Particle Transport, Deposition and Removal

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Review for Exam 1

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Outline

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- Drag Forces
- Cunningham Corrections
- Lift Forces
- Brownian Motion
- Diffusion Mechanisms
- Diffusion to a Cylinder

Hydrodynamic Forces

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Drag Forces

Stokes → $F = 3\pi\mu U d$

Drag Coefficient → $C_D = \frac{F_D}{\frac{1}{2}\rho U^2 A} = \frac{24}{Re}$

Hydrodynamic Forces

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Reynolds Number → $Re = \frac{\rho U d}{\mu}$

Drag Forces

$1 < Re < 1000$ → $C_D = \frac{24[1 + 0.15 Re^{0.687}]}{Re}$

Cunningham Correction

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For 1000 > Kn > 0

Stokes-Cunningham Drag → $F_D = \frac{3\pi\mu U d}{C_c}$

Cunningham Correction →

$$C_c = 1 + \frac{2\lambda}{d}[1.257 + 0.4e^{-1.1d/2\lambda}]$$

Cunningham Correction

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Variations of C_c with d for $\lambda = 0.07$ μm

Diameter, μm	C
10 μm	1.018
1 μm	1.176
0.1 μm	3.015
0.01 μm	23.775
0.001 μm	232.54

Droplets

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$$F_D = 3\pi\mu^f U d \frac{1 + 2\mu^f / 3\mu^p}{1 + \mu^f / \mu^p}$$

For Bubbles → $F_D = 2\pi\mu^f U d$

Non-Spherical Particles

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$$F_D = 3\pi\mu U d_e K$$

$$d_e = (\frac{6}{\pi}\text{Volume})^{1/3}$$

K=Correction Factor

Correction Factor

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Cluster Shape	Correction	Cluster Shape	Correction	Cluster Shape	Correction
00	K = 1.12	0000	K = 1.32	00 00	K = 1.17
000	K = 1.27	00000	K = 1.45	0 0 0 0 0	K = 1.19
0 0 0	K = 1.16	000000	K = 1.57	00 00 00	K = 1.17
000000 0 0	K = 1.64	0000000	K = 1.73		

Aerosols Particle Motion

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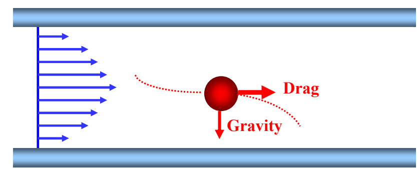

Equation of Motion

$$m\frac{d\mathbf{u}^p}{dt} = \frac{3\pi\mu d}{C_c}(\mathbf{u}^f - \mathbf{u}^p) + m\mathbf{g}$$

Aerosols Particle Motion

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$$\tau\frac{d\mathbf{u}^p}{dt} = (\mathbf{u}^f - \mathbf{u}^p) + \tau\mathbf{g}$$

Relaxation Time

$$\tau = \frac{mC_c}{3\pi\mu d} = \frac{d^2\rho^p C_c}{18\mu} = \frac{Sd^2C_c}{18\nu}$$

$$S = \frac{\rho^p}{\rho^f}$$

$$\tau(s) \approx 3\times 10^{-6} d^2(\mu m)$$

Terminal Velocity

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$$\mathbf{u}^p = (\mathbf{u}^f + \tau\mathbf{g})(1 - e^{-t/\tau})$$

Terminal Velocity = Equilibrium Velocity after Large Time

$$u^t = \tau g = \frac{\rho^p d^2 g C_c}{18\mu}$$

$$u^t(\mu m/s) \approx 30\, d^2(\mu m)$$

Stopping Distance

Stopping Distance = Penetration distance for an initial velocity of u_o

$$\mathbf{u}^p = \mathbf{u}_o e^{-t/\tau}$$

$$\mathbf{x}^p = \mathbf{u}_o^p \tau (1 - e^{-t/\tau})$$

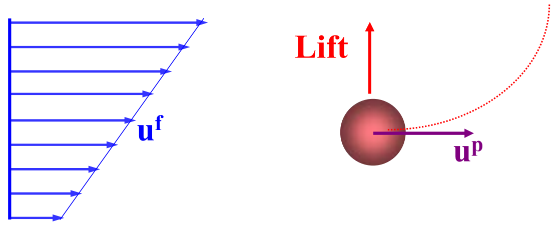
$$x^p = u_o^p \tau$$

$$x^p(\mu m) \approx 3 d^2 (\mu m)$$

Particle Path

$$\mathbf{x}^p = \mathbf{x}_o^p + \mathbf{u}_o^p \tau (1 - e^{-t/\tau}) + (\mathbf{u}^f + \tau \mathbf{g})[t - \tau(1 - e^{-t/\tau})]$$

Lift Force



Saffman (1965, 1968)

$$F_{L(Saff)} = 1.615 \rho \nu^{1/2} d^2 (u^f - u^p) \left| \frac{du^f}{dy} \right|^{1/2} \mathrm{sgn}\left(\frac{du^f}{dy}\right)$$

Saffman Lift Force Constraints

$$R_{es} = \frac{|u^f - u^p| d}{\nu} << 1$$

$$R_{e\Omega} = \frac{\Omega d^2}{\nu} << 1$$

$$R_{eG} = \frac{\dot{\gamma} d^2}{\nu} << 1$$

$$\varepsilon = \frac{R_{eG}^{1/2}}{R_{es}} >> 1$$

McLaughlin (1991)

Lift Force on a Particle Touching a Plane

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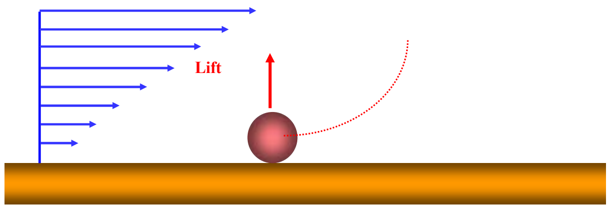

Leighton and Acrivos (1985)

$$F_{L(L-A)} = 0.576\rho d^4 \dot{\gamma}^2$$

Saffman

$$F_{L(Saff)} = 0.807\rho \nu^{1/2} d^3 \dot{\gamma}^{3/2}$$

Lift Force in Turbulent Boundary Layer

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Velocity Field in the Inertial Sublayer

$$u^+ = \frac{1}{\kappa}\ln y^+ + B$$

$$B \approx 5$$

$$30 < y^+ \le 300$$

Wall Units

$$u^+ = \frac{u}{u^*}$$

$$y^+ = \frac{u^* y}{\nu}$$

Viscous Sublayer

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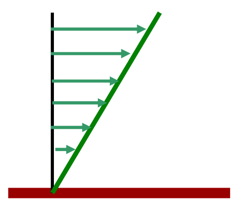
Turbulent stress is negligible

$$\tau_0 = \mu \frac{dU}{dy}$$

$$u^{*2} = \nu \frac{dU}{dy}$$



$$\frac{dU^+}{dy^+} = 1$$

$$u^+ = y^+$$

$$0 < y^+ \le 5$$

Brownian Motion

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Langevin Equation

$$\frac{du}{dt} + \beta u = n(t)$$

$$\beta = 3\pi\mu d / C_c m = 1/\tau$$

N(t) = White Noise

Spectral Intensity

$$S_{nn} = \frac{2kT\beta}{\pi m}$$

Brownian Motion

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Mass Diffusivity

$$D = \frac{1}{2}\frac{d}{dt}\overline{x^2}(t)$$

Diffusivity

$$D = \frac{kT}{\beta m} = \frac{kTC_c}{3\pi\mu d}$$

Computer Simulation Procedure

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- $G_1 = \sqrt{-2\ln U_1}\cos 2\pi U_2$

  $G_2 = \sqrt{-2\ln U_1}\sin 2\pi U_2$
- **Amplitude of the Brownian force is given by**

$$n(t_i) = G_i\sqrt{\frac{\pi S_{nn}}{\Delta t}}$$

Diffusion and Fick's Law

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Fick's Law

$$J = -D\frac{dc}{dx}$$

Diffusion Equation

$$\frac{\partial c}{\partial t} + \mathbf{v}\cdot\nabla c = D\nabla^2 c$$

Diffusivity

$$D = \frac{kTC_c}{3\pi\mu d}$$

Particle Diffusion to a Wall

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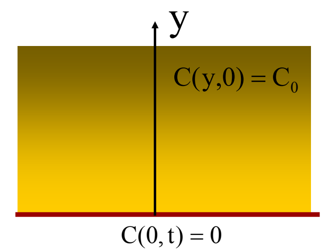
$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial y^2}$$



Similarity Variable

$$\eta = \frac{y}{\sqrt{4Dt}}$$

$$\frac{\partial c}{\partial y} = \frac{\partial c}{\partial \eta}\frac{\partial \eta}{\partial y} = \frac{\partial c}{\partial \eta}\frac{1}{\sqrt{4Dt}}$$

$$\frac{\partial^2 c}{\partial y^2} = \frac{\partial^2 c}{\partial \eta^2}\frac{1}{4Dt}$$

Particle Diffusion to a Wall

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$$\frac{\partial c}{\partial t}=\frac{\partial c}{\partial \eta}\frac{\partial \eta}{\partial t}=\frac{\partial c}{\partial \eta}\frac{-y}{2t\sqrt{4Dt}}=-\frac{\partial c}{\partial \eta}\frac{\eta}{2t}$$

Similarity Equation

$$\frac{d^2c}{d\eta^2}+2\eta\frac{dc}{d\eta}=0 \quad \Rightarrow \quad \ln\left(\frac{dc}{d\eta}\right)=-\eta^2+\ln A$$

$$\frac{dc}{d\eta}=Ae^{-\eta^2} \quad \Rightarrow \quad c=A\int_0^{\eta}e^{-\eta_1^2}d\eta_1+B$$

Particle Diffusion to a Wall

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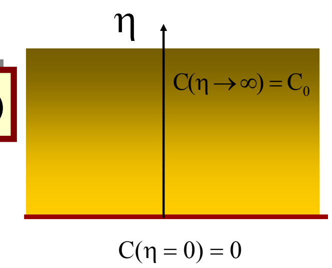
$$C(y,t)=C_0\,\mathrm{erf}\left(y/\sqrt{4Dt}\right)$$



$$\mathrm{erf}(\xi)=\frac{2}{\sqrt{\pi}}\int_0^{\xi}e^{-\xi^2}d\xi$$

$$\mathrm{erf}(0)=0 \qquad \mathrm{erf}(\infty)=1$$

Particle Diffusion to a Wall

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Diffusion Velocity

$$u_D=\frac{J}{C_0}=\sqrt{\frac{D}{\pi t}}=\frac{D}{\delta_c}$$

Diffusion Boundary Layer

$$\delta_c=\sqrt{\pi D t}$$

Diffusion Force

$$F_d=3\pi\mu d u_D/C_c$$

Convective Diffusion to a Flat Plate

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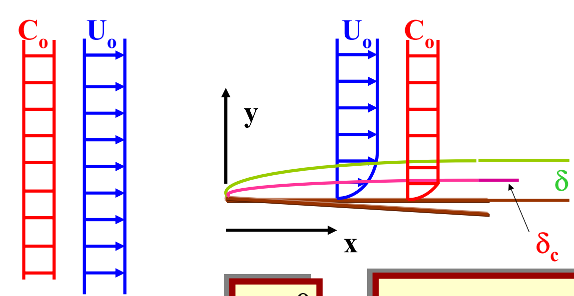

$y=0$: $u=v=c=0$

$y\to\infty$: $u=U_0,\ c=c_0$

Convective Diffusion to a Flat Plate

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Momentum → $u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{dy^2}$

Mass → $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$

Concentration → $u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}=D\frac{\partial^2 c}{\partial y^2}$

Flat Plate - Similarity Variables

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$\eta=y\sqrt{\frac{U_0}{\nu x}}$ $\frac{u}{U_0}=f'(\eta)$

$\psi=\sqrt{\nu U_o x}f(\eta)$ $c=c(\eta)$

Momentum/Mass

$ff''+2f'''=0$

Blasius Equation

Concentration

$c''+\frac{1}{2}S_c fc'=0$ $S_c=\frac{\nu}{D}$

Flat Plate - Similarity Variables

Clarkson University

Boundary Conditions

$f(0)=f'(0)=0$ $f'(\infty)=1$

$c(0)=0$ $c(\infty)=c_0$

Blasius Solution

$\delta=5\sqrt{\frac{\nu x}{U_0}}$ $f''(0)=\gamma=0.332$

Near the Plate → $f\simeq\frac{\gamma}{2}\eta^2+\ldots$

Concentration Profile

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$$C=\frac{C_0\int_0^\eta \exp(-\gamma_1 s_c z^3)dz}{\int_0^\infty \exp(-\gamma_1 s_c z^3)dz}$$

$$\gamma_1=\frac{\gamma}{12}$$

$$\frac{c}{c_0}=\frac{\sqrt[3]{\gamma_1 s_c}}{0.89}\int_0^\eta[\exp(-\gamma_1 s_c z^3)]dz$$

Diffusion in a Tube Flow

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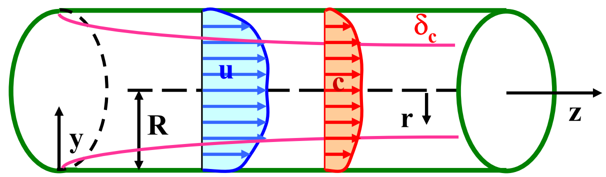

Laminar Flow → $u = u_0\left(1 - \frac{r^2}{R^2}\right)$

$y = R - r$

$u \simeq u_0 \frac{2y}{R} + \dots$

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Diffusion in a Tube Flow

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Diffusion Equation → $$\frac{2u_0}{R} y \frac{\partial c}{\partial x} = D \frac{\partial^2 c}{\partial y^2}$$

Boundary Condition

$y = 0$ $c = 0$

$y \to \infty$ $c = c_0$

$$\eta = \sqrt[3]{\frac{u_0}{DR}} \frac{y}{\sqrt[3]{x}}$$

Similarity Variable

$$c'' + \frac{2}{3}\eta^2 c' = 0$$

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Diffusion to a Cylinder

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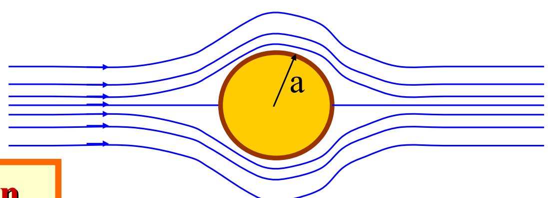

Diffusion Equation

$$\frac{v_\theta}{r}\frac{\partial c}{\partial \theta} + v_r \frac{\partial c}{\partial r} = D\left(\frac{\partial^2 c}{\partial r^2} + \frac{1}{r}\frac{\partial c}{\partial r}\right)$$

Boundary Conditions →

$r = a + \frac{d}{2},$ $c = 0$

$r = \infty,$ $c = c_\infty$

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Diffusion to a Cylinder

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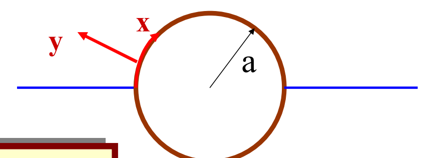
Diffusion Equation



$$u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} = D\frac{\partial^2 c}{\partial y^2}$$

Boundary Conditions →

$y = 0,$ $c = 0$

$y = \infty,$ $c = c_\infty$

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Direct Interception Limit

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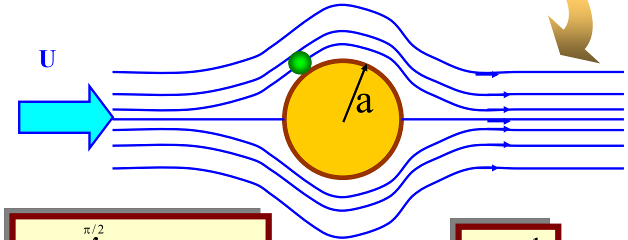
$P_e \to \infty$

No Diffusion



$$\eta_R = -\frac{\int_0^{\pi/2} v|_{y=d/2}\, dx}{Ua} = 2AR^2$$

$$R = \frac{d}{2a}$$

Diffusion to a Cylinder

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Let

$$\chi = \int \sin^{1/2} x_1 dx_1, \quad \psi_1 = \frac{\psi}{2AaU}$$

Diffusion Equation

$$\frac{\partial c}{\partial \chi} = \frac{D}{aAU}\frac{\partial}{\partial \psi_1}\left(\psi_1^{1/2}\frac{\partial c}{\partial \psi_1}\right)$$

$$\psi_1 = 0, \quad c = 0$$
$$\psi_1 = \infty, \quad c = c_\infty$$

Diffusion to a Cylinder

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Similarity Equation

$$\xi = \frac{\psi_1}{\chi^{2/3}}$$

$$-\frac{AP_e}{3}\xi\frac{dc}{d\xi} = \frac{d}{d\xi}\left(\xi^{1/2}\frac{dc}{d\xi}\right)$$

$$c = \frac{c_\infty (AP_e)^{1/3}}{1.45}\int_0^{\sqrt{\xi}} \exp\{-\frac{2}{9}AP_e z^3\}dz$$

$$P_e = \frac{2Ua}{D} = R_e \cdot S_c$$

Diffusion

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- **Similarity Method**
- **Separation of Variable Method**
- **Integral Method**