

Particle Transport,
Deposition and Removal

Clarkson University

INTRODUCTION TO AEROSOLS

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Outline

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- Introduction to Aerosols
- Drag Forces
- Cunningham Corrections
- Lift Forces
- Brownian Motion
- Particle Deposition Mechanisms
- Gravitational Sedimentation
- Aerosol Coagulation

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Definition

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- Aerosols are suspension of solid or liquid particles in a gas.
- Dust, smoke, mists, fog, haze, and smog are common aerosols.
- Aerosol particles are found in different shapes.



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Equivalent Diameters

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- Equivalent area diameters
- Feret's diameter (maximum distance edge to edge)
- Stokes' diameter (diameter of a sphere with the same density and the same velocity as the particle)
- Aerodynamic diameter (diameter of a sphere with the density of water and the same velocity as the particle)

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Aerosols in the Atmosphere Clarkson University

	Aerosols	Air
Number Density (Number/cm)	100-10 ⁵	10 ¹⁹
Mean Temperature (K)	240 – 310	240 – 310
Mean Free Path	Greater than 1 m	0.06 μm
Particle Radius	0.01 – 10 μm	2ϕ 10 ⁻⁴ μm
Particle Mass (g)	10 ⁻¹⁸ - 10 ⁻⁹	4.6 ϕ 10 ⁻²³
Particle Charge (Elementary Charge Units)	0 – 100	Weakly Ionized Single Charge

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Dimensionless Groups Clarkson University

Knudsen Number	$Kn = \frac{2\lambda}{d}$
Mach Number	$M = \frac{ v^p - v^f }{c^f}$
Schmidt Number	$Sc = \frac{v}{D} = \frac{n^f \lambda d^2}{4}$
Brown Number	$Br = \left(\frac{v^{p,2}}{v^{f,2}}\right)^{1/2} = \frac{ v^p }{ v^f }$
Reynolds Number	$Re = \frac{ v^p - v^f d}{\nu} = \frac{4M}{Kn}$

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Relevant Parameters Clarkson University

λ = Mean Free Path	ν = Kinematic Viscosity
d = Particle Diameter	D = Diffusivity
v = Particle Velocity	v' = Thermal Velocity
v = Fluid (Air) Velocity	n = Number Density
c = Speed of Sound	

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Mean Free Path Clarkson University

$$\lambda = \frac{1}{\sqrt{2}\pi n d_m^2} = \frac{kT}{\sqrt{2}\pi d_m^2 P}$$

$k = 1.38 \times 10^{-23} \text{ J/K}$

Molecular Diameter

Air → $\lambda(\mu\text{m}) = \frac{23.1T}{P}$

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Aerosols Characteristics Clarkson University

		Particle Diameter, μm									
		10^{-4}	10^{-3}	10^{-2}	10^{-1}	10^0	10^1	10^2	10^3	10^4	
Electro. Wave		← X-Ray →		← UV →	← Vis →	← Infrared →		← Microwaves →			
Definition	Solid Liquid	← Fume →			← Mist →	← Dust →		← Spray →			
Soil		← Clay →			← Silt →	← Sand →	← Gravel →				
Atmospheric		← Smog →			← Cloud/Fog →		← Mist →	← Rain →			
Typical Particles		← Viruses →		← Smoke →	← Bacteria →	← Coal Dust →	← Hair →	← Beach Sand →			
Size Analysis methods		← Electron Microscopy →					← Sieving →				
		← Ultra Centrifuge →			← Sedimentation →						

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Aerosols Characteristics Clarkson University

		Particle Diameter, μm									
		10^{-4}	10^{-3}	10^{-2}	10^{-1}	10^0	10^1	10^2	10^3	10^4	
Gas Cleaning Method		← Ultrasonic →			← Centrifuge →		← Settling Chamber →				
		← HE Air Filter →			← Air Filter →		← Impact Separator →				
		← Thermal Separator →									
		← Electrostatic Separator →									
Diffusion Coeff. cm^2/s	Air		5×10^{-2}		10^{-5}		2×10^{-9}		2×10^{-11}		
	Water		5×10^{-6}		5×10^{-8}		5×10^{-10}		5×10^{-12}		
Terminal Velocity cm/s $S=2$	Air		10^{-6}		2×10^{-4}		0.6		600		
	Water		10^{-10}		6×10^{-7}		6×10^{-3}		12		

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Hydrodynamic Forces Clarkson University

Drag Forces

Stokes $\Rightarrow F = 3\pi\mu Ud$

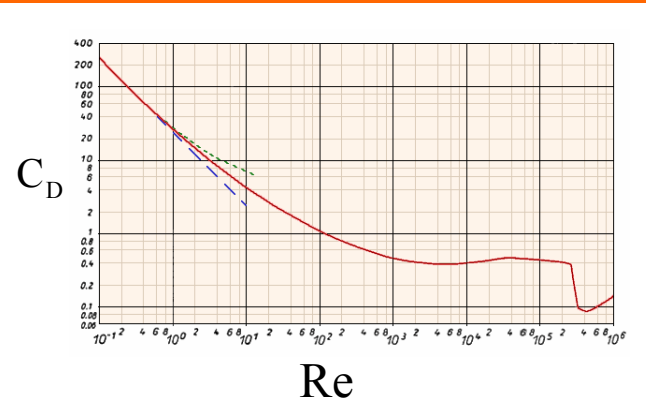
Drag Coefficient $\Rightarrow C_D = \frac{F_D}{\frac{1}{2}\rho U^2 A} = \frac{24}{Re}$

Reynolds Number $\Rightarrow Re = \frac{\rho Ud}{\mu}$

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Drag Force for a Sphere Clarkson University



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Hydrodynamic Forces Clarkson University

Drag Forces

Oseen $\Rightarrow C_D = \frac{24[1 + 3\text{Re}/16]}{\text{Re}}$

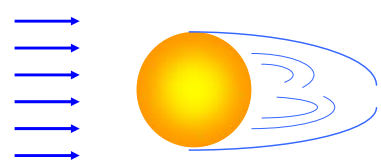
$1 < \text{Re} < 1000$ $\Rightarrow C_D = \frac{24[1 + 0.15\text{Re}^{0.687}]}{\text{Re}}$

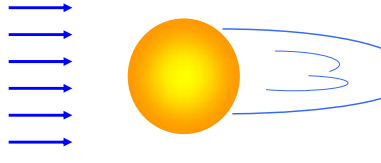
Newton

$10^3 < \text{Re} < 2.5 \times 10^5$ $\Rightarrow C_D = 0.4$

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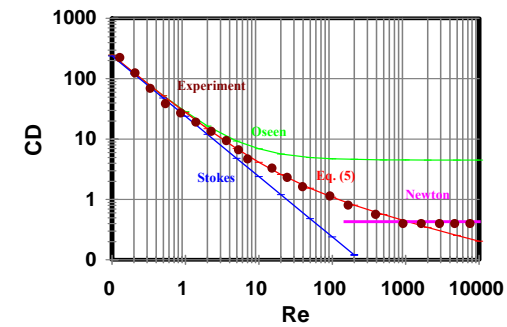
Wake Behind a Sphere Clarkson University

Laminar Boundary Layer 

Turbulent Boundary Layer 

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Drag Force for a Sphere Clarkson University

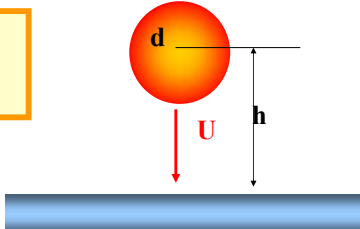


Predictions of various models for drag coefficient for a spherical particle.

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Wall Effects Clarkson University

Normal to the Wall (Bernner, 1961)

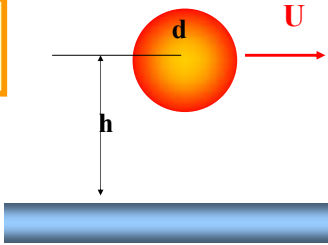


$$C_D = \frac{24}{\text{Re}} \left(1 + \frac{d}{2h}\right)$$

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Wall Effects Clarkson University

**Normal to the Wall
(Faxon, 1923)**



$$C_D = \frac{24}{Re} \left[1 - \frac{9}{16} \left(\frac{d}{2h} \right) + \frac{1}{8} \left(\frac{d}{2h} \right)^3 - \frac{45}{256} \left(\frac{d}{2h} \right)^4 + \frac{1}{16} \left(\frac{d}{2h} \right)^5 \right]^{-1}$$

Cunningham Correction Clarkson University

For $1000 > Kn > 0$

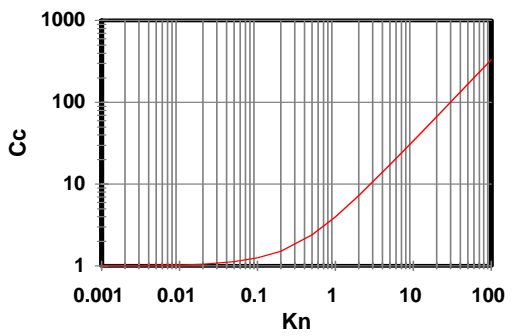
Stokes-Cunningham Drag

$$F_D = \frac{3\pi\mu U d}{C_c}$$

Cunningham Correction

$$C_c = 1 + \frac{2\lambda}{d} [1.257 + 0.4e^{-1.1d/2\lambda}]$$

Cunningham Correction Clarkson University



Variation of Cunningham correction with Knudsen number.

Cunningham Correction Clarkson University

Variations of C_c with d for $\lambda = 0.07 \mu\text{m}$

Diameter, μm	C_c
10 μm	1.018
1 μm	1.176
0.1 μm	3.015
0.01 μm	23.775
0.001 μm	232.54

Compressibility Effects Clarkson University

Henderson (1976)

M < 1

$$C_D = 24 \left[\text{Re} + S \left\{ 4.33 + 1.567 \times \exp\left(-0.247 \frac{\text{Re}}{S}\right) \right\} \right]^{-1} + \exp\left(-\frac{0.5M}{\sqrt{\text{Re}}}\right) \left[\frac{4.5 + 0.38(0.03\text{Re} + 0.48\sqrt{\text{Re}})}{1 + 0.03\text{Re} + 0.48\sqrt{\text{Re}}} + 0.1M^2 + 0.2M^8 \right] + \left[1 - \exp\left(-\frac{M}{\text{Re}}\right) \right] 0.6S$$

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Compressibility Effects Clarkson University

Henderson (1976)

M > 1

$$C_D = \frac{0.9 + \frac{0.34}{M^2} + 1.86 \left(\frac{M}{\text{Re}}\right)^{1/2} \left[2 + \frac{2}{S^2} + \frac{1.058}{S} - \frac{1}{S^4} \right]}{1 + 1.86 \left(\frac{M}{\text{Re}}\right)^{1/2}}$$

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Compressibility Effects Clarkson University

Carlson and Hoglund (1964)

$$C_D = \frac{24}{\text{Re}} \frac{1 + \exp\left(-\frac{0.427}{M^{4.63}} - \frac{3}{\text{Re}^{0.88}}\right)}{1 + \frac{M}{\text{Re}} \left\{ 3.82 + 1.28 \exp\left(-1.25 \frac{\text{Re}}{M}\right) \right\}}$$

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Droplets Clarkson University

$$F_D = 3\pi\mu^f Ud \frac{1 + 2\mu^f / 3\mu^p}{1 + \mu^f / \mu^p}$$

For Bubbles



$$F_D = 2\pi\mu^f Ud$$

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Non-Spherical Particles Clarkson University

$$F_D = 3\pi\mu U d_e K$$

$$d_e = \left(\frac{6}{\pi} \text{Volume}\right)^{1/3}$$

K=Correction Factor

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Correction Factor Clarkson University

Cluster Shape	Correction	Cluster Shape	Correction	Cluster Shape	Correction
oo	K = 1.12	oooo	K = 1.32	oo oo	K = 1.17
ooo	K = 1.27	oooooo	K = 1.45	o o o o o	K = 1.19
o o o	K = 1.16	oooooooo	K = 1.57	oo oo oo	K = 1.17
oooooo o o	K = 1.64		K = 1.73		

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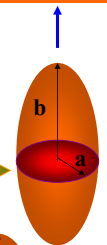
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Ellipsoidal Particles Clarkson University

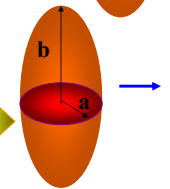
$$F_D = 6\pi\mu U a K'$$

$$\beta = \frac{b}{a}$$

$$K' = \frac{\frac{4}{3}(\beta^2 - 1)}{\frac{(2\beta^2 - 1)}{(\beta^2 - 1)^{1/2}} \ln[\beta + (\beta^2 - 1)^{1/2}] - \beta}$$



$$K' = \frac{\frac{8}{3}(\beta^2 - 1)}{\frac{(2\beta^2 - 3)}{(\beta^2 - 1)^{1/2}} \ln[\beta + (\beta^2 - 1)^{1/2}] + \beta}$$

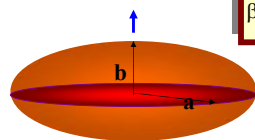


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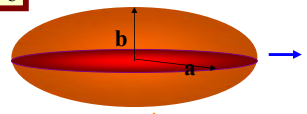
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Ellipsoidal Particles Clarkson University

$$\beta = \frac{a}{b}$$



$$K' = \frac{\frac{4}{3}(\beta^2 - 1)}{\frac{\beta(\beta^2 - 2)}{(\beta^2 - 1)^{1/2}} \tan^{-1}(\beta^2 - 1)^{1/2} + \beta}$$



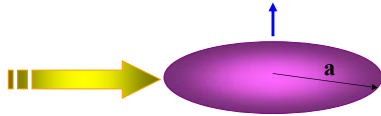
$$K' = \frac{\frac{8}{3}(\beta^2 - 1)}{\frac{\beta(3\beta^2 - 2)}{(\beta^2 - 1)^{1/2}} \tan^{-1}(\beta^2 - 1)^{1/2} - \beta}$$

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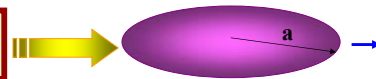
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Thin Disks

$$F_D = 16\mu aU$$

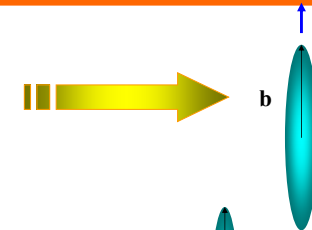


$$F_D = 32\mu aU/3$$

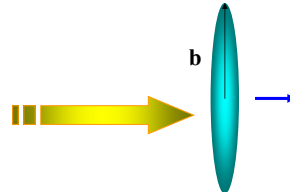


Thin Disks

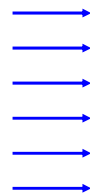
$$F_D = \frac{4\pi\mu Ub}{\ln 2\beta}$$



$$F_D = \frac{8\pi\mu Ub}{\ln 2\beta}$$



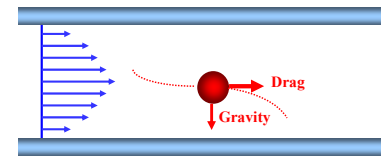
Cylindrical Needle



$$F_D = \frac{4\pi\mu U}{(2.002 - \ln R_e)}$$

$$R_e = \frac{2aU}{\nu}$$

Aerosols Particle Motion



Equation of Motion

$$m \frac{du^p}{dt} = \frac{3\pi\mu d}{C_c} (u^f - u^p) + mg$$

Aerosols Particle Motion Clarkson University

$$\tau \frac{du^p}{dt} = (u^f - u^p) + \tau g$$

Relaxation Time

$$\tau = \frac{mC_c}{3\pi\mu d} = \frac{d^2 \rho^p C_c}{18\mu} = \frac{Sd^2 C_c}{18\nu} \quad S = \frac{\rho^p}{\rho^f}$$

$$\tau(s) \approx 3 \times 10^{-6} d^2 (\mu m)$$

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Terminal Velocity Clarkson University

$$u^p = (u^f + \tau g)(1 - e^{-t/\tau})$$

Terminal Velocity = Equilibrium Velocity after Large Time

$$u^t = \tau g = \frac{\rho^p d^2 g C_c}{18\mu}$$

$$u^t (\mu m/s) \approx 30 d^2 (\mu m)$$

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Stopping Distance Clarkson University

Stopping Distance = Penetration distance for an initial velocity of u_0

$$u^p = u_0 e^{-t/\tau}$$

$$x^p = u_0^p \tau (1 - e^{-t/\tau})$$

$$X^p = u_0^p \tau$$

$$x^p (\mu m) \approx 3 d^2 (\mu m)$$

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Relaxation Time, Terminal Velocity and Stopping Distance Clarkson University

Diameter, μm	Terminal Velocity	τ sec	Stopping Distance $u=1$ m/s	Stopping Distance $u=10$ m/s
0.05	0.39 $\mu m/s$	4×10^{-8}	0.04 μm	0.0004 mm
0.1	0.93 $\mu m/s$	9.1×10^{-8}	0.092 μm	0.0009 mm
0.5	10.1 $\mu m/s$	1×10^{-6}	1.03 μm	0.0103 mm
1	35 $\mu m/s$	3.6×10^{-6}	3.6 μm	0.0357 mm
5	0.77 mm/s	7.9×10^{-5}	78.6 μm	0.786 mm
10	3.03 mm/s	3.1×10^{-4}	309 μm	3.09 mm
50	7.47 cm/s	7.6×10^{-3}	7.62 mm	76.2 mm

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Particle Path Clarkson University

$$\mathbf{x}^p = \mathbf{x}_0^p + \mathbf{u}_0^p \tau (1 - e^{-t/\tau}) + (\mathbf{u}^f + \tau \mathbf{g}) [t - \tau (1 - e^{-t/\tau})]$$

Components

$$x^p / u^f \tau = [t/\tau - (1 - e^{-t/\tau})]$$

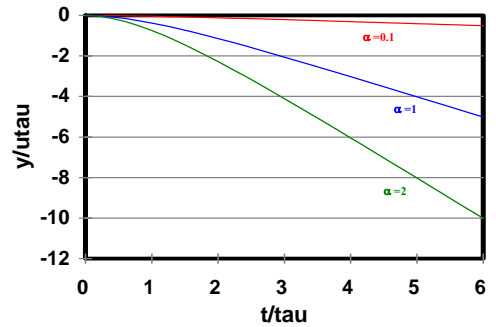
$$\alpha = \frac{\tau g}{u^f \tau}$$

$$y^p / u^f \tau = -\alpha [t/\tau - (1 - e^{-t/\tau})]$$

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Particle Path Clarkson University

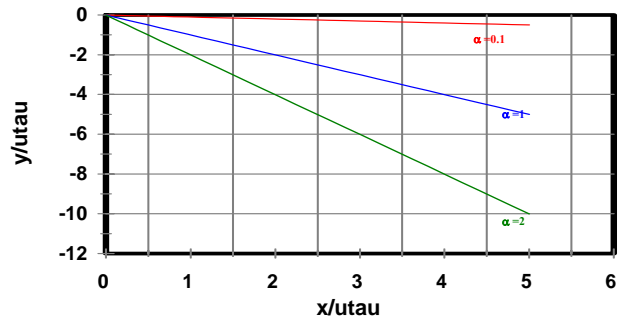


Variations of the particle vertical position with time.

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Particle Path Clarkson University



Sample particle trajectories.

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Buoyancy Effects Clarkson University

$$(m + m^a) \frac{d\mathbf{u}^p}{dt} = \frac{3\pi\mu d}{C_c} (\mathbf{u}^f - \mathbf{u}^p) + (m - m^f) \mathbf{g}$$

Fluid Mass



$$m^f = \frac{\pi d^3 \rho^f}{6}$$

Apparent Mass



$$m^a = \frac{1}{2} m^f$$

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Buoyancy Effects Clarkson University

$$\left(1 + \frac{1}{2S}\right) \tau \frac{d\mathbf{u}^p}{dt} = (\mathbf{u}^f - \mathbf{u}^p) + \tau \mathbf{g} \left(1 - \frac{1}{S}\right)$$

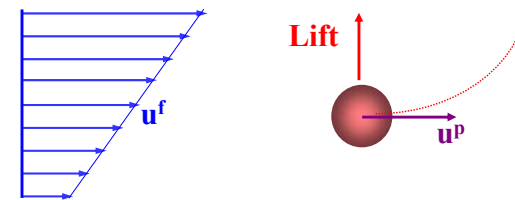
Terminal Velocity

$$\mathbf{u}^t = \tau \mathbf{g} \left(1 - \frac{1}{S}\right) = \frac{\rho^p d^2 g C_c}{18\mu} \left(1 - \frac{\rho^f}{\rho^p}\right)$$

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Lift Force Clarkson University



Saffman (1965, 1968)

$$F_{L(\text{Saff})} = 1.615 \rho \nu^{1/2} d^2 (\mathbf{u}^f - \mathbf{u}^p) \left| \frac{d\mathbf{u}^f}{dy} \right|^{1/2} \text{sgn}\left(\frac{d\mathbf{u}^f}{dy}\right)$$

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Saffman Lift Force Constraints Clarkson University

$$R_{es} = \frac{|\mathbf{u}^f - \mathbf{u}^p| d}{\nu} \ll 1$$

$$R_{e\Omega} = \frac{\Omega d^2}{\nu} \ll 1$$

$$R_{eG} = \frac{\dot{\gamma} d^2}{\nu} \ll 1$$

$$\varepsilon = \frac{R_{eG}^{1/2}}{R_{es}} \gg 1$$

McLaughlin (1991)

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Lift Force Clarkson University

Mai (1992)

$$\frac{F_L}{F_{L(\text{Saff})}} = \begin{cases} (1 - 0.3314\alpha^{1/2}) \exp(-R_{es}/10) + 0.3314\alpha^{1/2} & \text{for } R_{es} \leq 40 \\ 0.0524(\alpha R_{es})^{1/2} & \text{for } R_{es} > 40 \end{cases}$$

$$\alpha = \frac{\dot{\gamma} d}{2|\mathbf{u}^f - \mathbf{u}^p|} = \frac{R_{es} \varepsilon^2}{2} = \frac{R_{eG}}{2R_{es}}$$

$$\frac{F_L}{F_{L(\text{Saff})}} = 0.3 \{1 + \tanh[2.5 \log_{10}(\varepsilon + 0.191)]\} \{0.667 + \tanh[6(\varepsilon - 0.32)]\}$$

$$0.1 \leq \varepsilon \leq 20$$

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Lift Force Clarkson University

McLaughlin (1991)

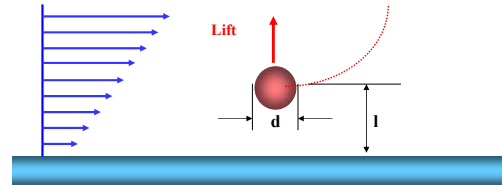
$$\frac{F_L}{F_{L(\text{Saff})}} = \begin{cases} 1 - 0.287\epsilon^{-2} & \text{for } \epsilon \gg 1 \\ -140\epsilon^5 \ln(\epsilon^{-2}) & \text{for } \epsilon \ll 1 \end{cases}$$

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Lift Force - Wall Effect Clarkson University

Cherukat and McLaughlin (1994)



$$F_{L(C-L)} = \rho V^2 d^2 I_L / 4$$

$$V = u^p - u^f = u^p - \dot{\gamma} l$$

$$I_L = I_L(\Lambda_G, K)$$

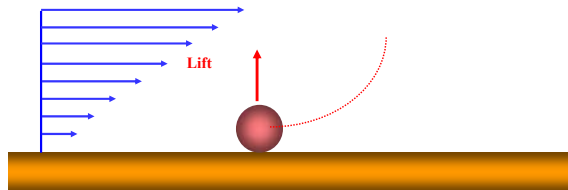
$$\Lambda_G = \frac{\dot{\gamma} d}{2V}$$

$$K = \frac{d}{2l}$$

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Lift Force on a Particle Touching a Plane Clarkson University



Leighton and Acrivos (1985)

$$F_{L(L-A)} = 0.576 \rho d^4 \dot{\gamma}^2$$

Saffman

$$F_{L(\text{Saff})} = 0.807 \rho v^{1/2} d^3 \dot{\gamma}^{3/2}$$

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Lift Force in Turbulent Boundary Layer Clarkson University

Velocity Field in the Inertial Sublayer

$$u^+ = \frac{1}{\kappa} \ln y^+ + B$$

$$B \approx 5$$

$$30 < y^+ \leq 300$$

Wall Units

$$u^+ = \frac{u}{u^*}$$

$$y^+ = \frac{u^* y}{\nu}$$

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Viscous Sublayer Clarkson University

Turbulent stress is negligible

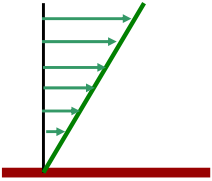
$$\tau_0 = \mu \frac{dU}{dy}$$

$$u^{*2} = \nu \frac{dU}{dy}$$

$$\frac{dU^+}{dy^+} = 1$$

$$u^+ = y^+$$

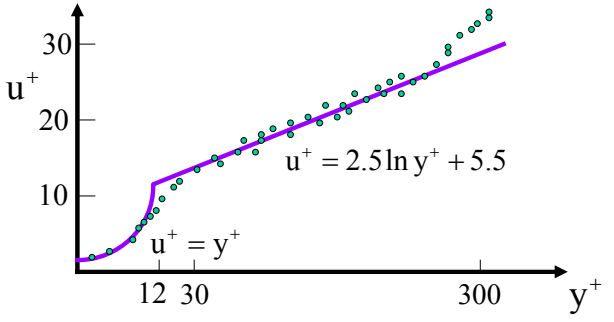
$$0 < y^+ \leq 5$$



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Velocity Near a Wall Clarkson University



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Lift Force in Turbulent Boundary Layer Clarkson University

$$\gamma = \frac{u^{*2}}{\nu}$$

$$F_L^+ = \frac{F_L}{\rho \nu^2}$$

$$d^+ = \frac{du^*}{\nu}$$

$$F_{L(L-A)}^+ = 0.576 d^{+4}$$

$$F_{L(Saff)}^+ = 0.807 d^{+3}$$

Hall (1988)

$$F_{L(Hall)}^+ = 4.21 d^{+2.31}$$

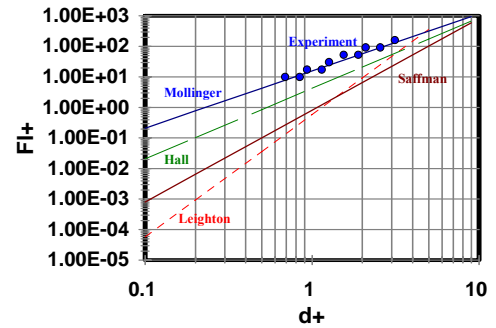
Mollinger and Nieuwstadt (1996)

$$F_{L(MN)}^+ = 15.57 d^{+1.87}$$

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