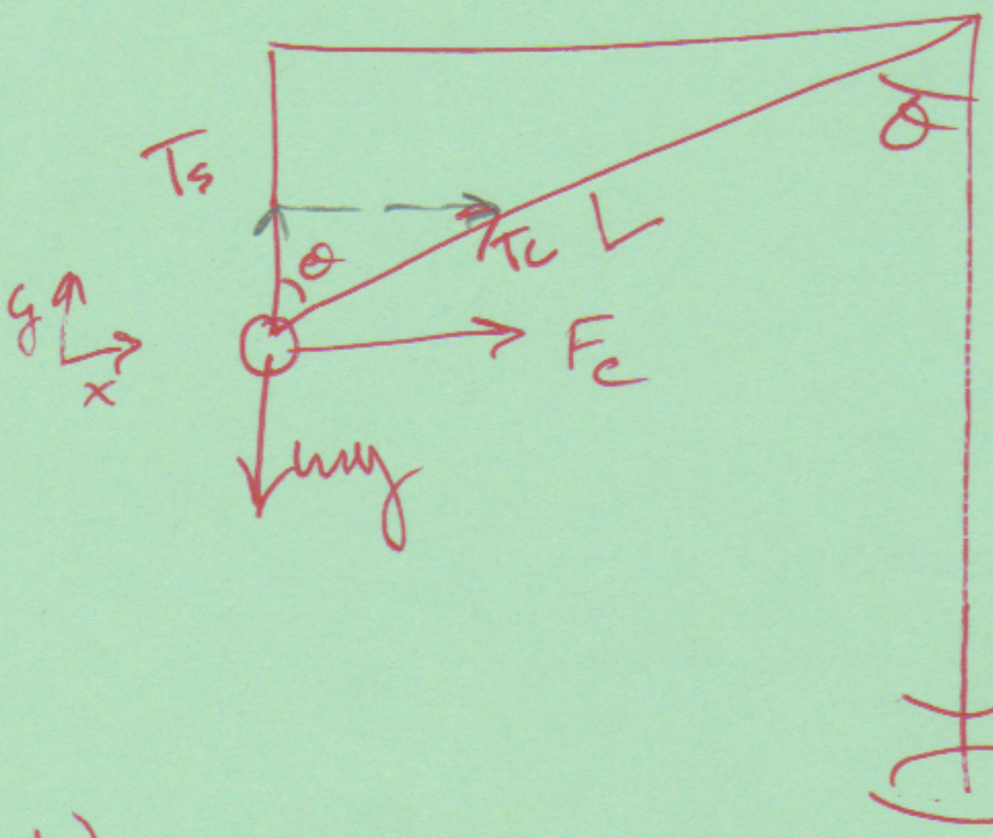
Problem 2 key



b) $\Sigma F_x = T_L \sin\theta = \frac{mv^2}{r}$

$\Sigma F_y = T_s + T_L \cos\theta - mg = 0$ (not moving up & down)

told $T_s = \frac{T_L}{2}$ so,

$\Sigma F_y = \frac{T_L}{2}\left(\frac{1}{2} + \cos\theta\right) = mg$

We want v interms of L, g, θ so,

$T_L = \frac{mg}{\frac{1}{2} + \cos\theta}$ Plug into $\longrightarrow \Sigma F_x, \Sigma\theta$

$T_L \sin\theta = \frac{mg\sin\theta}{\frac{1}{2} + \cos\theta} = \frac{mv^2}{r}$

$$\frac{g\sin\theta}{\frac{1}{2}+\cos\theta} = \frac{v^2}{r} \quad , r = L\sin\theta$$

$$\frac{g\sin\theta}{\frac{1}{2}+\cos\theta} = \frac{v^2}{L\sin\theta} \Rightarrow v^2 = \frac{Lg\sin^2\theta}{\frac{1}{2}+\cos\theta}$$

$$\Rightarrow v = \sqrt{\frac{Lg\sin^2\theta}{\frac{1}{2}+\cos\theta}} \approx 3.8\,\frac{m}{s}$$

c)

$$\Sigma F_x = T_L\sin\theta = m\frac{v^2}{r}$$

$$\Sigma F_y = \cancel{T_s} + T_L\cos\theta - mg = 0$$

T_s for T_s just disappears.

$\Sigma\theta$,

$$T_L = \frac{mg}{\cos\theta} \quad / \text{Plug into } \Sigma F_x$$

$$T_L\sin\theta = \frac{mg\sin\theta}{\cos\theta} = m\frac{v^2}{r} \quad , r = L\sin\theta$$

$$Lg\tan\theta\sin\theta = v^2$$

$$\Rightarrow v = \sqrt{gL\tan\theta\sin\theta}$$

$$\approx 5.4\,m/s$$

Bonus:

T_s is max if $T_L = 0$

$$\Sigma F_y = T_s + T_L \cos\theta - mg = 0$$

$$T_s = mg - \cancel{T_L}^{\to 0} \cos\theta$$

$$T_s = mg$$

so,

$$\Sigma F_x = T_L \sin\theta = m\frac{v^2}{r}$$

$T_L = 0$ so

$$0 = m\frac{v^2}{r} \Longrightarrow v = 0$$