Aerosols Charging and Their Kinetics

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Aerosols Charging

Outline

- Electrostatics
- Particle Charging
- Charged Particle Kinetic

Aerosols Charging and Their Kinetics

Most aerosol particles carry some electrical charges

Coulomb Force → $\mathbf{F}_E = q\mathbf{E}$

$q = ne$

Electric Charge

$e = 1.6\times10^{-19}$ coul

$e = 4.8\times10^{-10}$ statcoul

Particle Mobility: $qE = 3\pi\mu Ud/C_c$ → $u = Z^p = \frac{qC_c}{3\pi\mu d}$

Particle Charging

Boltzmann Equilibrium Charge Distribution →

$$f(n) = \frac{\exp\{-n^2e^2/dkT\}}{\sum_{n=-\infty}^{\infty}\exp\{-n^2e^2/dkT\}}$$

$$f(n) = \sqrt{\frac{e^2}{dkT\pi}}\exp\{-\frac{n^2e^2}{dkT}\}$$ $d > 0.02\mu$

$$f(n) = \frac{0.24}{\sqrt{d\pi}}\exp\{-\frac{0.05n^2}{d}\}$$ $d > 0.02\mu$

Particle Charging

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Average Number of Charge

$$\bar{n} = \sum_{-\infty}^{\infty} |n| f(n) \approx \int_{-\infty}^{\infty} |n| f(n) dn \approx \sqrt{\frac{dkT}{\pi e^2}}$$

$d > 0.02\mu$

$\bar{n} = \approx 2.36\sqrt{d}, \quad d(\mu m)$

Point Charge → $E = \frac{\gamma q}{4\pi r^2}$ $\quad \gamma = 4\pi/\varepsilon$ cgs

Air → $\varepsilon = 1$ $\quad \gamma = 4\pi$ $\quad \gamma = \frac{1}{\varepsilon_o \varepsilon}$ MKS

Coulomb's Law

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$$F = q'E = \frac{\gamma q' q}{4\pi r^2}$$

$$\gamma = \frac{1}{\varepsilon_o \varepsilon}$$

Permittivity → $\varepsilon_o = 8.859 \times 10^{-12} \frac{\text{amp} - \text{sec}}{\text{volt} - \text{meter}}$

Coulomb's Law → $F = \frac{q'q}{\varepsilon r^2}(9 \times 10^9)$

Field Charging

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$$n = [\frac{\pi e Z_i n_{i\infty} t}{\pi e Z_i n_{i\infty} t + 1}](1 + \frac{2(\varepsilon_p - 1)}{\varepsilon_p + 2})\frac{Ed^2}{4e}$$ cgs

$$n_\infty = [1 + \frac{2(\varepsilon_p - 1)}{\varepsilon_p + 2}]\frac{Ed^2}{4e} \text{ as } t \to \infty$$ cgs

$\varepsilon_p = 4.3$ for Quartz

Diffusion Charging

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$$n = \frac{dkT}{2e^2} \ln[1 + (\frac{2\pi}{m_i kT})^{1/2} n_{i\infty} d e^2 t]$$

$n_{i\infty} t \simeq 10^8$ ion sec/ cm^3

Field and Diffusion Charging

Diameter	Number of Units of Charge		
μm	Diffusion	Field	Combined
0.01	0.276	0.0007	0.277
0.02	0.672	0.0027	0.675
0.03	1	0.0062	1.12
0.05	2.1	0.02	2.12
0.1	5	0.07	5.1
0.5	32	2	34
1	69	7	76
2	149	28	177
3	234	63	297
5	414	174	588
10	889	694	1583
20	1901	2778	4679
50	5162	17361	22523
100	10954	69583	80537
200	23121	277778	300899
300	35767	625000	660767

Particle Drift in an Electric Field

Equation of Motion

$$m\frac{d\mathbf{u}^p}{dt} = \mathbf{F}_D + \mathbf{F}_G + \mathbf{F}_E$$

$$\tau\frac{d\mathbf{u}^p}{dt} = \mathbf{u}^f - \mathbf{u}^p + \tau\mathbf{g} - Eq\frac{\tau}{m}$$

$$\tau\frac{d\mathbf{u}^p}{dt} + \mathbf{u}^p = \mathbf{u}_o - Eq\frac{\tau}{m}$$

$$\mathbf{u}_o = \mathbf{u}^f + \tau\mathbf{g}$$

Particle Drift in an Electric Field

For $\mathbf{u}_o \parallel \mathbf{E}$

$$\tau\frac{d\hat{u}^p}{dt} + \hat{u}^p = 1 - \Gamma$$

$$\hat{u}^p = \frac{u^p}{u_0}$$

$$\Gamma = \frac{Eq\tau}{mu_0}$$

For $|\Gamma| >> 1$, neglecting inertia

$$\hat{U}^p = -\Gamma$$

$$u^p = -Eq\frac{\tau}{m}$$

Electrical Forces

Coulomb Force — Image Force — Polarization Force

$$F_e = qE - \frac{q^2}{16\pi\epsilon_0 y^2} + \frac{qEd^3}{16y^3} - \frac{3}{128}\frac{\pi\epsilon_0 d^6 E^2}{y^4},$$

Dipole Interactions