Modified Time Delay of Arrival for Biomedical and Environmental Applications

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***Abstract*—Irregular heartbeats at the root of atrial fibrillation (AFib) have been observed as reentrant electric rhythms induced by spiral waves. Eliminating the center of the rotor or wave core using treatments such as ablation therapy has been shown to be effective in treating AFib, but its success relies crucially on accurately locating the core. Here, we present a modified time difference measurement based algorithm (mTDOA) to estimate the core. Our mTDOA algorithm, which is a modified version of the time delay of arrival (TDOA) algorithm, can solve the localization problem without requiring knowledge of the time of origin of the reentrant rhythm, or the speed of propagation of electrical signals along the surface of the heart. We validate the effectiveness of mTDOA in FitzHugh–Nagumo type heart simulation models. mTDOA can also be extended to applications such as in forest fires to estimate sources and rates of growth.**

***Index Terms*—Atrial Fibrillation, rotors, TDOA, Forest fire, Localization**

I. Introduction

Atrial fibrillation (AFib) is a potential indicator of stroke [1], [2] and dementia [3]. Atrial fibrillation is an irregular heartbeat that occurs when the atria experience chaotic electrical signals [4]. With a sufficiently large initial stimulus, a wave spreads out from a source location in the cardiac tissue and is the prime cause of the generation and sustenance of AFib [5]. To mitigate AFib, treatment methods target the source location with surgical ablation therapy to burn out the affected cardiac tissue [6]. However, identifying the source is not a simple matter owing to the complex spatial patterns of spiral waves along with the intricate structure of the heart [7].

The pattern of movement of the affected wave is generally modeled as a rotor, with clockwise or anticlockwise rotating patterns [8]–[10]. An example of an evolving AFib signal simulated on the FitzHugh–Nagumo model is shown in Figure 1 [11]. Approaches to estimate the center of the rotor include visual techniques, pattern recognition based on a theoretical model [12], using the recordings from high-density electrode arrays [13]–[15], recording from a basket catheter [16], and by mappings taken from body surface electrodes [17], [18].

With Conventional ablation with or without Focal Impulse & Rotor Modulation (CONFIRM) trial [16], it was established that nearly in all patients with paroxysmal, persistent, and long-standing persistent AFib had rotors and focal sources. Ablation of these rotors increased recovery from AFib by a factor of two and it was shown to last for 3 years [19]. Multi-electrode contact basket catheters in real-time can be used to record AFib [20]. Subsequently, with algorithms incorporating the dynamic response of repolarization and conduction, rotors can be identified in these patients [21], [22].

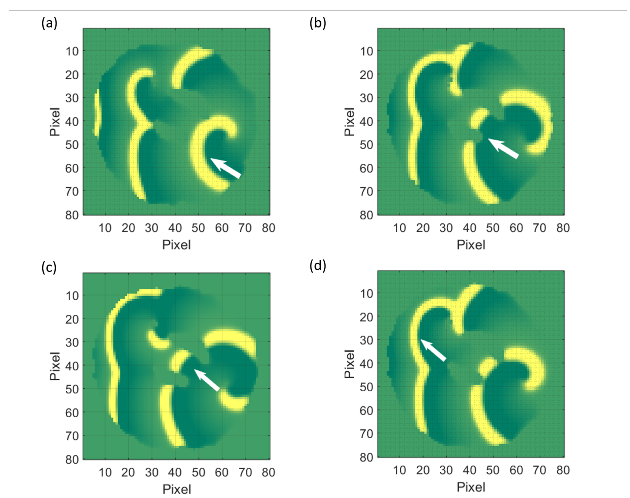

Fig. 1. A cardiac wave is simulated using a modified FitzHugh–Nagumo model [11]. Each sub-figure shows an evolution of the system, with the rotor model being visible in the stripe identified by the white arrow.

Though it is possible to map stationary macro-reentry with a minimal number of electrodes [12], fibrillatory wobbles in animal models [23] and humans [24] show that it is difficult to identify the location of the AFib rotor core as it follows a complex path inside a stable region bounded by limited numbers of electrodes. Hence it is essential to map panoramically to include rotor trajectories as thoroughly as possible.

In [11], triangulation based localization is proposed, which is a variant of time difference of arrival (TDOA). Here, sensors are placed in the path of the spiral waves. The speed of propagation of the waves is assumed to be known. Three probes are placed at known locations along the pathway of the

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wave. The arrival times of the activation waves at the probes are recorded, and using the speed of the propagating wave and the locations of the probes, the center of the spiral wave is estimated as the intersection of two hyperbolas associated with two pairs of probes. Though triangulation has the potential to localize rotor cores, it may result in multiple solutions if the hyperbola associated with pair of probes intersect or no prediction if they do not intersect.

TDOA is one of the most widely used range based passive localization algorithm, where anchors utilize the emitted signal from source to identify its location [25]–[27]. TDOA operates under the assumption that signal is propagating with a known constant speed [28]. The algorithm measures the difference in propagation time at which line of sight (LOS) signals arrive at anchor nodes [29] for source localization. This kind of algorithm is useful in reconnaissance and in target localization. However, in scenarios where we do not know the signal propagation speed of the transmitting device, target localization becomes challenging.

In this paper, we present a modified time difference measurement based algorithm (mTDOA) that jointly estimates the source of pulsating waves, the origin time, and the speed of propagation. Our mTDOA algorithm, which is a modified version of the time delay of arrival (TDOA) algorithm, can solve the localization problem without knowing the time of origin of the reentrant rhythm or the speed of propagation of electric signals along the surface of the heart. We present two solutions for the mTDOA algorithm. We validate the effectiveness of both algorithms using FitzHugh–Nagumo based heart simulation models. Our algorithms can also be extended to other applications such as forest fires to estimate the source locations and rate of fire growth respectively.

The rest of this paper is organized as follows. Section II introduces the problem being solved. The proposed localization algorithms are described in Section III. In Section IV, simulations are shown to validate our algorithms. Finally, concluding remarks are presented in Section V.

II. Problem Formulation

One of the root causes of atrial fibrillation (AFib) is the occurrence of chaotic electrical signals leading to a muscular spasm wave. There have been many studies to capture the dynamics of heart [30], [31] and FitzHugh–Nagumo model has been shown to capture the dynamics of the heart closely [32], [33]. Hence, a modified FitzHugh–Nagumo (FHN) model [34], [35] has been used to model the dynamics of spiral waves to generate pulsating waves moving outwards from a rotor core. The modified FHN is given by (1-2):

$$\frac{\partial v}{\partial t} = \frac{1}{\epsilon}\left(v - \frac{v^3}{3} - w\right) + I_p + D\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{1}$$

$$\frac{\partial w}{\partial t} = \epsilon(v + \beta - \gamma w)\left(\frac{w_h - w_l}{1 + e^{-4v}} + w_l\right) \tag{2}$$

where v and w represent voltage and recovery variables, w_h represents duration of the action potential, w_l affects the duration of the refractory period, I_p represents pacemaker current. The model is solved and simulated with an initial condition at a set of coordinates, which is responsible for the origin of spiral waves. A snapshot from the resulting simulation is shown in Figure 2.

In this paper, we aim to estimate the center of the rotor core. We propose to place sensors on the surface of the heart at known locations. These sensors record the times at which the spiral waves pass them. The recorded timestamps will be used by localization algorithms to estimate the rotor core. Though the timestamps are recorded, there is no information about the origin of signal propagation time, and medium in which the wave propagates. In Section III, mTDOA is proposed to work without the knowledge of the speed of propagation.

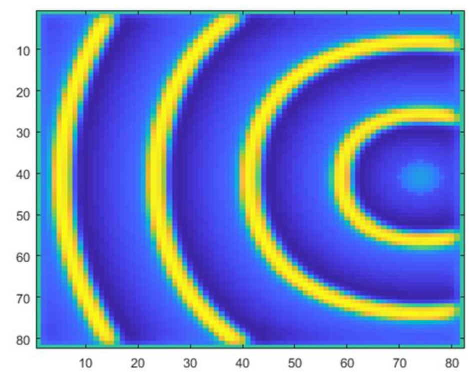

Fig. 2. A snapshot from resulting simulation of modified FHN Model.

III. Localization

Time Difference of Arrival (TDOA) is a popular localization technique. This method measures the time of signal reception at each sensor, without knowing the time of origin, and calculating the difference of arrival time measurements, can be used to estimate the target location. The signal propagation speed is assumed to be known [36], [37]. With N anchors, the TDOA solves for unknown target location and unknown start time by solving (3):

$$\mathbf{x} = \underset{r_0, t_0}{\operatorname{argmin}} \sum_{l=1}^{N} \left[c^2(t_l - t_o)^2 - \|\mathbf{r}_l - \mathbf{r}_0\|^2\right]^2, \tag{3}$$

where $\mathbf{x}$ contains the estimates of the target location r_0, and the unknown start time t_0. The location of the l-th anchor is $\mathbf{r}_l$, and the time at which the signal passes it is t_l.

However, when the signal propagation speed is not known, traditional TDOA falls short. In several biomedical and environmental problems, the speed of propagation is not known. In these cases, the TDOA problem has to be modified in such a way that localization can still be carried out while estimating the speed of propagation as well.

To do this, we design an algorithm that jointly estimates the source of the pulsating waves, the origin time, and the speed of propagation. Sensors are placed at known locations on the surface of the heart. The sensors record the times when the spiral waves pass them. With N anchors placed, our proposed modified TDOA (mTDOA) algorithm jointly estimates the source location, speed of propagation and initial time of the signal by solving the following optimization problem:

$$\mathbf{x} = \underset{r_0, t_0, c}{\operatorname{argmin}} \sum_{l=1}^{N} \left[c^2 (t_l - t_o)^2 - \|\mathbf{r}_l - \mathbf{r}_0\|^2 \right]^2, \tag{4}$$

where $\mathbf{x}$ contains the estimates of the target location r_0, unknown signal propagation speed c, and unknown start time t_0. The location of the l-th anchor is $\mathbf{r}_l$, and the time at which the signal passes it is t_l. We solve (4) using the standard Nelder-Mead simplex direct search [38] via MATLAB.

This newly recast optimization problem can also be solved simply using a least-squares approach (mTDOA-LS). Let $\mathbf{H}$ be the observation matrix given by

$$\mathbf{H} = \begin{bmatrix} 2(\mathbf{r}_2 - \mathbf{r}_1) & -2(t_{21} - t_{10}) & (t_{21}^2 - t_{10}^2) \\ 2(\mathbf{r}_3 - \mathbf{r}_1) & -2(t_{31} - t_{10}) & (t_{31}^2 - t_{10}^2) \\ 2(\mathbf{r}_4 - \mathbf{r}_1) & -2(t_{41} - t_{10}) & (t_{41}^2 - t_{10}^2) \\ . & . & . \\ . & . & . \\ . & . & . \\ 2(\mathbf{r}_N - \mathbf{r}_1) & -2(t_{N1} - t_{10}) & (t_{N1}^2 - t_{10}^2) \end{bmatrix}, \tag{5}$$

where t_{lk} is the estimate of difference of time of arrival measurements between the l-th and k-th sensors. Let $\mathbf{b}$, be the measurement vector given by

$$\mathbf{b} = \frac{1}{2} \begin{bmatrix} \|\mathbf{r}_2\|^2 - \|\mathbf{r}_1\|^2 \\ \|\mathbf{r}_3\|^2 - \|\mathbf{r}_1\|^2 \\ \|\mathbf{r}_4\|^2 - \|\mathbf{r}_1\|^2 \\ . \\ . \\ . \\ \|\mathbf{r}_N\|^2 - \|\mathbf{r}_1\|^2 \end{bmatrix}. \tag{6}$$

The vector of unknowns $\mathbf{x}$, containing the target location r_0, the unknown signal propagation speed c, and the unknown start time t_0, can be calculated as

$$\mathbf{x} = \mathbf{H}^{\#}\mathbf{b}. \tag{7}$$

where $\mathbf{H}^{\#}$ is the pseudo-inverse of $\mathbf{H}$.

IV. Results

The mTDOA algorithm can be applied to solve localization problems with unknown start times, signal propagation speeds, and target locations. It can be used to estimate the spiral wave core for atrial fibrillation cases, and in the case of forest fires, it can be used to solve for the origin of the forest fire as well as the rate of growth.

A. Atrial Fibrillation

The cardiac dynamics of spiral waves can be simulated using a modified FitzHugh–Nagumo (FHN) model [34], [35], to generate pulsating waves moving outwards from a rotor core. In our simulations, the rotor core is fixed in an 80mm×80mm square. Five anchors are placed in randomly selected known locations and the rotor center is estimated. The error between the estimated location of the rotor center core and the actual location is calculated. The process is repeated 1000 times, and the error is averaged. The experiment is then repeated for 6, 7, and 10 anchors.

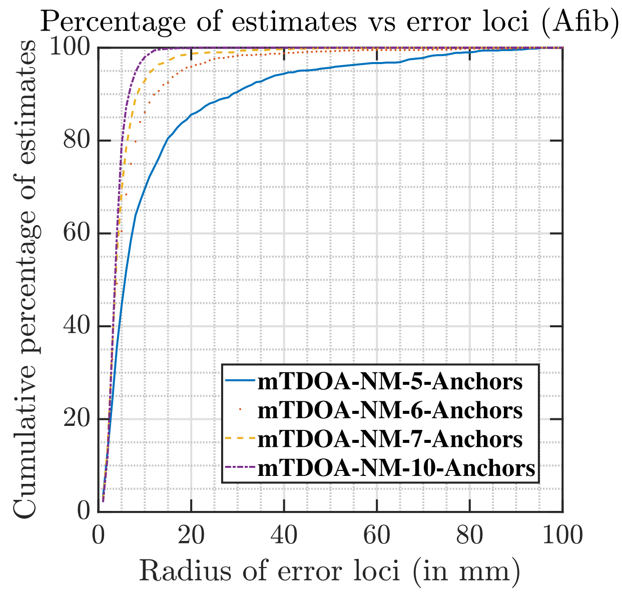

Fig. 3. To estimate the accuracy of the algorithm, the number of estimates within a circle of a given radius are counted. The percentage of estimates for each radius value is plotted. The faster the curve reaches 100%, indicates a more accurate estimator. The algorithm is tested by placing varying number anchors in random locations. The process is repeated one thousand times, and the results are averaged.

The effectiveness of mTDOA-NM is evaluated by counting the number of estimates within a circle of a given radius. The percentage of estimates for each radius value is plotted in Figure 3. It can be seen that for five sensors, 80% of the estimates lie within a radius of 20mm from the rotor center, and the performance improves as the number anchors are increased. With six and seven anchors, almost 90% of the estimates lie within a radius of 15mm. For 10 sensors, 95% estimates lie within a radius of 10mm and is the best performer of those evaluated.

Then the performance of mTDOA-NM and mTDOA-LS is evaluated together. A set of 15 anchors is randomly selected with known locations and the rotor center is estimated. The cumulative percentage of estimates for each case is plotted in Figure 4. It can be observed that for mTDOA LS with 15 anchors, more than 95 % of the estimates lie within a radius of 20 mm from the spiral wave center. However, more than 95 % of the estimates lie within a radius of 7 mm from the spiral wave center for mTDOA-NM. Overall, mTDOA-NM results give the best estimate of the rotor location.

The Monte-Carlo results from mTDOA-NM simulations are plotted in a scatter plot and shown in Figure 5. During all the simulations, the anchors are placed at random unknown times during the simulation of spiral wave. From the results, it is

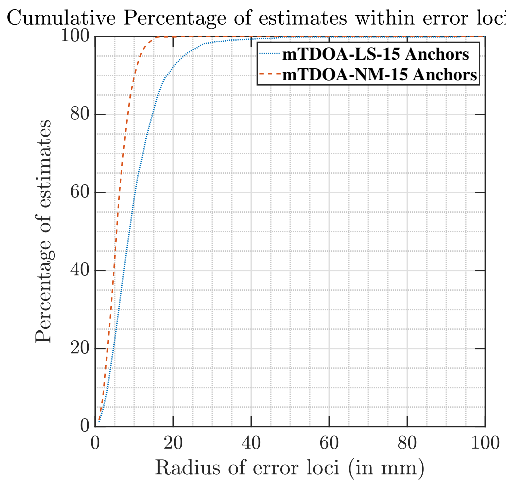

Fig. 4. The results from mTDOA-NM and mTDOA-LS are compared for 15 set of anchors. The experiment is performed 1000 times and the anchors are randomly placed in each simulation. The results show that mTDOA-NM performs better compared to mTDOA-LS

clearly visible that most of the estimates fall in close vicinity of the true AFib core.

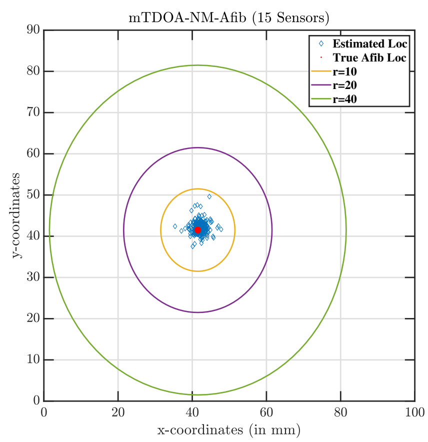

Fig. 5. The rotor center is fixed as shown with the red dot. 15 anchors are placed in randomly selected known locations in a 80mm×80mm square and the rotor center is estimated. The process is repeated one thousand times. All one thousand estimates are plotted as blue diamonds.

B. Environmental Applications

The mTDOA algorithm can be extended to solve for the origin of forest fires and to estimate the rate of growth of the fire. The forest fire model is simulated [39] with a single source of origin and unknown growth rate, in a space of size 300m×300m. Anchors are simulated as thermal sensors in the path of the fire. mTDOA for forest fires is tested with 5, 6, 7, and 10 sensors.

With 5 sensors, 80% of the estimates lie within a radius of 100m from the fire source. With 6 and 7 anchors, performance improves by more than a factor of 2 and 80% percent of estimates now lie within a radius 40m. For 10 sensors, 95%

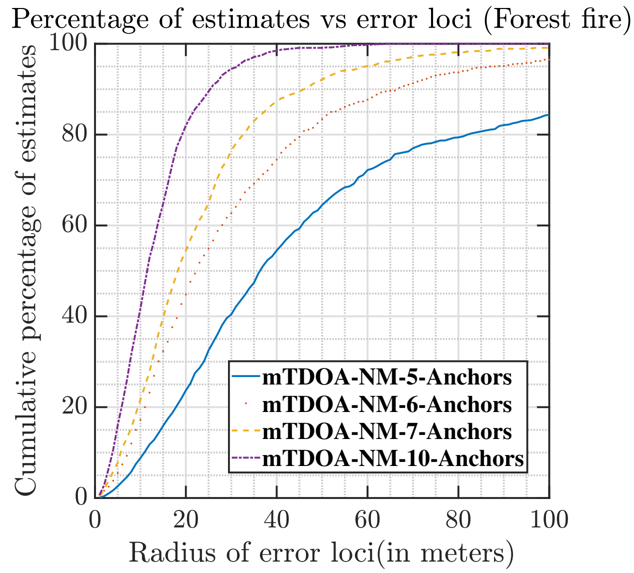

Fig. 6. To estimate the accuracy of the algorithm, the number of estimates within a circle of a given radius are counted. The percentage of estimates for each radius value is plotted. The faster the curve reaches 100%, indicates a more accurate estimator. The algorithm is tested by randomly placing varying number anchors in co-ordinate plane. The process is repeated one thousand times, and the results are averaged.

estimates lie within a radius of 25m. The fire propagation speed measurements and corresponding estimates are shown in Table I.

TABLE I
FOREST FIRE PROPAGATION SPEED COMPARISONS WITH VARYING ANCHORS. THE ACTUAL FIRE PROPAGATION SPEED IS 0.8 M/S. AS THE NUMBER OF ANCHORS INCREASES, THE ESTIMATE GETS CLOSER TO THE ACTUAL VALUE.

Number of anchors	5	6	7	10
Actual speed (in m/s)	0.8	0.8	0.8	0.8
Estimated speed (in m/s)	0.8306	0.8455	0.8034	0.8012

V. CONCLUSIONS

In this paper, we presented a modified time difference of arrival algorithm to jointly estimate the location of a signal source, the speed of propagation of the signal, and the origin time of the signal. Sensors are placed at random known locations and the algorithm is evaluated by varying the number of anchors. The algorithm can be used to estimate the rotor core center in the case of AFib, and to estimate the source and growth rates of forest fires. The problem can be solved using both least square and Nelder-Mead direct search approach. Overall, mTDOA-NM resulted in the best performance.

Future work includes developing algorithms for estimating the centers of spiral waves having non-constant velocities or with obstacles blocking the spiral waves or spiral waves with multiple cores. It also can be extended to other similar problems such as tracking the wave velocity of tsunamis.

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