Particle Transport, Deposition and Removal

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Particle Adhesion and Detachment Models

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Outline

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- Particle Detachment Mechanisms
- JKR Adhesion Model
- DMT Adhesion Model
- Maugis-Pollock Model
- Maximum Moment Resistance

Particle Adhesion and Detachment Models

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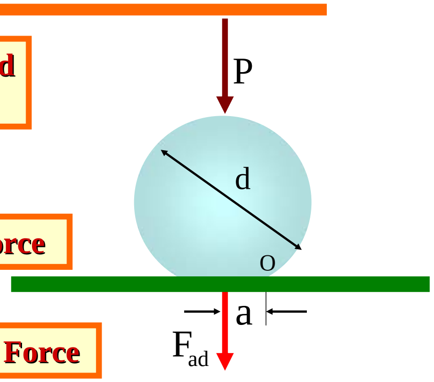
Sphere Attached to a Surface

a = Contact

P = Exerted Force

F_{ad} = Adhesion Force



JKR Model

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Johnson-Kandall-Roberts (1971)

$$a^3 = \frac{d}{2K}\left[P + \frac{3}{2}W_A \pi d + \sqrt{3\pi W_A dP + \left(\frac{3\pi W_A d}{2}\right)^2}\right]$$

$$K = \frac{4}{3}\left[\frac{1-\upsilon_1^2}{E_1} + \frac{1-\upsilon_2^2}{E_2}\right]^{-1}$$

Hertz Model

$$a^3 = \frac{dP}{2K}$$

JKR Model

Pull-Off Force → $$F_{po}^{JKR} = \frac{3}{4}\pi W_A d$$

Contact Radius at Zero Force → $$a_0 = \left(\frac{3\pi W_A d^2}{2K}\right)^{\frac{1}{3}}$$

Contact Radius at Separation → $$a = \left(\frac{3\pi W_A d^2}{8K}\right)^{\frac{1}{3}} = \frac{a_0}{4^{1/3}}$$

DMT Model

Derjaguin-Muller-Toporov (1975)

Pull-Off Force → $$F_{Po}^{DMT} = \pi W_A d$$

$$F_{Po}^{DMT} = \frac{4}{3} F_{Po}^{JKR}$$

Contact Radius at Zero Force → $$a_0 = \left(\frac{\pi W_A d^2}{2K}\right)^{\frac{1}{3}}$$

Contact Radius at Separation → $$a = 0$$

Maugis-Pollock Model

$$P + \pi W_A d = \pi a^2 H$$

$$H = 3Y$$

$$a_0 \sim d^{\frac{2}{3}}$$ ↔ **Elastic**

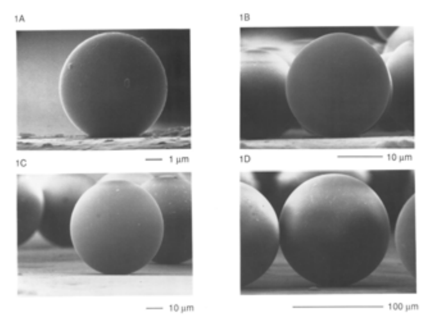
$$a_0 \sim d^{\frac{1}{2}}$$ ↔ **Plastic**

Examples of Adhesion-Induced Deformations - JKR Systems

Rimai et al.

Polystyrene on Polyurethane

High elastic modulus spherical particles on elastomeric substrates.



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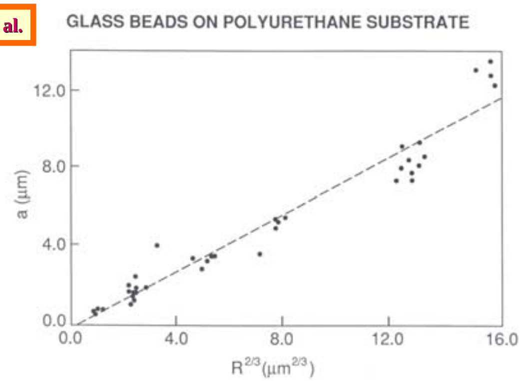
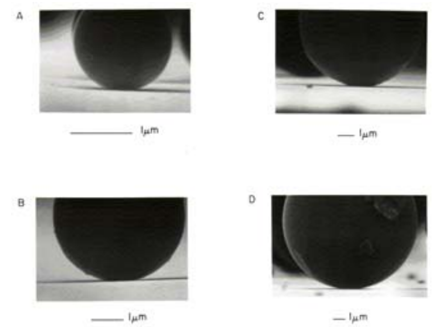


Non-JKR Systems

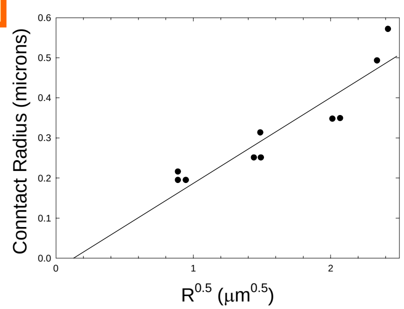
Rimai et al.

Polystyrene particles on a silicon wafer



Non-JKR Systems

Rimai et al.



Thermodynamic Work of Adhesion

$$W_A = \frac{A}{12\pi z_0^2}$$

JKR Model

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$$a^{*3} = 1 - P^* + \sqrt{1 - 2P^*}$$

$$P^* = -\frac{P}{\frac{3}{2}\pi W_A d}$$

$$a^* = \frac{a}{\left(\frac{3\pi W_A d^2}{4K}\right)^{\frac{1}{3}}}$$

$$a_0^* = 1.26$$

JKR Model

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$$M^{*JKR} = P^* a^* = P^* (1 - P^* + \sqrt{1 - 2P^*})^{1/3}$$

Maximum Moment → $M^{*JKR}_{max} = 0.42$

$$P^*_{max} = F^{*JKR}_{po} = \frac{F^{JKR}}{\frac{3\pi}{2} W_A d} = 0.5$$ → $M^{*JKR} = 0.397$

$$P^*_{max} a_0^* = 0.63$$

DMT Model

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Contact Radius → $a^3 \approx \frac{d}{2K}(P + \pi W_A d)$

Nondimensional Contact Radius $a^{*3} = \left(\frac{a}{3\pi W_A d^2 / 4K}\right)^3 = -P^* + \frac{2}{3}$

Resistance Moment $M^{*DMT} = P^* (2/3 - P^*)^{1/3}$

Maximum Moment → $M^{*DMT}_{max} = 0.28$

JKR-DMT Models

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$$P^*_{max} = F^{*DMT}_{po} = \frac{F^{DMT}}{\frac{3\pi}{2} W_A d} = \frac{2}{3}$$

$$P^*_{max} a_0^* = 0.58$$

$$M^{*DMT} = 0$$

Maximum Moments → $M^{*JKR}_{max} = 0.42 = 1.5 M^{*DMT}_{max}$

$$M^{JKR}_{max} = 2.63 \frac{W_A^{\frac{4}{3}} d^{\frac{5}{3}}}{\kappa^{\frac{1}{3}}}$$

$$M^{DMT}_{max} = 1.83 \frac{W_A^{\frac{4}{3}} d^{\frac{5}{3}}}{\kappa^{\frac{1}{3}}}$$

JKR-DMT Models

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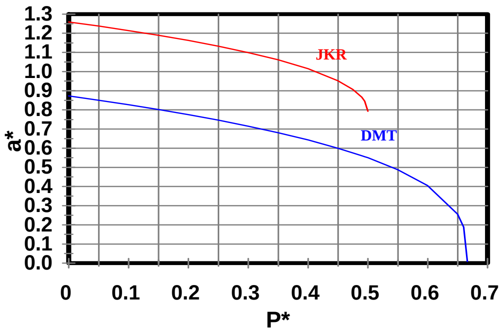

Variations of contact radius with the exerted force.

JKR-DMT Models

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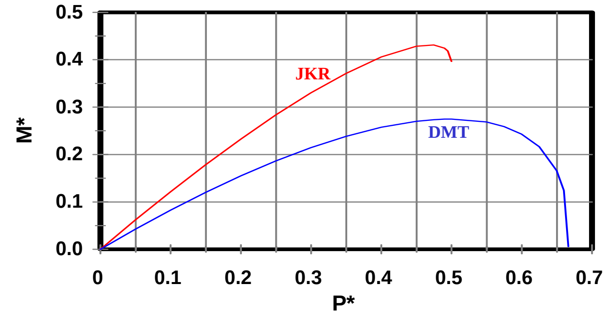

Variations of resistance moment with the exerted force.

Conclusions

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- Spherical particles are removed by overcoming the adhesion rolling resistance
- JKR, DMT and Maugis-Pollock Models
- Contact radius varies differently with d for elastic and plastic deformations

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Thank you!

Questions?