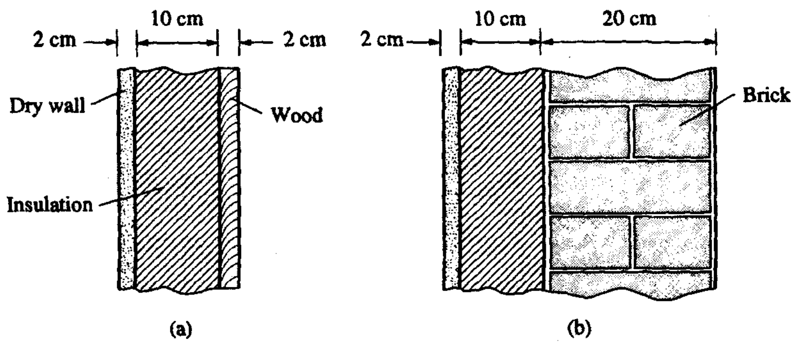
Int. To Heat Transfer (ME 411) Quiz 1 July 5, 2005
Name: (1 Hour)

Problem 1 (10 points): The cross section of the wall of a wooden house is shown here. To save heating costs, the outside wood is to be replaced with a brick wall. Find the reduction in heat loss from the house. Data: Conductivities for dry wall, insulation, wood, and brick are k_d = 0.2, k_i = 0.1, k_w = 0.2, and k_b = 0.7 W/m.K, the inside and outside coefficients of heat transfer are h_i =10, and h_o = 30 W/m^2 .K.



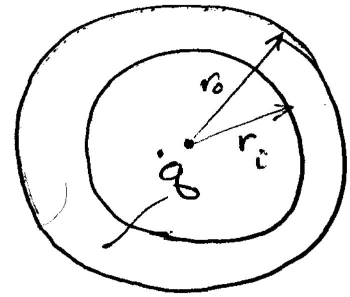
Problem 2 (10 points): Derive an expression for the temperature distribution in a hollow cylinder with heat sources which vary according to the following linear relation,

$\dot{q} = a + b.r$

with $\dot{q}_i$ the generation rate per unit volume at r=r_i. The inside and outside temperatures are T=T_i at r=r_i, and T=T_o at r=r_o.

Problem 2.

$\dot{q} = a + br$, $\dot{q}_i = a + br_i$



$T(r_i) = T_i$, $T(r_o) = T_o$

Temperature distribution?

Starting with heat equation in cylinderical coordinate

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \underbrace{\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)}_{0\ (1D)} + \underbrace{\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)}_{0\ (1D)} + \dot{q} = \underbrace{\rho c_p \frac{\partial T}{\partial t}}_{0\ S.S.}$$

for constant conductivity,

$$\frac{k}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \dot{q} = 0$$

or $$\frac{d}{dr}\left(r\frac{dt}{dr}\right) = -\frac{\dot{q}r}{k} = -\frac{ar + br^2}{k}$$

integrate once $$r\frac{dT}{dr} = -\frac{1}{k}\left(\frac{\dot{q}}{2}r^2 + \frac{b}{3}r^3\right) + C_1$$

or $$\frac{dT}{dr} = -\frac{1}{k}\left(\frac{ar}{2} + \frac{br^2}{3}\right) + \frac{C_1}{r}$$

integrate again $$T = -\frac{1}{k}\left(\frac{a}{2}\frac{r^2}{2} + \frac{b}{3}\frac{r^3}{3}\right) + C_1 \ln r + C_2$$

or $$T = -\frac{1}{k}\left(\frac{a}{4}r^2 + \frac{b}{9}r^3\right) + C_1 \ln(r) + C_2$$

use the BC's , $T(r_i) = T_i$, $T(r_o) = T_o$

it results in

PROBLEM 1

Case (a) Wooden wall. $A_{\infty,i} = A_d = A_i = A_w = A_{\infty,o} = \underline{\underline{A}}$

$$R_{\infty,i} = \frac{1}{h_{\infty,i} A_{\infty,i}} = \frac{1}{10 \times A} = \frac{0.1}{A}$$

$$R_d = \frac{\Delta x_d}{K_d A_d} = \frac{2 \times 10^{-2}\ m}{0.2\ W/mK \cdot A} = \frac{0.1}{A}$$

$$R_i = \frac{\Delta x_i}{K_i A_i} = \frac{10 \times 10^{-2}}{0.1 \times A} = \frac{1}{A}$$

$$R_W = \frac{\Delta x_w}{K_w A_w} = \frac{2 \times 10^{-2}}{0.2\ A} = \frac{0.1}{A}$$

$$R_{\infty,o} = \frac{1}{h_o A_{\infty,o}} = \frac{1}{30\ A} = 0.0333 \frac{1}{A}$$

All in series then

$$R_{tot,a} = \sum R = \frac{1}{A}(0.1 + 0.1 + 1 + 0.1 + 0.0333)$$

$$R_{tot,a} = 1.333/A$$

Case (b) Brick Wall

only R_W has to be replaced with $R_B = \dfrac{\Delta x_B}{K_B A_B} = \dfrac{20 \times 10^{-2}}{0.7 \times A}$

$\rightarrow R_B = 0.2857/A$

$\rightarrow R_{tot,b} = \frac{1}{A}(1.333 - 0.1 + 0.2857) = 1.519/A$

Now $q = \dfrac{\Delta T}{R} = \dfrac{T_o - T_i}{R} \rightarrow \begin{cases} q_a = \dfrac{\Delta T}{R_a} \\ q_b = \dfrac{\Delta T}{R_b} \end{cases} \rightarrow \dfrac{q_a}{q_b} = \dfrac{R_b}{R_a} = \dfrac{\cancel{0.2857}/A}{1.333/A}$

$\rightarrow q_a = 1.139\ q_b$ $\underline{\underline{\text{Ans.}}}$

Problem 2 (cont'd)

$$c_1 = \frac{(T_i - T_o) - \frac{1}{K}\left(\frac{a(r_o^2 - r_i^2)}{4} + \frac{b(r_o^3 - r_i^3)}{9}\right)}{\ln(r_i/r_o)}$$

$$c_2 = T_o + \frac{1}{K}\left(\frac{a r_o^2}{4} + \frac{b r_o^3}{9}\right) + \frac{(T_i - T_o) - \frac{1}{K}\left(\frac{a(r_o^2 - r_i^2)}{4} + \frac{b(r_o^3 - r_i^3)}{9}\right)}{\ln r_o}$$