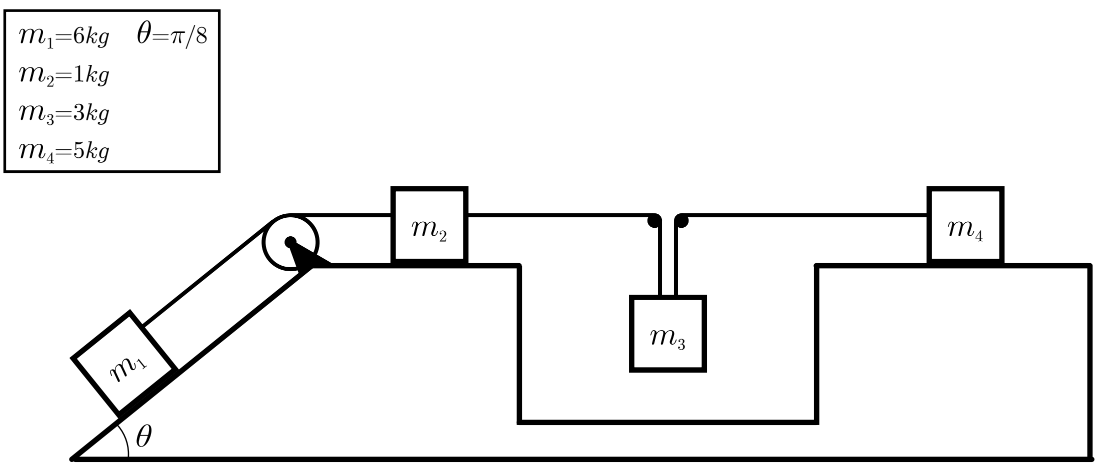
Clarkson University
PH 131 Exam 2
Problem 1

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Name and Section: ______________________________

1. In the following system m_1 accelerates downward in the *negative* direction. Consider all pulleys to be massless and frictionless and consider both cords to be massless.



   (a) (5 points) Draw and label all forces acting on each block.

   (b) (12 points) Find a simplified mathematical model describing the magnitude of the acceleration of the system.

Solution: We first sum the forces acting on each mass.

$$
\begin{aligned}
m_1 &: \sum F_{x_1} = m_1 g \sin(\theta) - T_1 \quad && \sum F_{y_1} = N_1 - m_1 g \cos(\theta) = 0 \\
m_2 &: \sum F_{x_2} = T_1 - T_2 && \sum F_{y_2} = N_2 - m_2 g = 0 \\
m_3 &: \sum F_{x_3} = T_2 + T_3 - m_3 g && \sum F_{y_3} = 0 \\
m_4 &: \sum F_{x_4} = -T_3 && \sum F_{y_4} = N_4 - m_4 g = 0
\end{aligned}
$$

We now sum equation $\sum F_{x_i}$, giving:

$$a_x \sum m_i = m_1 g \sin(\theta) - m_3 g$$

Therefore a_x:

$$a_x = \frac{m_1 \sin(\theta) - m_3}{\sum m_i} g$$

(c) (1 point) Find the magnitude of the tension in each cord for the values given.

Solution: Using the work of part (b), and $a_x \approx -.4598 \frac{m}{s^2}$,

$$\begin{aligned} T_1: &\quad T_1 = m_1(g \sin(\theta) - a_x) \approx 25.26N \\ T_2: &\quad T_2 = m_1(g \sin(\theta) - a_x) - m_2 a_x \approx 25.72N \\ T_3: &\quad T_3 = -m_4 a_x \approx 2.299N \end{aligned}$$

(d) (2 points) Find how long it takes m_3 to reach the bottom the pit, given the depth, d, of the pit is 3m, and the system starts from rest .

Solution: Using the result of part (b), we see that a_x is constant so we can use,

$$\begin{aligned} x_f &= x_i - \frac{1}{2} a_x t^2 \\ -x_i &= -\frac{1}{2} a_x t^2 \\ t &= \pm \sqrt{\frac{2\Delta x}{a_x}} \end{aligned}$$

Since we start at $t = 0$ we want the positive solution,

$$\begin{aligned} t &= \sqrt{\frac{2\Delta x}{a_x}} \\ t &\approx 3.612s \end{aligned}$$

(e) (3 points (bonus)) Find a value for, θ, to keep the system from accelerating when released.

Solution: Using the result of part b, for a static system $a_x = 0$

$$\begin{aligned} a_x &= \frac{m_1 \sin(\theta) - m_3}{\sum m_i} g = 0 \\ m_3 &= m_1 \sin(\theta) \\ \theta &= \sin^{-1}\left(\frac{m_3}{m_1}\right) \\ \theta &= \frac{\pi}{6} \end{aligned}$$