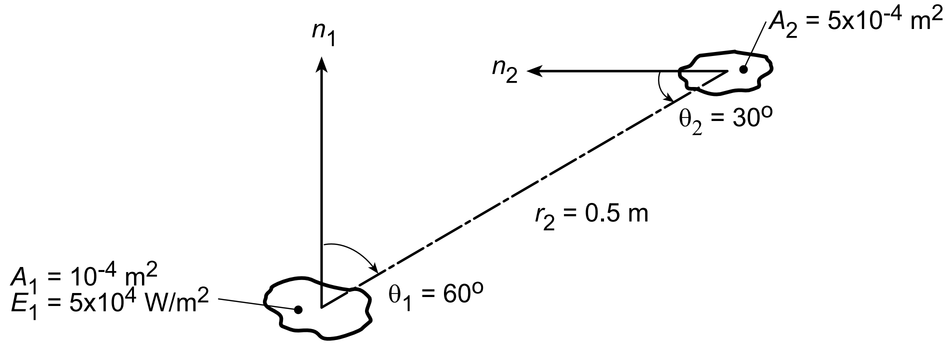
PROBLEM 12.2

KNOWN: A diffuse surface of area $A_1 = 10^{-4} m^2$ emits diffusely with total emissive power $E = 5 \times 10^4$ W/m^2 .

FIND: (a) Rate this emission is intercepted by small surface of area $A_2 = 5 \times 10^{-4}$ m^2 at a prescribed location and orientation, (b) Irradiation G_2 on A_2, and (c) Compute and plot G_2 as a function of the separation distance r_2 for the range $0.25 \le r_2 \le 1.0$ m for zenith angles $\theta_2 = 0$, 30 and 60°.

SCHEMATIC:



ASSUMPTIONS: (1) Surface A_1 emits diffusely, (2) A_1 may be approximated as a differential surface area and that $A_2 / r_2^2 << 1$.

ANALYSIS: (a) The rate at which emission from A_1 is intercepted by A_2 follows from Eq. 12.5 written on a total rather than spectral basis.

$$q_{1\to 2} = I_{e,1}(\theta,\phi) A_1 \cos\theta_1 d\omega_{2-1}. \quad (1)$$

Since the surface A_1 is diffuse, it follows from Eq. 12.13 that

$$I_{e,1}(\theta,\phi) = I_{e,1} = E_1/\pi \quad . \quad (2)$$

The solid angle subtended by A_2 with respect to A_1 is

$$d\omega_{2-1} \approx A_2 \cdot \cos\theta_2 / r_2^2 \quad . \quad (3)$$

Substituting Eqs. (2) and (3) into Eq. (1) with numerical values gives

$$q_{1\to 2} = \frac{E_1}{\pi} \cdot A_1 \cos\theta_1 \cdot \frac{A_2 \cos\theta_2}{r_2^2} = \frac{5\times 10^4 \, W/m^2}{\pi \, sr} \times \left(10^{-4} m^2 \times \cos 60^\circ\right) \times \left[\frac{5\times 10^{-4} m^2 \times \cos 30^\circ}{(0.5m)^2}\right] sr \quad (4)$$

$$q_{1\to 2} = 15,915 \, W/m^2 sr \times \left(5\times 10^{-5} m^2\right) \times 1.732\times 10^{-3} sr = 1.378\times 10^{-3} W . \quad <$$

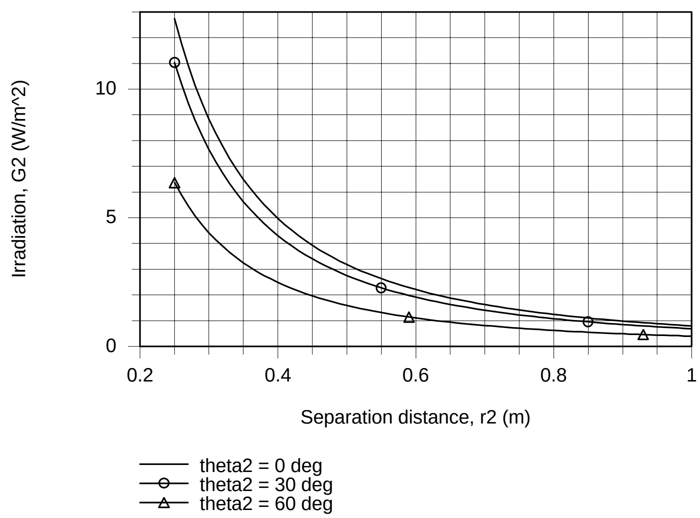
(b) From section 12, 2.3, the irradiation is the rate at which radiation is incident upon the surface per unit surface area,

$$G_2 = \frac{q_{1\to 2}}{A_2} = \frac{1.378\times 10^{-3} W}{5\times 10^{-4} m^2} = 2.76 \, W/m^2 \quad (5) <$$

(c) Using the IHT workspace with the foregoing equations, the G_2 was computed as a function of the separation distance for selected zenith angles. The results are plotted below.

Continued...

PROBLEM 12.2 (Cont.)



For all zenith angles, G_2 decreases with increasing separation distance r_2 . From Eq. (3), note that $d\omega_{2-1}$ and, hence G_2, vary inversely as the square of the separation distance. For any fixed separation distance, G_2 is a maximum when $\theta_2 = 0°$ and decreases with increasing θ_2, proportional to cos θ_2.

COMMENTS: (1) For a diffuse surface, the intensity, I_e, is independent of direction and related to the emissive power as $I_e = E/\pi$. Note that π has the units of $[\mathrm{sr}]$ in this relation.

(2) Note that Eq. 12.5 is an important relation for determining the radiant power leaving a surface in a prescribed manner. It has been used here on a total rather than spectral basis.

(3) Returning to part (b) and referring to Figure 12.10, the irradiation on A2 may be expressed as

$$G_2 = I_{i,2} \cos\theta_2 \frac{A_1 \cos\theta_1}{r_2^2}$$

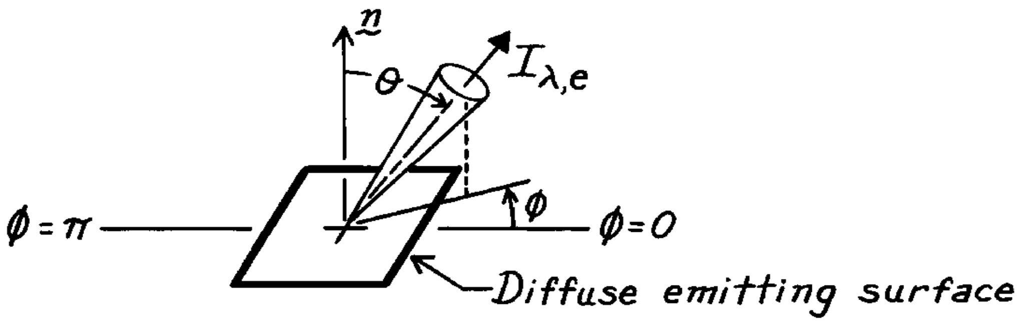
Show that the result is $G_2 = 2.76\ \mathrm{W/m^2}$. Explain how this expression follows from Eq. (12.15).

PROBLEM 12.9

KNOWN: Emissive power of a diffuse surface.

FIND: Fraction of emissive power that leaves surface in the directions $\pi/4 \le \theta \le \pi/2$ and $0 \le \phi \le \pi$.

SCHEMATIC:



ASSUMPTIONS: (1) Diffuse emitting surface.

ANALYSIS: According to Eq. 12.12, the total, hemispherical emissive power is

$$E = \int_0^\infty \int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,e}(\lambda,\theta,\phi)\cos\theta \sin\theta \, d\theta \, d\phi \, d\lambda.$$

For a diffuse surface $I_{\lambda,e}$ (λ, θ, ϕ) is independent of direction, and as given by Eq. 12.14,

$$E = \pi I_e.$$

The emissive power, which has directions prescribed by the limits on θ and ϕ, is

$$\Delta E = \int_0^\infty I_{\lambda,e}(\lambda)\, d\lambda \left[\int_0^\pi d\phi\right]\left[\int_{\pi/4}^{\pi/2} \cos\theta \sin\theta \, d\theta\right]$$

$$\Delta E = I_e[\phi]_0^\pi \times \left[\frac{\sin^2\theta}{2}\right]_{\pi/4}^{\pi/2} = I_e[\pi]\left[\frac{1}{2}\left(1 - 0.707^2\right)\right]$$

$$\Delta E = 0.25\pi I_e.$$

It follows that

$$\frac{\Delta E}{E} = \frac{0.25\pi I_e}{\pi I_e} = 0.25.$$ <

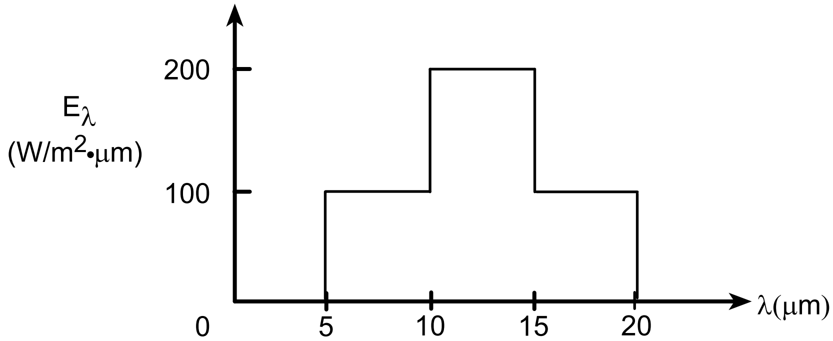
COMMENTS: The diffuse surface is an important concept in radiation heat transfer, and the directional independence of the intensity should be noted.

PROBLEM 12.10

KNOWN: Spectral distribution of E_λ for a diffuse surface.

FIND: (a) Total emissive power E, (b) Total intensity associated with directions $\theta = 0^o$ and $\theta = 30^o$, and (c) Fraction of emissive power leaving the surface in directions $\pi/4 \le \theta \le \pi/2$.

SCHEMATIC:



ASSUMPTIONS: (1) Diffuse emission.

ANALYSIS: (a) From Eq. 12.11 it follows that

$$E = \int_0^\infty E_\lambda(\lambda)\,d\lambda = \int_0^5 (0)\,d\lambda + \int_5^{10} (100)\,d\lambda + \int_{10}^{15} (200)\,d\lambda + \int_{15}^{20} (100)\,d\lambda + \int_{20}^\infty (0)\,d\lambda$$

$$E = 100\ W/m^2 \cdot\mu m\ (10 - 5)\ \mu m + 200 W/m^2 \cdot\mu m\ (15 - 10)\ \mu m + 100\ W/m^2 \cdot\mu m\ (20 - 15)\ \mu m$$

$$E = 2000\ W/m^2$$ <

(b) For a diffuse emitter, I_e is *independent* of θ and Eq. 12.14 gives

$$I_e = \frac{E}{\pi} = \frac{2000\ W/m^2}{\pi\ sr}$$

$$I_e = 637\ W/m^2 \cdot sr$$ <

(c) Since the surface is diffuse, use Eqs. 12.10 and 12.14,

$$\frac{E(\pi/4 \to \pi/2)}{E} = \frac{\int_0^{2\pi} \int_{\pi/4}^{\pi/2} I_e \cos\theta \sin\theta\, d\theta\, d\phi}{\pi I_e}$$

$$\frac{E(\pi/4 \to \pi/2)}{E} = \frac{\int_{\pi/4}^{\pi/2} \cos\theta \sin\theta\, d\theta \int_0^{2\pi} d\phi}{\pi} = \frac{1}{\pi}\left[\frac{\sin^2\theta}{2}\right]_{\pi/4}^{\pi/2} \phi\Big|_0^{2\pi}$$

$$\frac{E(\pi/4 \to \pi/2)}{E} = \frac{1}{\pi}\left[\frac{1}{2}(1^2 - 0.707^2)(2\pi - 0)\right] = 0.50$$ <

COMMENTS: (1) Note how a spectral integration may be performed in parts.

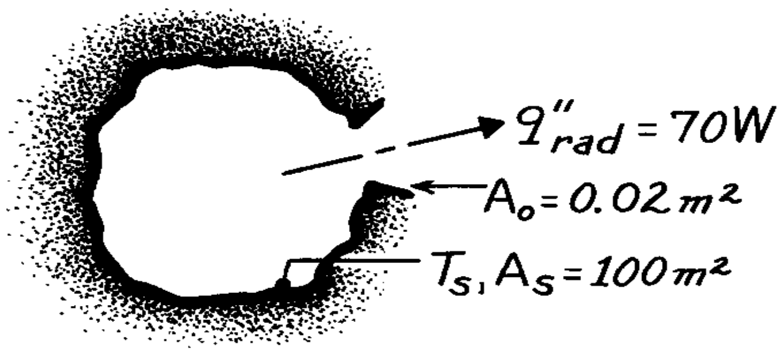
(2) In performing the integration of part (c), recognize the significance of the diffuse emission assumption for which the intensity is uniform in all directions.

PROBLEM 12.16

KNOWN: Isothermal enclosure of surface area, A_s, and small opening, A_o, through which 70W emerges.

FIND: (a) Temperature of the interior enclosure wall if the surface is black, (b) Temperature of the wall surface having $\varepsilon = 0.15$.

SCHEMATIC:



ASSUMPTIONS: (1) Enclosure is isothermal, (2) $A_o << A_s$.

ANALYSIS: A characteristic of an isothermal enclosure, according to Section 12.3, is that the radiant power emerging through a small aperture will correspond to blackbody conditions. Hence

$$q_{rad} = A_o E_b(T_s) = A_o \sigma T_s^4$$

where q_{rad} is the radiant power leaving the enclosure opening. That is,

$$T_s = \left(\frac{q_{rad}}{A_o \sigma}\right)^{1/4} = \left(\frac{70\text{W}}{0.02\text{m}^2 \times 5.670 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4}\right)^{1/4} = 498\text{K}. \qquad <$$

Recognize that the radiated power will be independent of the emissivity of the wall surface. As long as $A_o << A_s$ *and* the enclosure is isothermal, then the radiant power will depend only upon the temperature.

COMMENTS: It is important to recognize the unique characteristics of isothermal enclosures. See Fig. 12.12 to identify them.

PROBLEM 12.20

KNOWN: Various surface temperatures.

FIND: (a) Wavelength corresponding to maximum emission for each surface, (b) Fraction of solar emission in UV, VIS and IR portions of the spectrum.

ASSUMPTIONS: (1) Spectral distribution of emission from each surface is approximately that of a blackbody, (2) The sun emits as a blackbody at 5800 K.

ANALYSIS: (a) From Wien's law, Eq. 12.27, the wavelength of maximum emission for blackbody radiation is

$$l_{max} = \frac{C_3}{T} = \frac{2897.6\ mm \cdot K}{T}.$$

For the prescribed surfaces

Surface	Sun (5800K)	Tungsten (2500K)	Hot metal (1500K)	Skin (305K)	Cool metal (60K)	
$\lambda_{max}(\mu m)$	0.50	1.16	1.93	9.50	48.3	<

(b) From Fig. 12.3, the spectral regions associated with each portion of the spectrum are

Spectrum	**Wavelength limits, mm**
UV	**0.0 – 0.4**
VIS	**0.4 – 0.7**
IR	**0.7 - 100**

For T = 5800K and each of the wavelength limits, from Table 12.1 find:

$\lambda(\mu m)$	10^{-2}	0.4	0.7	10^2
$\lambda T(\mu m \cdot K)$	58	2320	4060	5.8×10^5
$F_{(0 \to \lambda)}$	0	0.125	0.491	1

Hence, the fraction of the solar emission in each portion of the spectrum is:

$F_{UV} = 0.125 - 0 = 0.125$ <

$F_{VIS} = 0.491 - 0.125 = 0.366$ <

$F_{IR} = 1 - 0.491 = 0.509.$ <

COMMENTS: (1) Spectral concentration of surface radiation depends strongly on surface temperature.

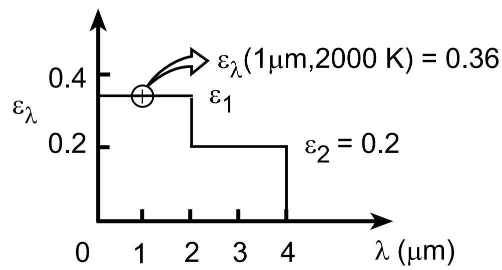
(2) Much of the UV solar radiation is absorbed in the earth's atmosphere.

PROBLEM 12.32

KNOWN: Metallic surface with prescribed spectral, directional emissivity at 2000 K and 1 μm (see Example 12.6) and additional measurements of the spectral, hemispherical emissivity.

FIND: (a) Total hemispherical emissivity, ε, and the emissive power, E, at 2000 K, (b) Effect of temperature on the emissivity.

SCHEMATIC:



ANALYSIS: (a) The total, hemispherical emissivity, ε, may be determined from knowledge of the spectral, hemispherical emissivity, ε_λ, using Eq. 12.38.

$$\varepsilon(\mathrm{T}) = \int_0^\infty \varepsilon_\lambda(\lambda) \mathrm{E}_{\lambda,\mathrm{b}}(\lambda,\mathrm{T})\, d\lambda / \mathrm{E}_\mathrm{b}(\mathrm{T}) = \varepsilon_1 \int_0^{2\mu\mathrm{m}} \frac{\mathrm{E}_{\lambda,\mathrm{b}}(\lambda,\mathrm{T}) d\lambda}{\mathrm{E}_\mathrm{b}(\mathrm{T})} + \varepsilon_2 \int_{2\mu\mathrm{m}}^{4\mu\mathrm{m}} \frac{\mathrm{E}_{\lambda,\mathrm{b}}(\lambda,\mathrm{T}) d\lambda}{\mathrm{E}_\mathrm{b}(\mathrm{T})}$$

or from Eqs. 12.28 and 12.30,

$$\varepsilon(\mathrm{T}) = \varepsilon_1 \mathrm{F}_{(0\to\lambda_1)} + \varepsilon_2 \left[\mathrm{F}_{(0\to\lambda_2)} - \mathrm{F}_{(0\to\lambda_1)} \right]$$

From Table 12.1,

$$\lambda_1 = 2\,\mu\mathrm{m}, \quad \mathrm{T} = 2000\ \mathrm{K}: \quad \lambda_1\mathrm{T} = 4000\,\mu\mathrm{m}\cdot\mathrm{K}, \quad \mathrm{F}_{(0\to\lambda_1)} = 0.481$$

$$\lambda_2 = 4\,\mu\mathrm{m}, \quad \mathrm{T} = 2000\ \mathrm{K}: \quad \lambda_2\mathrm{T} = 8000\,\mu\mathrm{m}\cdot\mathrm{K}, \quad \mathrm{F}_{(0\to\lambda_2)} = 0.856$$

Hence,

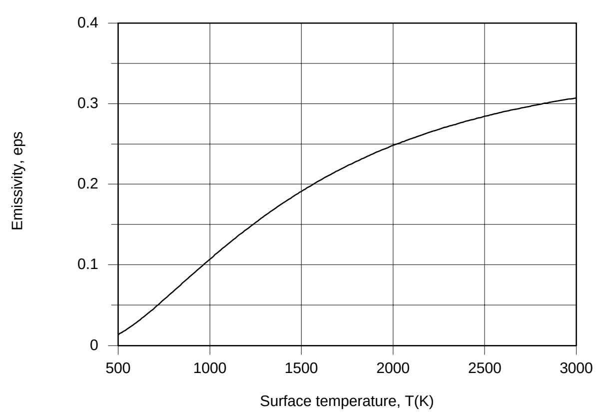
$$\varepsilon(\mathrm{T}) = 0.36 \times 0.481 + 0.20(0.856 - 0.481) = 0.25$$ <

From Eqs. 12.28 and 12.37, the total emissive power at 2000 K is

$$\mathrm{E}(2000\ \mathrm{K}) = \varepsilon\,(2000\ \mathrm{K}) \cdot \mathrm{E}_\mathrm{b}\,(2000\ \mathrm{K})$$

$$\mathrm{E}(2000\ \mathrm{K}) = 0.25 \times 5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4} \times (2000\ \mathrm{K})^4 = 2.27 \times 10^5\ \mathrm{W/m^2}.$$ <

(b) Using the *Radiation* Toolpad of IHT, the following result was generated.



Continued...

PROBLEM 12.32 (Cont.)

At T ≈ 500 K, most of the radiation is emitted in the far infrared region ($\lambda > 4$ μm), in which case $\varepsilon \approx 0$. With increasing T, emission is shifted to lower wavelengths, causing ε to increase. As $T \to \infty$, $\varepsilon \to 0.36$.

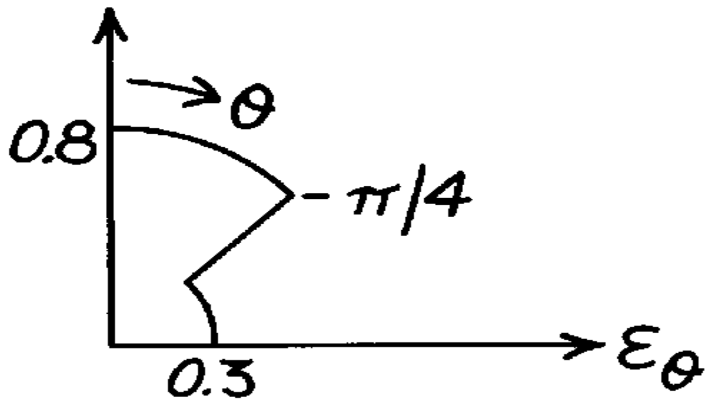
COMMENTS: Note that the value of ε_λ for $0 < \lambda \le 2$ μm cannot be read directly from the ε_λ distribution provided in the problem statement. This value is calculated from knowledge of $\varepsilon_{\lambda,\theta}(\theta)$ in Example 12.6.

PROBLEM 12.35

KNOWN: Directional emissivity, ε_θ, of a selective surface.

FIND: Ratio of the normal emissivity, ε_n, to the hemispherical emissivity, ε.

SCHEMATIC:



ASSUMPTIONS: Surface is isotropic in ϕ direction.

ANALYSIS: From Eq. 12.36 written on a total, rather than spectral, basis, the hemispherical emissivity is

$$e = 2\int_0^{p/2} e_q\,(q)\cos q\,\sin q\,\mathrm{d}\,q.$$

Recognizing that the integral can be expressed in two parts, find

$$e = 2\left[\int_0^{p/4} e(q)\cos q\,\sin q\,\mathrm{d}q + \int_{p/4}^{p/2} e(q)\cos q\,\sin q\,\mathrm{d}q\right]$$

$$e = 2\left[0.8\int_0^{p/4}\cos q\,\sin q\,\mathrm{d}q + 0.3\int_{p/4}^{p/2}\cos q\,\sin q\,\mathrm{d}q\right]$$

$$e = 2\left[0.8\frac{\sin^2 q}{2}\Bigg|_0^{p/4} + 0.3\frac{\sin^2 q}{2}\Bigg|_{p/4}^{p/2}\right]$$

$$e = 2\left[0.8\frac{1}{2}(0.50-0) + 0.3\times\frac{1}{2}(1-0.50)\right] = 0.550.$$

The ratio of the normal emissivity (ε_n) to the hemispherical emissivity is

$$\frac{e_n}{e} = \frac{0.8}{0.550} = 1.45.$$

<

COMMENTS: Note that Eq. 12.36 assumes the directional emissivity is independent of the ϕ coordinate. If this is not the case, then Eq. 12.35 is appropriate.

PROBLEM 12.37

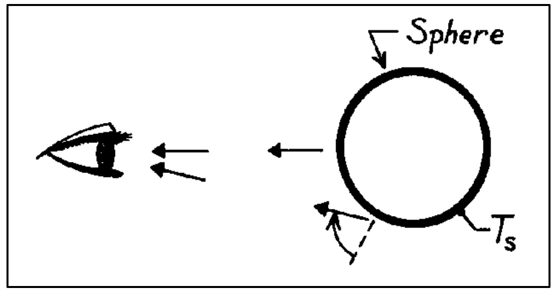
KNOWN: Incandescent sphere suspended in air within a darkened room exhibiting these characteristics:

initially: brighter around the rim
after some time: brighter in the center

FIND: Plausible explanation for these observations.

ASSUMPTIONS: (1) The sphere is at a uniform surface temperature, T_s.

ANALYSIS: Recognize that in observing the sphere by eye, emission from the central region is in a nearly normal direction. Emission from the rim region, however, has a large angle from the normal to the surface.



Note now the directional behavior, ε_θ, for conductors and non-conductors as represented in Fig. 12.17.

Assume that the sphere is fabricated from a *metallic* material. Then, the rim would appear brighter than the central region. This follows since ε_θ is higher at higher angles of emission.

If the metallic sphere oxidizes with time, then the ε_θ characteristics change. Then ε_θ at small angles of θ become larger than at higher angles. This would cause the sphere to appear brighter at the center portion of the sphere.

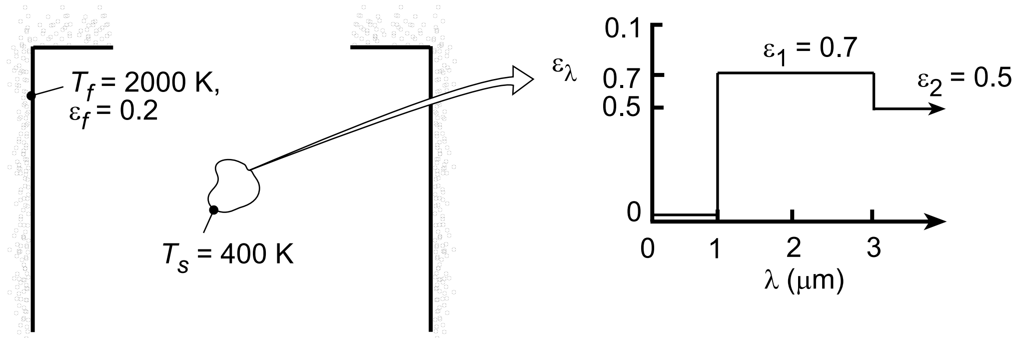
COMMENTS: Since the emissivity of non-conductors is generally larger than for metallic materials, you would also expect the oxidized sphere to appear brighter for the same surface temperature.

PROBLEM 12.44

KNOWN: Temperature and spectral emissivity of small object suspended in large furnace of prescribed temperature and total emissivity.

FIND: (a) Total surface emissivity and absorptivity, (b) Reflected radiative flux and net radiative flux to surface, (c) Spectral emissive power at $\lambda = 2$ μm, (d) Wavelength $\lambda_{1/2}$ for which one-half of total emissive power is in spectral region $\lambda \geq \lambda_{1/2}$.

SCHEMATIC:



ASSUMPTIONS: (1) Surface is opaque and diffuse, (2) Walls of furnace are much larger than object.

ANALYSIS: (a) The emissivity of the object may be obtained from Eq. 12.38, which is expressed as

$$\varepsilon(T_s) = \frac{\int_o^\infty \varepsilon_\lambda(\lambda) E_{\lambda,b}(\lambda, T_s)\, d\lambda}{E_b(T)} = \varepsilon_1\left[F_{(0\to 3\mu m)} - F_{(0\to 1\mu m)}\right] + \varepsilon_2\left[1 - F_{(0\to 3\mu m)}\right]$$

where, with $\lambda_1 T_s = 400$ μm·K and $\lambda_2 T_s = 1200$ μm·K, $F_{(0\to 1\mu m)} = 0$ and $F_{(0\to 3\mu m)} = 0.002$. Hence,

$$\varepsilon(T_s) = 0.7(0.002) + 0.5(0.998) = 0.500$$ <

The absorptivity of the surface is determined by Eq. 12.46,

$$\alpha = \frac{\int_o^\infty \alpha_\lambda(\lambda) G_\lambda(\lambda)\, d\lambda}{\int_o^\infty G_\lambda(\lambda)\, d\lambda} = \frac{\int_o^\infty \alpha_\lambda(\lambda) E_{\lambda,b}(\lambda, T_f)\, d\lambda}{E_b(T_f)}$$

Hence, with $\lambda_1 T_f = 2000$ μm·K and $\lambda_2 T_f = 6000$ μm·K, $F_{(0\to 1\mu m)} = 0.067$ and $F_{(0\to 3\mu m)} = 0.738$. It follows that

$$\alpha = \alpha_1\left[F_{(0\to 3\mu m)} - F_{(0\to 1\mu m)}\right] + \alpha_2\left[1 - F_{(0\to 3\mu m)}\right] = 0.7\times 0.671 + 0.5\times 0.262 = 0.601$$ <

(b) The reflected radiative flux is

$$G_{ref} = \rho G = (1-\alpha) E_b(T_f) = 0.399 \times 5.67\times 10^{-8}\, W/m^2\cdot K^4\, (2000\, K)^4 = 3.620\times 10^5\, W/m^2$$ <

The net radiative flux to the surface is

$$q''_{rad} = G - \rho G - \varepsilon E_b(T_s) = \alpha E_b(T_f) - \varepsilon E_b(T_s)$$

$$q''_{rad} = 5.67\times 10^{-8}\, W/m^2\cdot K^4\left[0.601(2000\, K)^4 - 0.500(400\, K)^4\right] = 5.438\times 10^5\, W/m^2$$ <

(c) At $\lambda = 2$ μm, $\lambda T_s = 800$ K and, from Table 12.1, $I_{\lambda,b}(\lambda,T)/\sigma T^5 = 0.991 \times 10^{-7}$ (μm·K·sr)$^{-1}$. Hence,

Continued...

PROBLEM 12.44 (Cont.)

$$I_{\lambda,b} = 0.991\times10^{-7} \times 5.67\times10^{-8} \frac{\text{W}/\text{m}^2\cdot\text{K}^4}{\mu\text{m}\cdot\text{K}\cdot\text{sr}} \times (400\,\text{K})^5 = 0.0575 \frac{\text{W}}{\text{m}^2\cdot\mu\text{m}\cdot\text{sr}}$$

Hence, with $E_\lambda = \varepsilon_\lambda E_{\lambda,b} = \varepsilon_\lambda \pi I_{\lambda,b}$,

$$E_\lambda = 0.7(\pi\,\text{sr})0.0575\,\text{W}/\text{m}^2\cdot\mu\text{m}\cdot\text{sr} = 0.126\,\text{W}/\text{m}^2\cdot\mu\text{m}$$ <

(d) From Table 12.1, $F_{(0\rightarrow\lambda)} = 0.5$ corresponds to $\lambda T_s \approx 4100\ \mu\text{m}\cdot\text{K}$, in which case,

$$\lambda_{1/2} \approx 4100\,\mu\text{m}\cdot\text{K}/400\,\text{K} \approx 10.3\,\mu\text{m}$$ <

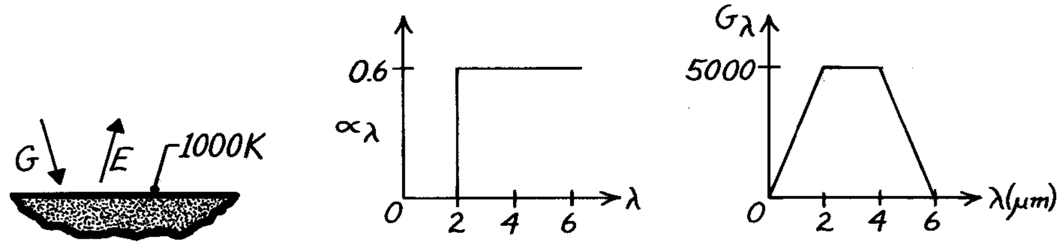
COMMENTS: Because of the significant difference between T_f and T_s, $\alpha \neq \varepsilon$. With increasing $T_s \rightarrow T_f$, ε would increase and approach a value of 0.601.

PROBLEM 12.50

KNOWN: Spectral distribution of the absorptivity and irradiation of a surface at 1000 K.

FIND: (a) Total, hemispherical absorptivity, (b) Total, hemispherical emissivity, (c) Net radiant flux to the surface.

SCHEMATIC:



ASSUMPTIONS: (1) $\alpha_\lambda = \varepsilon_\lambda$.

ANALYSIS: (a) From Eq. 12.46,

$$\alpha = \frac{\int_0^\infty \alpha_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda} = \frac{\int_0^{2\mu m} \alpha_\lambda G_\lambda d\lambda + \int_2^{4\mu m} \alpha_\lambda G_\lambda d\lambda + \int_4^{6\mu m} \alpha_\lambda G_\lambda d\lambda}{\int_0^{2\mu m} G_\lambda d\lambda + \int_2^{4\mu m} G_\lambda d\lambda + \int_4^{6\mu m} G_\lambda d\lambda}$$

$$\alpha = \frac{0\times 1/2(2-0)5000 + 0.6(4-2)5000 + 0.6\times 1/2(6-4)5000}{1/2(2-0)5000 + (4-2)(5000) + 1/2(6-4)5000}$$

$$\alpha = \frac{9000}{20,000} = 0.45.$$ <

(b) From Eq. 12.38,

$$\varepsilon = \frac{\int_0^\infty \varepsilon_\lambda E_{\lambda,b} d\lambda}{E_b} = \frac{0\int_0^{2\mu m} E_{\lambda,b} d\lambda}{E_b} + \frac{0.6\int_2^\infty E_{\lambda,b} d\lambda}{E_b}$$

$$\varepsilon = 0.6 F_{(2\mu m \to \infty)} = 0.6\left[1 - F_{(0\to 2\,\mu m)}\right].$$

From Table 12.1, with $\lambda T = 2\ \mu m \times 1000K = 2000\ \mu m\cdot K$, find $F_{(0 \to 2\ \mu m)} = 0.0667$. Hence,

$$\varepsilon = 0.6[1 - 0.0667] = 0.56.$$ <

(c) The net radiant heat flux *to* the surface is

$$q''_{rad,net} = \alpha G - E = \alpha G - \varepsilon \sigma T^4$$

$$q''_{rad,net} = 0.45\left(20,000 W/m^2\right) - 0.56\times 5.67\times 10^{-8} W/m^2\cdot K^4 \times (1000K)^4$$

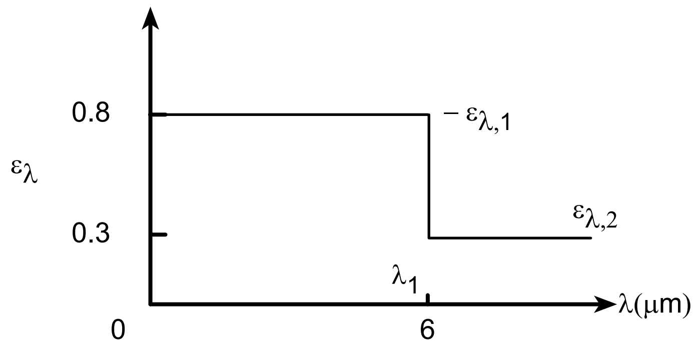
$$q''_{rad,net} = (9000 - 31,751) W/m^2 = -22,751 W/m^2.$$ <

PROBLEM 12.53

KNOWN: Spectral emissivity of an opaque, diffuse surface.

FIND: (a) Total, hemispherical emissivity of the surface when maintained at 1000 K, (b) Total, hemispherical absorptivity when irradiated by large surroundings of emissivity 0.8 and temperature 1500 K, (c) Radiosity when maintained at 1000 K and irradiated as prescribed in part (b), (d) Net radiation flux into surface for conditions of part (c), and (e) Compute and plot each of the parameters of parts (a)-(c) as a function of the surface temperature T_s for the range $750 < T_s \leq 2000$ K.

SCHEMATIC:



ASSUMPTIONS: (1) Surface is opaque, diffuse, and (2) Surroundings are large compared to the surface.

ANALYSIS: (a) When the surface is maintained at 1000 K, the total, hemispherical emissivity is evaluated from Eq. 12.38 written as

$$\varepsilon = \int_0^\infty \varepsilon_\lambda E_{\lambda,b}(T)\,d\lambda / E_b(T) = \varepsilon_{\lambda,1}\int_0^{\lambda_1} E_{\lambda,b}(T)\,d\lambda / E_b(T) + \varepsilon_{\lambda,2}\int_{\lambda_1}^\infty E_{\lambda,b}(T)\,d\lambda / E_b(T)$$

$$\varepsilon = \varepsilon_{\lambda,1} F_{(0-\lambda_1 T)} + \varepsilon_{\lambda,2}\left(1 - F_{(0-\lambda_1 T)}\right)$$

where for $\lambda T = 6\mu m \times 1000\ K = 6000 \mu m \cdot K$, from Table 12.1, find $F_{0-\lambda T} = 0.738$. Hence,

$$\varepsilon = 0.8 \times 0.738 + 0.3(1 - 0.738) = 0.669. \quad <$$

(b) When the surface is irradiated by large surroundings at $T_{sur} = 1500$ K, $G = E_b(T_{sur})$.
From Eq. 12.46,

$$\alpha = \int_0^\infty \alpha_\lambda G_\lambda\,d\lambda \Big/ \int_0^\infty G_\lambda\,d\lambda = \int_0^\infty \varepsilon_\lambda E_{\lambda,b}(T_{sur})\,d\lambda / E_b(T_{sur})$$

$$\alpha = \varepsilon_{\lambda,1} F_{(0-\lambda_1 T_{sur})} + \varepsilon_{\lambda,2}\left(1 - F_{(0-\lambda_1 T_{sur})}\right)$$

where for $\lambda_1 T_{sur} = 6\ \mu m \times 1500\ K = 9000\ \mu m \cdot K$, from Table 12.1, find $F_{(0-\lambda T)} = 0.890$. Hence,

$$\alpha = 0.8 \times 0.890 + 0.3\,(1 - 0.890) = 0.745. \quad <$$

Note that $\alpha_\lambda = \varepsilon_\lambda$ for all conditions and the emissivity of the surroundings is irrelevant.

(c) The radiosity for the surface maintained at 1000 K and irradiated as in part (b) is

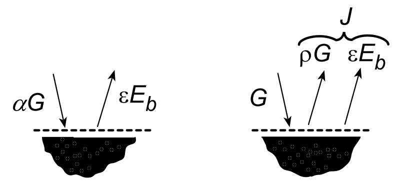
$$J = \varepsilon E_b(T) + \rho G = \varepsilon E_b(T) + (1 - \alpha) E_b(T_{sur})$$

$$J = 0.669 \times 5.67 \times 10^{-8}\ W/m^2 \cdot K^4\,(1000\ K)^4 + (1 - 0.745)\,5.67 \times 10^{-8}\ W/m^2 \cdot K^4\,(1500\ K)^4$$

$$J = (37{,}932 + 73{,}196)\ W/m^2 = 111{,}128\ W/m^2 \quad <$$

Continued...

(d) The net radiation flux into the surface with $G = \sigma T_{sur}^4$ is

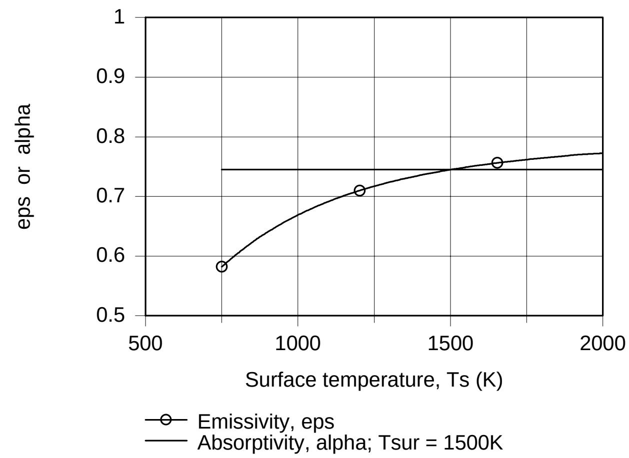


$$q''_{rad,in} = \alpha G - \varepsilon E_b(T) = G - J$$

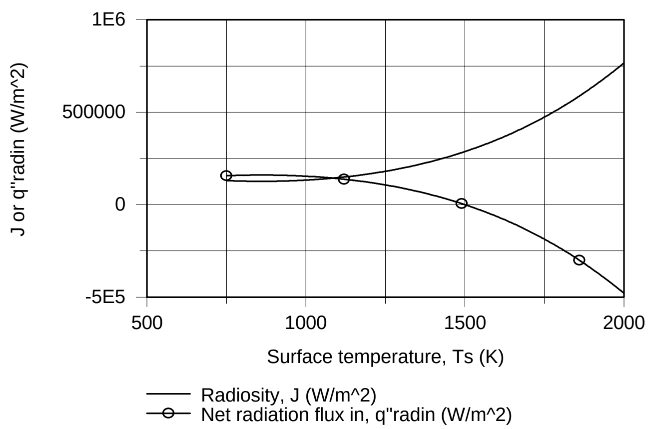
$$q''_{rad,in} = 5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K}\ (1500\ \mathrm{K})^4 - 111{,}128\ \mathrm{W/m^2}$$

$$q''_{rad,in} = 175{,}915\ \mathrm{W/m^2}.$$ <

(e) The foregoing equations were entered into the IHT workspace along with the *IHT Radiaton Tool, Band Emission Factor*, to evaluate $F_{(0-\lambda T)}$ values and the respective parameters for parts (a)-(d) were computed and are plotted below.



Note that the absorptivity, $\alpha = \alpha(\alpha_\lambda, T_{sur})$, remains constant as T_s changes since it is a function of α_λ (or ε_λ) and Tsur only. The emissivity $\varepsilon = \varepsilon(\varepsilon_\lambda, T_s)$ is a function of T_s and increases as T_s increases. Could you have surmised as much by looking at the spectral emissivity distribution? At what condition is $\varepsilon = \alpha$?



The radiosity, J_1 increases with increasing T_s since $E_b(T)$ increases markedly with temperature; the reflected irradiation, $(1 - \alpha)E_b(T_{sur})$ decreases only slightly as T_s increases compared to $E_b(T)$. Since G is independent of T_s, it follows that the variation of $q''_{rad,in}$ will be due to the radiosity change; note the sign difference.

COMMENTS: We didn't use the emissivity of the surroundings ($\varepsilon = 0.8$) to determine the irradiation onto the surface. Why?