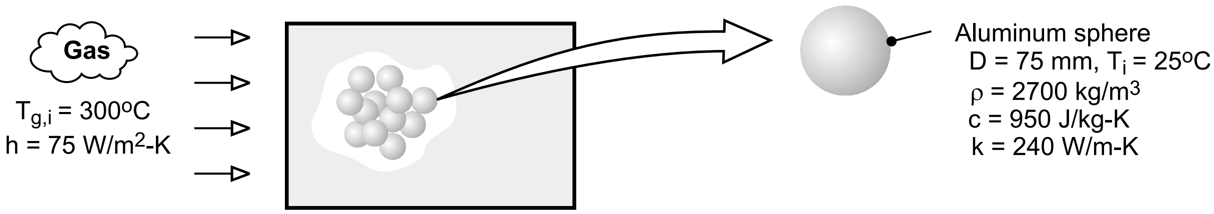
The results agree to within 6%. The lumped capacitance method underestimates the actual time, since the response at the centerline lags that at any other location in the shaft.

PROBLEM 5.11

KNOWN: Diameter, density, specific heat and thermal conductivity of aluminum spheres used in packed bed thermal energy storage system. Convection coefficient and inlet gas temperature.

FIND: Time required for sphere to acquire 90% of maximum possible thermal energy and the corresponding center temperature. Potential advantage of using copper in lieu of aluminum.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat transfer to or from a sphere by radiation or conduction due to contact with other spheres, (2) Constant properties.

ANALYSIS: To determine whether a lumped capacitance analysis can be used, first compute Bi = $h(r_o/3)/k = 75\ W/m^2\cdot K\ (0.025m)/150\ W/m\cdot K = 0.013 < 0.1$. Hence, the lumped capacitance approximation may be made, and a uniform temperature may be assumed to exist in the sphere at any time. From Eq. 5.8a, achievement of 90% of the maximum possible thermal energy storage corresponds to

$$\frac{Q}{\rho c V \theta_i} = 0.90 = 1 - \exp\left(-t/\tau_t\right)$$

where $\tau_t = \rho Vc/hA_s = \rho Dc/6h = 2700\,kg/m^3 \times 0.075m \times 950\,J/kg\cdot K/6\times 75\,W/m^2\cdot K = 427s$. Hence,

$$t = -\tau_t \ln(0.1) = 427s \times 2.30 = 984s \qquad <$$

From Eq. (5.6), the corresponding temperature at any location in the sphere is

$$T(984s) = T_{g,i} + \left(T_i - T_{g,i}\right)\exp\left(-6ht/\rho Dc\right)$$

$$T(984s) = 300°C - 275°C\ \exp\left(-6\times 75\,W/m^2\cdot K\times 984s/2700\,kg/m^3\times 0.075m\times 950\,J/kg\cdot K\right)$$

$$T(984)s = 272.5°C \qquad <$$

Obtaining the density and specific heat of copper from Table A-1, we see that $(\rho c)_{Cu} \approx 8900\ kg/m^3 \times 400\ J/kg\cdot K = 3.56 \times 10^6\ J/m^3\cdot K > (\rho c)_{Al} = 2.57 \times 10^6\ J/m^3\cdot K$. Hence, for an equivalent sphere diameter, the copper can store approximately 38% more thermal energy than the aluminum.

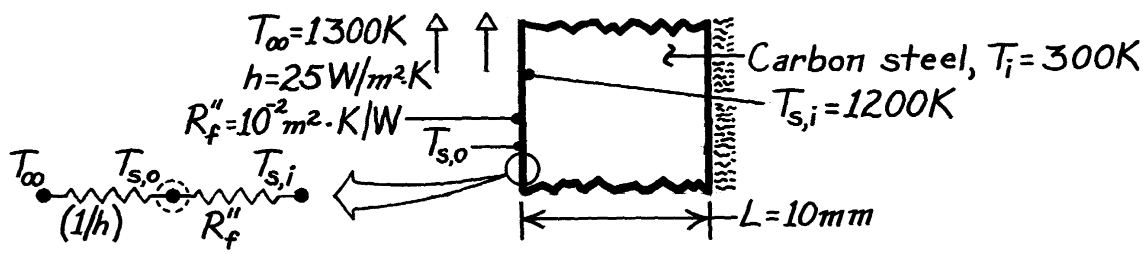
COMMENTS: Before the packed bed becomes fully charged, the temperature of the gas decreases as it passes through the bed. Hence, the time required for a sphere to reach a prescribed state of thermal energy storage increases with increasing distance from the bed inlet.

PROBLEM 5.15

KNOWN: Thickness and properties of furnace wall. Thermal resistance of film on surface of wall exposed to furnace gases. Initial wall temperature.

FIND: (a) Time required for surface of wall to reach a prescribed temperature, (b) Corresponding value of film surface temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Negligible film thermal capacitance, (3) Negligible radiation.

PROPERTIES: Carbon steel (given): ρ = 7850 kg/m^3, c = 430 J/kg·K, k = 60 W/m·K.

ANALYSIS: The overall coefficient for heat transfer from the surface of the steel to the gas is

$$U = \left(R''_{tot}\right)^{-1} = \left(\frac{1}{h} + R''_f\right)^{-1} = \left(\frac{1}{25\ \text{W/m}^2\cdot\text{K}} + 10^{-2}\,\text{m}^2\cdot\text{K/W}\right)^{-1} = 20\ \text{W/m}^2\cdot\text{K}.$$

Hence,

$$Bi = \frac{UL}{k} = \frac{20\ \text{W/m}^2\cdot\text{K}\times 0.01\ \text{m}}{60\ \text{W/m}\cdot\text{K}} = 0.0033$$

and the lumped capacitance method can be used.

(a) It follows that

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left(-t/t_t\right) = \exp\left(-t/RC\right) = \exp\left(-Ut/r\,Lc\right)$$

$$t = -\frac{r\,Lc}{U}\ln\frac{T - T_\infty}{T_i - T_\infty} = -\frac{7850\ \text{kg/m}^3\left(0.01\ \text{m}\right)430\ \text{J/kg}\cdot\text{K}}{20\ \text{W/m}^2\cdot\text{K}}\ln\frac{1200 - 1300}{300 - 1300}$$

$$t = 3886\text{s} = 1.08\text{h}.$$ <

(b) Performing an energy balance at the outer surface (s,o),

$$h\left(T_\infty - T_{s,o}\right) = \left(T_{s,o} - T_{s,i}\right)/R''_f$$

$$T_{s,o} = \frac{hT_\infty + T_{s,i}/R''_f}{h + \left(1/R''_f\right)} = \frac{25\ \text{W/m}^2\cdot\text{K}\times 1300\ \text{K} + 1200\ \text{K}/10^{-2}\,\text{m}^2\cdot\text{K/W}}{\left(25 + 100\right)\text{W/m}^2\cdot\text{K}}$$

$$T_{s,o} = 1220\ \text{K}.$$ <

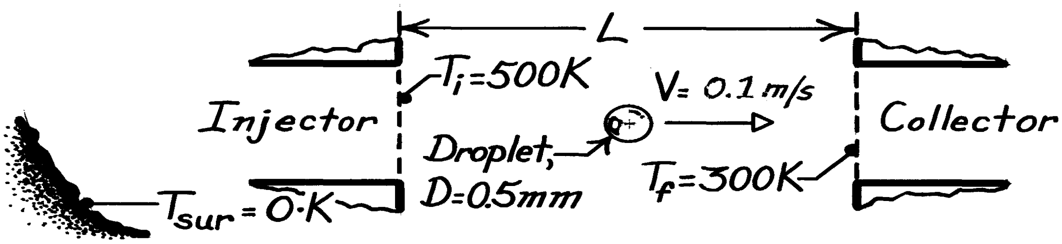
COMMENTS: The film increases t_t by increasing R_t but not C_t.

PROBLEM 5.22

KNOWN: Droplet properties, diameter, velocity and initial and final temperatures.

FIND: Travel distance and rejected thermal energy.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Negligible radiation from space.

PROPERTIES: Droplet (given): $\rho = 885$ kg/m^3, c = 1900 J/kg·K, k = 0.145 W/m·K, $\varepsilon = 0.95$.

ANALYSIS: To assess the suitability of applying the lumped capacitance method, use Equation 1.9 to obtain the maximum radiation coefficient, which corresponds to $T = T_i$.

$$h_r = \varepsilon\sigma T_i^3 = 0.95\times5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(500\text{ K}\right)^3 = 6.73\text{ W/m}^2\cdot\text{K}.$$

Hence

$$\text{Bi}_r = \frac{h_r\left(r_o/3\right)}{k} = \frac{\left(6.73\text{ W/m}^2\cdot\text{K}\right)\left(0.25\times10^{-3}\text{ m/3}\right)}{0.145\text{ W/m}\cdot\text{K}} = 0.0039$$

and the lumped capacitance method can be used. From Equation 5.19,

$$t = \frac{L}{V} = \frac{\rho c\left(\pi D^3/6\right)}{3\varepsilon\left(\pi D^2\right)\sigma}\left(\frac{1}{T_f^3} - \frac{1}{T_i^3}\right)$$

$$L = \frac{\left(0.1\text{ m/s}\right)885\text{ kg/m}^3\left(1900\text{ J/kg}\cdot\text{K}\right)0.5\times10^{-3}\text{ m}}{18\times0.95\times5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4}\left(\frac{1}{300^3} - \frac{1}{500^3}\right)\frac{1}{\text{K}^3}$$

$$L = 2.52\text{ m}. \qquad <$$

The amount of energy rejected by each droplet is equal to the change in its internal energy.

$$E_i - E_f = \rho Vc\left(T_i - T_f\right) = 885\text{ kg/m}^3\pi\frac{\left(5\times10^{-4}\text{m}\right)^3}{6}1900\text{ J/kg}\cdot\text{K}\left(200\text{ K}\right)$$

$$E_i - E_f = 0.022\text{ J}. \qquad <$$

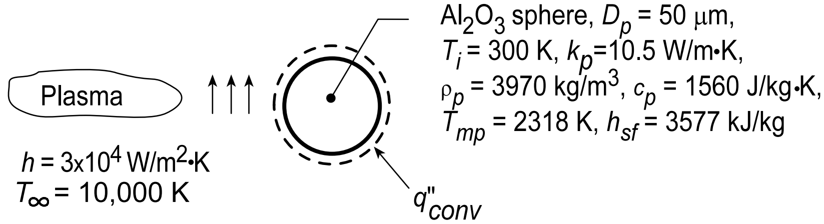
COMMENTS: Because some of the radiation emitted by a droplet will be intercepted by other droplets in the stream, the foregoing analysis overestimates the amount of heat dissipated by radiation to space.

PROBLEM 5.24

KNOWN: Diameter and thermophysical properties of alumina particles. Convection conditions associated with a two-step heating process.

FIND: (a) Time-in-flight ($t_{i\text{-}f}$) required for complete melting, (b) Validity of assuming negligible radiation.

SCHEMATIC:



ASSUMPTIONS: (1) Particle behaves as a lumped capacitance, (2) Negligible radiation, (3) Constant properties.

ANALYSIS: (a) The two-step process involves (i) the time t_1 to heat the particle to its melting point and (ii) the time t_2 required to achieve complete melting. Hence, $t_{i\text{-}f} = t_1 + t_2$, where from Eq. (5.5),

$$t_1 = \frac{\rho_p V c_p}{h A_s} \ln \frac{\theta_i}{\theta} = \frac{\rho_p D_p c_p}{6h} \ln \frac{T_i - T_\infty}{T_{mp} - T_\infty}$$

$$t_1 = \frac{3970\,\text{kg}/\text{m}^3 \left(50\times10^{-6}\,\text{m}\right) 1560\,\text{J}/\text{kg}\cdot\text{K}}{6\left(30,000\,\text{W}/\text{m}^2\cdot\text{K}\right)} \ln \frac{(300-10,000)}{(2318-10,000)} = 4\times10^{-4}\,\text{s}$$

Performing an energy balance for the second step, we obtain

$$\int_{t_1}^{t_1+t_2} q_{conv}\,dt = \Delta E_{st}$$

where $q_{conv} = hA_s(T_\infty - T_{mp})$ and $\Delta E_{st} = \rho_p V h_{sf}$. Hence,

$$t_2 = \frac{\rho_p D_p}{6h} \frac{h_{sf}}{\left(T_\infty - T_{mp}\right)} = \frac{3970\,\text{kg}/\text{m}^3 \left(50\times10^{-6}\,\text{m}\right)}{6\left(30,000\,\text{W}/\text{m}^2\cdot\text{K}\right)} \times \frac{3.577\times10^6\,\text{J/kg}}{(10,000-2318)\,\text{K}} = 5\times10^{-4}\,\text{s}$$

Hence $t_{i-f} = 9\times10^{-4}\,\text{s} \approx 1\,\text{ms}$ <

(b) Contrasting the smallest value of the convection heat flux, $q''_{conv,min} = h\left(T_\infty - T_{mp}\right) = 2.3\times10^8\ \text{W}/\text{m}^2$ to the largest radiation flux, $q''_{rad,max} = \varepsilon\sigma\left(T_{mp}^4 - T_{sur}^4\right) = 6.5\times10^5\ \text{W/m}^2$, we conclude that radiation is, in fact, negligible.

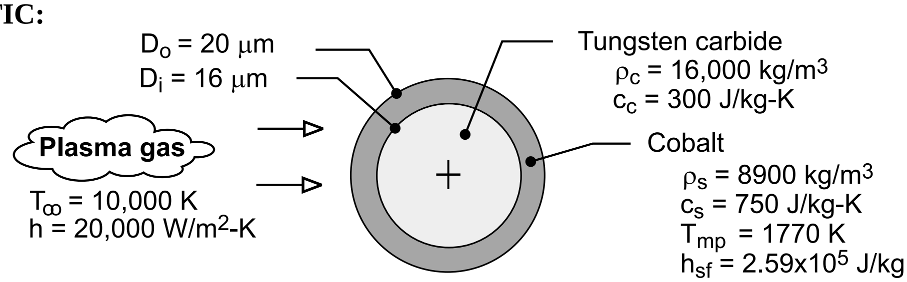
COMMENTS: (1) Since Bi = $(hr_p/3)/k \approx 0.05$, the lumped capacitance assumption is good. (2) In an actual application, the droplet should impact the substrate in a superheated condition ($T > T_{mp}$), which would require a slightly larger $t_{i\text{-}f}$.

PROBLEM 5.25

KNOWN: Diameters, initial temperature and thermophysical properties of WC and Co in composite particle. Convection coefficient and freestream temperature of plasma gas. Melting point and latent heat of fusion of Co.

FIND: Times required to reach melting and to achieve complete melting of Co.

SCHEMATIC:



ASSUMPTIONS: (1) Particle is isothermal at any instant, (2) Radiation exchange with surroundings is negligible, (3) Negligible contact resistance at interface between WC and Co, (4) Constant properties.

ANALYSIS: From Eq. (5.5), the time required to reach the melting point is

$$t_1 = \frac{(\rho V c)_{tot}}{h \pi D_o^2} \ln \frac{T_i - T_\infty}{T_{mp} - T_\infty}$$

where the total heat capacity of the composite particle is

$$(\rho V c)_{tot} = (\rho V c)_c + (\rho V c)_s = 16{,}000\,kg/m^3 \left[\pi \left(1.6\times10^{-5}\,m\right)^3/6\right] 300\,J/kg\cdot K$$

$$+8900\,kg/m^3 \left\{\pi/6\left[\left(2.0\times10^{-5}\,m\right)^3 - \left(1.6\times10^{-5}\,m\right)^3\right]\right\} 750\,J/kg\cdot K$$

$$= \left(1.03\times10^{-8} + 1.36\times10^{-8}\right) J/K = 2.39\times10^{-8}\,J/K$$

$$t_1 = \frac{2.39\times10^{-8}\,J/K}{\left(20{,}000\,W/m^2\cdot K\right)\pi\left(2.0\times10^{-5}\,m\right)^2} \ln \frac{(300 - 10{,}000)K}{(1770 - 10{,}000)K} = 1.56\times10^{-4}\,s$$

<

The time required to melt the Co may be obtained by applying the first law, Eq. (1.11b) to a control surface about the particle. It follows that

$$E_{in} = h\pi D_o^2 \left(T_\infty - T_{mp}\right) t_2 = \Delta E_{st} = \rho_s (\pi/6)\left(D_o^3 - D_i^3\right) h_{sf}$$

$$t_2 = \frac{8900\,kg/m^3 (\pi/6)\left[\left(2\times10^{-5}\,m\right)^3 - \left(1.6\times10^{-5}\,m\right)^3\right] 2.59\times10^5\,J/kg}{\left(20{,}000\,W/m^2\cdot K\right)\pi\left(2\times10^{-5}\,m\right)^2 (10{,}000 - 1770)K} = 2.28\times10^{-5}\,s$$

<

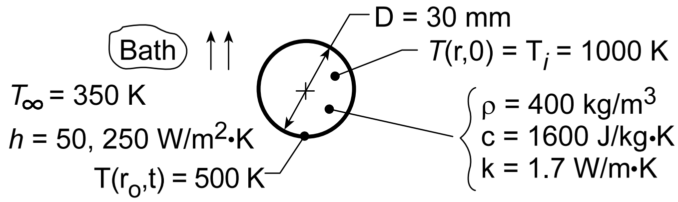
COMMENTS: (1) The largest value of the radiation coefficient corresponds to $h_r = \varepsilon\sigma\,(T_{mp} + T_{sur})\left(T_{mp}^2 + T_{sur}^2\right)$. For the maximum possible value of $\varepsilon = 1$ and $T_{sur} = 300K$, $h_r = 378\ W/m^2\cdot K \ll h = 20{,}000\ W/m^2\cdot K$. Hence, the assumption of negligible radiation exchange is excellent. (2) Despite the large value of h, the small values of D_o and D_i and the large thermal conductivities (~ 40 W/m·K and 70 W/m·K for WC and Co, respectively) render the lumped capacitance approximation a good one. (3) A detailed treatment of plasma heating of a composite powder particle is provided by Demetriou, Lavine and Ghoniem (Proc. 5th ASME/JSME Joint Thermal Engineering Conf., March, 1999).

PROBLEM 5.49

KNOWN: A long cylinder, initially at a uniform temperature, is suddenly quenched in a large oil bath.

FIND: (a) Time required for the surface to reach 500 K, (b) Effect of convection coefficient on surface temperature history.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional radial conduction, (2) Constant properties, (3) Fo > 0.2.

ANALYSIS: (a) Check first whether lumped capacitance method is applicable. For h = 50 W/m$^2\cdot$K,

$$Bi_c = \frac{hL_c}{k} = \frac{h(r_o/2)}{k} = \frac{50\,W/m^2\cdot K(0.015\,m/2)}{1.7\,W/m\cdot K} = 0.221.$$

Since $Bi_c > 0.1$, method is not suited. Using the approximate series solution for the infinite cylinder,

$$\theta^*(r^*, Fo) = C_1 \exp\left(-\zeta_1^2 Fo\right) \times J_0\left(\zeta_1 r^*\right) \tag{1}$$

Solving for Fo and setting $r^* = 1$, find

$$Fo = -\frac{1}{\zeta_1^2}\ln\left[\frac{\theta^*}{C_1 J_0(\zeta_1)}\right]$$

where $\theta^* = (1, Fo) = \dfrac{T(r_o, t_o) - T_\infty}{T_i - T_\infty} = \dfrac{(500-350)K}{(1000-350)K} = 0.231$.

From Table 5.1, with Bi = 0.441, find $\zeta_1 = 0.8882$ rad and $C_1 = 1.1019$. From Table B.4, find $J_0(\zeta_1) = 0.8121$. Substituting numerical values into Eq. (2),

$$Fo = -\frac{1}{(0.8882)^2}\ln[0.231/1.1019 \times 0.8121] = 1.72.$$

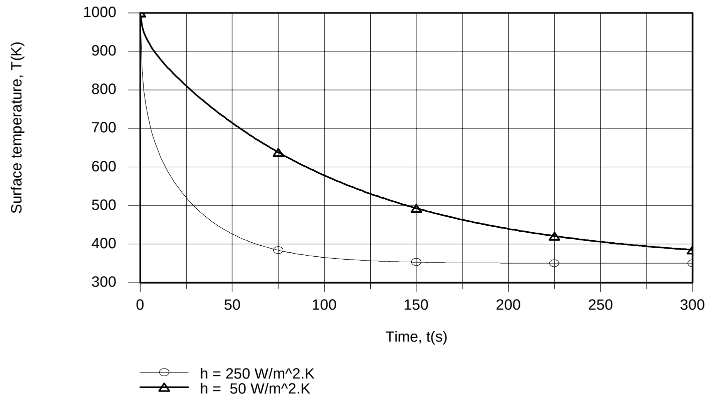
From the definition of the Fourier number, Fo = $\alpha t/r_o^2$, and $\alpha = k/\rho c$,

$$t = Fo\frac{r_o^2}{\alpha} = Fo \cdot r_o^2\,\frac{\rho c}{k}$$

$$t = 1.72(0.015\,m)^2 \times 400\,kg/m^3 \times 1600\,J/kg\cdot K/1.7\,W/m\cdot K = 145s.$$ <

(b) Using the IHT *Transient Conduction Model* for a *Cylinder*, the following surface temperature histories were obtained.

Continued...



Increasing the convection coefficient by a factor of 5 has a significant effect on the surface temperature, greatly accelerating its approach to the oil temperature. However, even with h = 250 $W/m^2 \cdot K$, Bi = 1.1 and the convection resistance remains significant. Hence, in the interest of accelerated cooling, additional benefit could be achieved by further increasing the value of h.

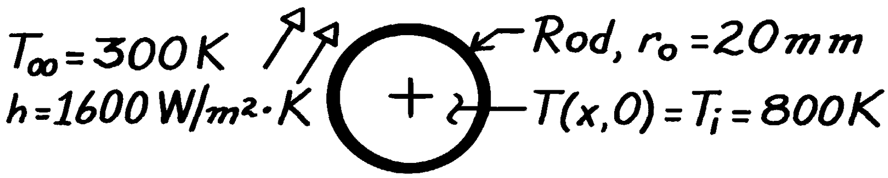
COMMENTS: For Part (a), note that, since Fo = 1.72 > 0.2, the approximate series solution is appropriate.

PROBLEM 5.51

KNOWN: Sapphire rod, initially at a uniform temperature of 800K is suddenly cooled by a convection process; after 35s, the rod is wrapped in insulation.

FIND: Temperature rod reaches after a long time following the insulation wrap.

SCHEMATIC:



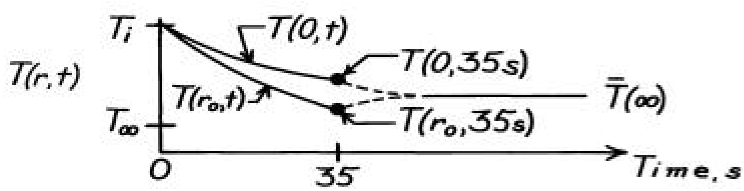
ASSUMPTIONS: (1) One-dimensional radial conduction, (2) Constant properties, (3) No heat losses from the rod when insulation is applied.

PROPERTIES: *Table A-2*, Aluminum oxide, sapphire (550K): $\rho = 3970\ kg/m^3$, $c = 1068\ J/kg\cdot K$, $k = 22.3\ W/m\cdot K$, $\alpha = 5.259\times10^{-5}\ m^2/s$.

ANALYSIS: First calculate the Biot number with $L_c = r_o/2$,

$$Bi = \frac{h\,L_c}{k} = \frac{h\,(r_o/2)}{k} = \frac{1600\ W/m^2\cdot K\,(0.020\ m/2)}{22.3\ W/m\cdot K} = 0.72.$$

Since Bi > 0.1, the rod cannot be approximated as a lumped capacitance system. The temperature distribution during the cooling process, $0 \le t \le 35s$, and for the time following the application of insulation, $t > 35s$, will appear as



Eventually ($t \to \infty$), the temperature of the rod will be uniform at $\bar{T}(\infty)$. To find $\bar{T}(\infty)$, write the conservation of energy requirement for the rod on a *time interval* basis, $E_{in} - E_{out} = \Delta E \equiv E_{final} - E_{initial}$.

Using the nomenclature of Section 5.5.3 and basing energy relative to T_∞, the energy balance becomes

$$-Q = r\ cV\left(\bar{T}(\infty) - T_\infty\right) - Q_o$$

where $Q_o = \rho c V(T_i - T_\infty)$. Dividing through by Q_o and solving for $\bar{T}(\infty)$, find

$$\bar{T}(\infty) = T_\infty + (T_i - T_\infty)(1 - Q/Q_o).$$

From the Groeber chart, Figure D.6, with

$$Bi = \frac{hr_o}{k} = \frac{1600\ W/m^2\cdot K \times 0.020m}{22.3\ W/m\cdot K} = 1.43$$

$$Bi^2Fo = Bi^2\left(a\ t/r_o^2\right) = (1.43)^2\ \left(5.259\times10^{-6}\ m^2/s \times 35s/(0.020m)^2\right) = 0.95.$$

find $Q/Q_o \approx 0.57$. Hence,

$$\bar{T}(\infty) = 300K + (800 - 300)K\ (1\text{-}0.57) = 515\ K.$$ <

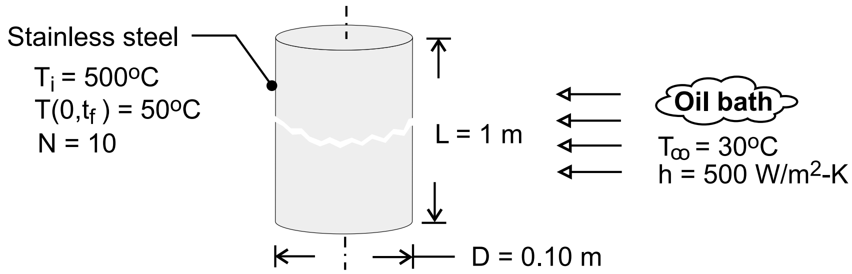
COMMENTS: From use of Figures D.4 and D.5, find $T(0,35s) = 525K$ and $T(r_o,35s) = 423K$.

PROBLEM 5.54

KNOWN: Diameter and initial temperature of roller bearings. Temperature of oil bath and convection coefficient. Final centerline temperature. Number of bearings processed per hour.

FIND: Time required to reach centerline temperature. Cooling load.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, radial conduction in rod, (2) Constant properties.

PROPERTIES: *Table A.1*, St. St. 304 $\left(\overline{T} = 548\,K\right)$: ρ=7900 kg/m^3, k = 19.0 W/m·K, c_p = 546 J/kg·K, α = 4.40 × 10^{-6} m^2/s.

ANALYSIS: With Bi = h (r_o/2)/k = 0.658, the lumped capacitance method can not be used. From the one-term approximation of Eq. 5.49 c for the centerline temperature,

$$\theta_o^* = \frac{T_o - T_\infty}{T_i - T_\infty} = \frac{50-30}{500-30} = 0.0426 = C_1 \exp\left(-\zeta_1^2 Fo\right) = 1.1382 \exp\left[-\left(0.9287\right)^2 Fo\right]$$

where, for Bi = hr_o/k = 1.316, C_1 = 1.1382 and ζ_1 = 0.9287 from Table 5.1.

$$Fo = -\ell n\left(0.0374\right)/0.863 = 3.81$$

$$t_f = Fo\, r_o^2/\alpha = 3.81\left(0.05\,m\right)^2/4.40\times10^{-6} = 2162\,s = 36\,min \qquad <$$

From Eqs. 5.44 and 5.51, the energy extracted from a single rod is

$$Q = \rho c V\left(T_i - T_\infty\right)\left[1 - \frac{2\theta_o^*}{\zeta_1} J_1\left(\zeta_1\right)\right]$$

With J_1 (0.9287) = 0.416 from Table B.4,

$$Q = 7900\,kg/m^3 \times 546\,J/kg\cdot K\left[\pi\left(0.05\,m\right)^2 1m\right]470\,K\left[1 - \frac{0.0852\times0.416}{0.9287}\right] = 1.53\times10^7\ J$$

The nominal cooling load is

$$\overline{q} = \frac{N\,Q}{t_f} = \frac{10\times1.53\times10^7\ J}{2162\,s} = 70{,}800\,W = 7.08\ kW \qquad <$$

COMMENTS: For a centerline temperature of 50°C, Eq. 5.49b yields a surface temperature of

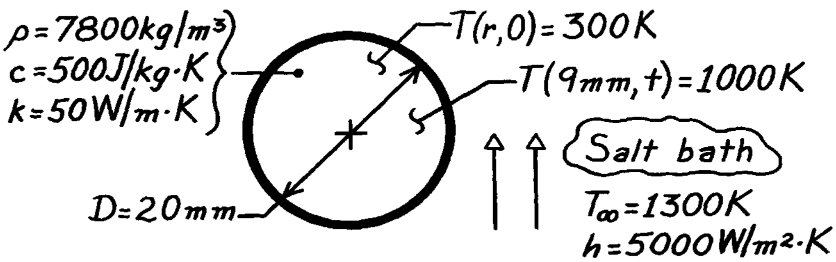
$$T\left(r_o, t\right) = T_\infty + \left(T_i - T_\infty\right)\theta_o^*\ J_o\left(\zeta_1\right) = 30°C + 470°C\times0.0426\times0.795 = 45.9°C$$

PROBLEM 5.58

KNOWN: A ball bearing is suddenly immersed in a molten salt bath; heat treatment to harden occurs at locations with T > 1000K.

FIND: Time required to harden outer layer of 1mm.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional radial conduction, (2) Constant properties, (3) Fo ≥ 0.2.

ANALYSIS: Since any location within the ball whose temperature exceeds 1000K will be hardened, the problem is to find the time when the location r = 9mm reaches 1000K. Then a 1mm outer layer will be hardened. Begin by finding the Biot number.

$$\mathrm{Bi} = \frac{h\, r_o}{k} = \frac{5000\ \mathrm{W/m^2\cdot K}\,(0.020\mathrm{m}/2)}{50\ \mathrm{W/m\cdot K}} = 1.00.$$

Using the one-term approximate solution for a sphere, find

$$\mathrm{Fo} = -\frac{1}{\zeta_1^2}\ln\left[\theta^* / C_1 \frac{1}{\zeta_1 r^*}\sin\left(\zeta_1 r^*\right)\right].$$

From Table 5.1 with Bi = 1.00, for the sphere find $\zeta_1 = 1.5708$ rad and C_1 = 1.2732. With r* = r/r_o = (9mm/10mm) = 0.9, substitute numerical values.

$$\mathrm{Fo} = \frac{-1}{(1.5708)^2}\ln\left[\frac{(1000-1300)\mathrm{K}}{(300-1300)\mathrm{K}} / 1.2732 \frac{1}{1.5708\times 0.9}\sin\left(1.5708\times 0.9\ \mathrm{rad}\right)\right] = 0.441.$$

From the definition of the Fourier number with α = k/ρc,

$$t = \mathrm{Fo}\frac{r_o^2}{\alpha} = \mathrm{Fo}\cdot r_o^2\frac{\rho\, c}{k} = 0.441\times\left[\frac{0.020\mathrm{m}}{2}\right]^2 7800\frac{\mathrm{kg}}{\mathrm{m^3}}\times 500\frac{\mathrm{J}}{\mathrm{kg\cdot K}} / 50\ \mathrm{W/m\cdot K} = 3.4\mathrm{s}.$$ <

COMMENTS: (1) Note the very short time required to harden the ball. At this time it can be easily shown the center temperature is T(0,3.4s) = 871 K.

(2) The Heisler charts can also be used. From Fig. D.8, with Bi^{-1} = 1.0 and r/r_o = 0.9, read θ/θ_o = 0.69(±0.03). Since

$$\theta = T - T_\infty = 1000 - 1300 = -300\mathrm{K} \qquad \theta_i = T_i - T_\infty = -1000\mathrm{K}$$

it follows that

$$\frac{\theta}{\theta_i} = 0.30. \quad \text{Since} \quad \frac{\theta}{\theta_i} = \frac{\theta}{\theta_o}\cdot\frac{\theta_o}{\theta_i}, \quad \text{then} \quad \frac{\theta}{\theta_i} = 0.69\ \frac{\theta_o}{\theta_i}$$

and $\quad \theta_o / \theta_i = 0.30 / 0.69 = 0.43\ (\pm 0.02).$

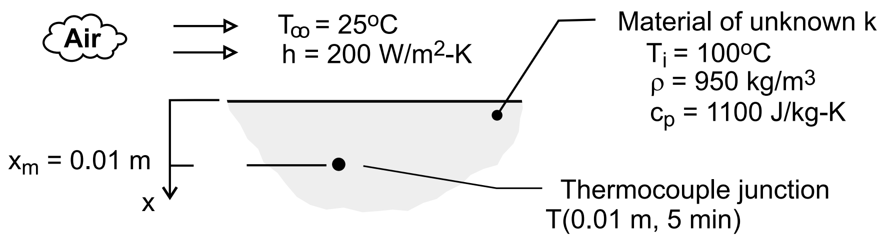
From Fig. D.7 at θ_o/θ_i=0.43, Bi^{-1}=1.0, read Fo = 0.45 (±0.03) and t = 3.5 (±0.2)s. Note the use of tolerances associated with reading the charts to ±5%.

PROBLEM 5.79

KNOWN: Initial temperature, density and specific heat of a material. Convection coefficient and temperature of air flow. Time for embedded thermocouple to reach a prescribed temperature.

FIND: Thermal conductivity of material.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in x, (2) Sample behaves as a semi-infinite modium, (3) Constant properties.

ANALYSIS: The thermal response of the sample is given by Case 3, Eq. 5.60,

$$\frac{T(x,t)-T_i}{T_\infty - T_i} = \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \left[\exp\left(\frac{hx}{k}+\frac{h^2\alpha t}{k^2}\right)\right]\left[\operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}+\frac{h\sqrt{\alpha t}}{k}\right)\right]$$

where, for x = 0.01m at t = 300 s, $[T(x,t) - T_i]/(T_\infty - T_i) = 0.533$. The foregoing equation must be solved iteratively for k, with $\alpha = k/\rho c_p$. The result is

$$k = 0.45\ \text{W/m}\cdot\text{K} \qquad <$$

with $\alpha = 4.30 \times 10^{-7}\ \text{m}^2/\text{s}$.

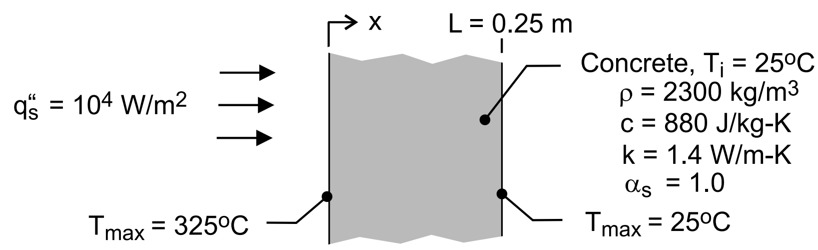
COMMENTS: The solution may be effected by inserting the *Transient Conduction*/*Semi-infinite Solid*/*Surface Conduction Model* of *IHT* into the work space and applying the *IHT Solver*. However, the ability to obtain a converged solution depends strongly on the initial guesses for k and α.

PROBLEM 5.82

KNOWN: Thickness, initial temperature and thermophysical properties of concrete firewall. Incident radiant flux and duration of radiant heating. Maximum allowable surface temperatures at the end of heating.

FIND: If maximum allowable temperatures are exceeded.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in wall, (2) Validity of semi-infinite medium approximation, (3) Negligible convection and radiative exchange with the surroundings at the irradiated surface, (4) Negligible heat transfer from the back surface, (5) Constant properties.

ANALYSIS: The thermal response of the wall is described by Eq. (5.60)

$$T(x,t) = T_i + \frac{2\,q_o''(\alpha t/\pi)^{1/2}}{k}\exp\left(\frac{-x^2}{4\alpha t}\right) - \frac{q_o'' x}{k}\operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

where, $\alpha = k/\rho c_p = 6.92\times10^{-7}\,m^2/s$ and for $t = 30\ min = 1800s$, $2q_o''(\alpha t/\pi)^{1/2}/k = 284.5\,K$. Hence, at x = 0,

$$T(0, 30\ min) = 25°C + 284.5°C = 309.5°C < 325°C \qquad <$$

At $x = 0.25m$, $\left(-x^2/4\alpha t\right) = -12.54$, $q_o''x/k = 1,786K$, and $x/2(\alpha t)^{1/2} = 3.54$. Hence,

$$T(0.25m,\ 30\,min) = 25°C + 284.5°C\left(3.58\times10^{-6}\right) - 1786°C\times(\sim 0) \approx 25°C \qquad <$$

Both requirements are met.

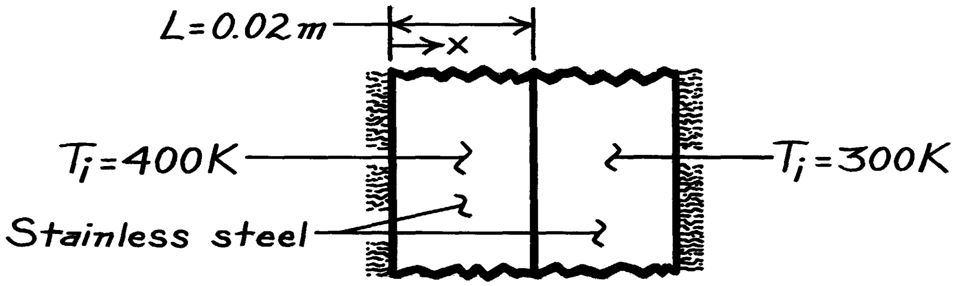
COMMENTS: The foregoing analysis is conservative since heat transfer at the irradiated surface due to convection and net radiation exchange with the environment have been neglected. If the emissivity of the surface and the temperature of the surroundings are assumed to be ε = 1 and T_{sur} = 298K, radiation exchange at T_s = 309.5°C would be $q_{rad}'' = \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) = 6,080\ W/m^2\cdot K$, which is significant (~ 60% of the prescribed radiation).

PROBLEM 5.84

KNOWN: Initial temperatures, properties, and thickness of two plates, each insulated on one surface.

FIND: Temperature on insulated surface of one plate at a prescribed time after they are pressed together.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) Negligible contact resistance.

PROPERTIES: Stainless steel (given): ρ = 8000 kg/m^3, c = 500 J/kg·K, k = 15 W/m·K.

ANALYSIS: At the instant that contact is made, the plates behave as semi-infinite slabs and, since the (ρkc) product is the same for the two plates, Equation 5.63 yields a surface temperature of

$$T_s = 350\text{K}.$$

The interface will remain at this temperature, even after thermal effects penetrate to the insulated surfaces. The transient response of the hot wall may therefore be calculated from Equations 5.40 and 5.41. At the insulated surface (x* = 0), Equation 5.41 yields

$$\frac{T_o - T_s}{T_i - T_s} = C_1 \exp\left(-z_1^2 Fo\right)$$

where, in principle, $h \to \infty$ and $T_\infty \to T_s$. From Equation 5.39c, Bi $\longrightarrow \infty$ yields z_1 = 1.5707, and from Equation 5.39b

$$C_1 = \frac{4\sin z_1}{2z_1 + \sin(2z_1)} = 1.273$$

Also, $$Fo = \frac{\alpha t}{L^2} = \frac{3.75\times10^{-6}\text{m}^2/\text{s}(60\text{s})}{(0.02\text{ m})^2} = 0.563.$$

Hence, $$\frac{T_o - 350}{400 - 350} = 1.273\exp\left(-1.5707^2 \times 0.563\right) = 0.318$$

$$T_o = 365.9\text{ K}.$$ <

COMMENTS: Since Fo > 0.2, the one-term approximation is appropriate.