Lift Force

Small particles in a shear field as shown in Figure 2 experience a lift force perpendicular to the direction of flow. The shear lift originates from the inertia effects in the viscous flow around the particle and is fundamentally different from aerodynamic lift force. The expression for the inertia shear lift was first obtained by Saffman (1965, 1968). That is

$$F_{L(Saff)} = 1.615 \rho v^{1/2} d^2 (u^f - u^p)|\frac{du^f}{dy}|^{1/2} \text{sgn}\left(\frac{du^f}{dy}\right)$$

Here $u^f$ is the fluid velocity at the location of mass center of the particle, $u^p$ is the particle velocity, $\dot{\gamma} = \frac{du^f}{dt}$ is the shear rate, $d$ is the particle diameter and $\rho$ and $\nu$ are the fluid density and viscosity. Note that $F_L$ is in the positive $y$-direction if $u^f > u^p$.

Equation (1) is subjected to the following constraints:

$$R_{es} = \frac{|u^f - u^p| d}{\nu} << 1$$
$$R_{c\Omega} = \frac{\Omega d^2}{\nu} << 1$$
$$R_{cG} = \frac{\dot{\gamma} d^2}{\nu} << 1$$
$$\varepsilon = \frac{R_{cG}^{1/2}}{R_{es}} \gg 1$$

Here $\Omega$ is the rotational speed of the sphere. Dandy & Dwyer (1990) found that the Saffman lift force is approximately valid at larger $R_{es}$ and small $\varepsilon$. McLaughlin (1991) showed that the lift force decreases as $\varepsilon$ decreases. Based on these studies Mei (1992) suggested the following empirical fit to the results of Dandy and Dwyer and McLaughlin. For large $\varepsilon$ and $R_{es}$,
\[
\frac{F_{L}}{F_{L(Saff)}} = \begin{cases} 
(1 - 0.3314\alpha^{1/2})\exp(-R_{es}/10) + 0.3314\alpha^{1/2} & \text{for } R_{es} \leq 40 \\
0.0524(\alpha R_{es})^{1/2} & \text{for } R_{es} > 40
\end{cases}
\]

where
\[
\alpha = \frac{\dot{\gamma}d}{2|u^f - u^p|} = R_{es}^{2/3} = \frac{R_{ef}}{2R_{es}}
\]

For \(0.1 \leq \epsilon \leq 20\)

\[
\frac{F_{L}}{F_{L(Saff)}} = 0.3\{1 + \tanh[2.5\log_{10}(\epsilon + 0.191)]\} \{0.667 + \tan[6(\epsilon - 0.32)]\}
\]

For large and small \(\epsilon\) McLaughlin obtained the following expressions

\[
\frac{F_{L}}{F_{L(Saff)}} = \begin{cases} 
1 - 0.287\epsilon^{-2} & \text{for } \epsilon >> 1 \\
-140\epsilon^5 \ln(\epsilon^{-2}) & \text{for } \epsilon << 1
\end{cases}
\]

Note the change in sign of the lift force for small values of \(\epsilon\).

McLaughlin (1993) included the effects of presence of the wall in his analysis of the lift force. The results for particles in a shear field but not too close to the wall were given in tabulated forms. Cherukat and McLaughlin (1994) analyzed the lift force acting on spherical particles near a wall as shown in Figure 2. Accordingly

\[
F_{L(C-L)} = \frac{\rho V^2 d^3 I_L}{4}
\]

where
\[
V = u^p - u^f = u^p - \dot{\gamma}l
\]

and for non-rotating spheres,

\[
I_L = (1.7716 + 0.216K - 0.7292K^2 + 0.4854K^3) \\
- (3.2397/K + 1.145 + 2.084K - 0.9059K^2) \chi_G \\
+ (2.0069 + 1.0575 - 2.4007K^2 + 1.3174K^3) \chi_G^2
\]
For rotating (freely) spheres,

\[
I_L = (1.7631s + 0.3561K - 1.1837K^2 + 0.845163K^3) \\
- (3.24139/K + 2.6760 + 0.8248K - 0.4616K^2) \wedge_G \\
+ (1.8081 + 0.879585K - 1.9009K^2 + 0.98149K^3) \wedge_G^2
\]  

(8)

Here

\[
K = \frac{d}{2l}, \quad \wedge_G = \frac{\dot{\gamma}d}{2v}.
\]  

(9)

**Lift Force on a Particle Touching a Plane**

Leighton and Acrivos (1985) obtain the expression for the lift on the spherical particles resting on a plane substrate as shown in Figure 4. They found

\[
F_{L,(L-A)} = 0.576p d^{1/4} \dot{\gamma}^2
\]  

(10)

which is always point away from the wall. Note that the Saffman expression given by (1) may be restated as

\[
F_{L,(Saff)} = 0.807p v^{1/2} d^{3} \dot{\gamma}^{3/2}
\]  

(11)
Equation (6) with $I_L$ given by (7) reduces to (10) for $K = 1$, $^\Lambda_G = -1$.

For small particles in turbulent flows, using

$$u^+ = y^+ , \quad u^+ = \frac{u}{u^*} , \quad y^+ = \frac{yu^*}{v} , \quad \gamma = \frac{u'^2}{v}$$

(12)

where $u^*$ is the shear velocity, equations (10) and (11) become

$$F_{L(L-A)}^+ = 0.576d^{+4}$$

(13)

$$F_{L(Saff)}^+ = 0.807d^{+3}$$

(14)

where

$$F_L^+ = \frac{F_L}{\rho v^2} , \quad d^+ = \frac{du^*}{v}$$

(15)

Experimental studies of lift force were performed for generally larger particles in the range of 100 to several hundred µm. Hall (1988) found

$$F_{L(Hall)}^+ = 4.21d^{+2.31} \quad \text{for} \quad d^+ > 1.5$$

(16)

Mollinger and Nieuwstadt (1996) found

$$F_{L(MN)}^+ = 15.57d^{+1.87} \quad \text{for} \quad 0.15 < d^+ < 1$$

(17)

Figure 4 compares the model predictions with the experimental data of Hall. It is seen that the experimental data is generally much higher than the theoretical models.
Figure 4. Comparison of model predictions with the experimental data.