ME 437/537 **Exam 1**

Problem 1. (35 points) Consider a one-dimensional diffusion process with a space dependent diffusivity in the absence of a flow field. The governing equation is given by

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial y}\left(D\frac{\partial c}{\partial y}\right)$$

Assume that $D = \alpha y$, and an initially uniform concentration of aerosols in the neighborhood of an absorbing wall. The initial and boundary conditions are:

$$C(y,0) = C_0 \qquad \boxed{C(0,t) = 0} \qquad C(\infty, t) = C_o.$$

a) Find the appropriate similarity variable $\eta = \frac{y}{\alpha t}$, reduce the governing equation and boundary conditions to the similarity form. b) Does this equation accept a similarity solution? c) Justify your solution.

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial y}\left(\alpha y \frac{\partial c}{\partial y}\right) \qquad 1 = a \qquad a = 1$$

$$t - t' \, , \quad y = y^a \qquad \eta = \frac{y}{\alpha t} \, , \quad y = \alpha t \eta$$

$$\frac{\partial c}{\partial t} = \frac{\partial c}{\partial \eta}\frac{\partial \eta}{\partial t} = \frac{dc}{d\eta}\frac{-y}{\alpha t^2} = -\frac{dc}{d\eta}\frac{\eta}{t}$$

$$\frac{\partial c}{\partial y} = \frac{\partial c}{\partial \eta}\frac{\partial \eta}{\partial y} = \frac{dc}{d\eta}\frac{1}{\alpha t}$$

$$-\frac{\eta}{t}\frac{dc}{d\eta} = \frac{1}{\alpha t}\frac{\partial}{\partial \eta}\left(\frac{\cancel{\alpha^2} \cancel{t} \eta}{\cancel{\alpha} \cancel{t}}\frac{\partial c}{\partial \eta}\right)$$

$$-\frac{\eta}{t}\frac{dc}{d\eta} = \frac{1}{t}\frac{d}{d\eta}\left(\eta \frac{dc}{d\eta}\right) \Rightarrow \frac{d}{d\eta}\left(\eta \frac{dc}{d\eta}\right) + \eta \frac{dc}{d\eta} = 0$$

$$\eta \frac{d^2 c}{d\eta^2} + (1+\eta)\frac{dc}{d\eta} = 0 \Rightarrow \frac{c''}{c'} = -\frac{1}{\eta} - 1 \Rightarrow \ln c' = -\ln \eta - \eta + \ln A$$

$$c' = A\frac{e^{-\eta}}{\eta} \Rightarrow \boxed{c = A\int_0^{\eta} \frac{e^{-\eta_1}}{\eta_1} d\eta_1 + B}$$

B.C. $C(y,0) = C_0$, $C(0,t) = 0$, $C(\infty,t) = C_0$

$\eta = y/\alpha t$ $C(\infty) = C_0$, $C(0) = 0$ $\rightarrow$ $B = 0$

Yes, Accepts similarity

$$A = \frac{C_0}{\int_0^\infty \frac{e^{-\eta}}{\eta}\, d\eta}$$

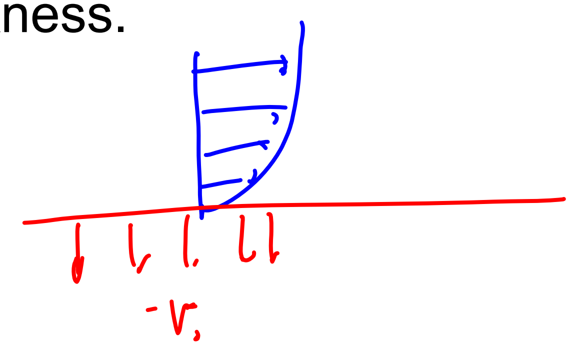
$$C = \frac{\int_0^\eta \frac{e^{-\eta_1}}{\eta_1}\, d\eta_1}{\int_0^\infty \frac{e^{-\eta_1}}{\eta_1}\, d\eta_1}$$

Problem 2. (35 points) Consider a fully developed laminar boundary layer flow of a dilute gas-solid mixture over a flat plate with suction. Assume that the free stream velocity is U_o, the free stream concentration is C_o, and the suction velocity at the surface of the plate is $-V_o$.

a) Show that $u = U_o(1 - e^{-yV_0/\nu})$, $V = -V_o$, are the exact velocity field in the boundary layer.

b) Find the fully developed concentration profile C(y) in terms of particle diffusivity and suction velocity $-V_o$.

c) Evaluate the expression for particle deposition velocity and concentration boundary layer thickness.



$$u\cancel{\frac{\partial u}{\partial x}} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$

$$u = u_0\left(1 - e^{-yV_0/\nu}\right)$$

$$v = -V_0$$

$$\cancel{\frac{\partial u}{\partial x}} + \cancel{\frac{\partial v}{\partial y}} = 0$$

a)

$$-V_0 u_0\left(\frac{V_0}{\cancel{\nu}} e^{-yV_0/\nu}\right) = \cancel{\nu}\frac{V_0 u_0}{\cancel{\nu}}\left(-\frac{V_0}{\cancel{\nu}}\right)e^{-yV_0/\nu}$$

yes

b)

$$u\cancel{\frac{\partial c}{\partial x}} + v\frac{\partial c}{\partial y} = D\frac{\partial^2 c}{\partial y^2}$$

$$c(y)$$

$$-V_0\frac{dc}{dy} = D\frac{d^2 c}{dy^2}$$

B.C. At $y=0$ $c=0$

$y=\infty$ $c=c_0$

$$\frac{dc}{dy} + \frac{V_0}{D}c = A$$

$$c = Be^{-\frac{V_0 y}{D}} + A_1$$

$$A_1 = c_0, \quad B = -c_0$$

$$c = c_0\left(1 - e^{-V_0 y/D}\right)$$

$$J = D \frac{dc}{dy}\Big|_{y=0} = D C_0 \frac{V_0}{D} e^{-V_0 y/D}\Big|_{y=0} = C_0 V_0$$

$$U_d = J/C_0 = V_0$$

$$\delta = \frac{D}{U_d} = \frac{D}{V_0}$$

Problem 3.(30 points) Consider a 0.03 μm particles of density of 2000 kg/m3 in air under normal condition.

i. Find the terminal velocity with and without Cunningham correction.
ii. Determine the diffusivity.
iii. Find the intensity of Brownian excitation for a Δt of 10-6 s.
iv. When the particle is falling in a shear field of 1000 s-1, find the Saffman lift force.

$\rho_{air} = 1.2\ kg/m^3$

(Assume a kinematic viscosity of 1.5×10^{-5} m2/s, a temperature of 300 K, and λ=0.07μm. For other needed parameters assume typical values.)

i) $$u_t = \frac{\rho^p d^2 g C_c}{18\mu}$$

$$C_c = 1 + \frac{2\lambda}{d}\left(1.257 + 0.4 e^{-1.1 d/2\lambda}\right) = 8.34$$

$d = 0.03 \times 10^{-6}$, $\lambda = 0.07 \times 10^{-6}$

$$u_t = \frac{2000\,(0.03 \times 10^{-6})^2\, 9.81\,(8.34)}{18 \times 1.8 \times 10^{-5}} = 4.5 \times 10^{-7}\ m/s \quad \text{with correction}$$

$U_t = 5.45 \times 10^{-8}$ (No correction)

ii) $D = \dfrac{kT\, C_c}{3\pi \mu d} = \dfrac{1.38 \times 10^{-23} (300)\ 8.34}{3\pi \times 1.8 \times 10^{-5} (0.03 \times 10^{-6})} = 6.8 \times 10^{-9}\ m^2/s$

iii) $S_{nn} = \dfrac{6 kT \mu d}{C_c\ m^2} = \dfrac{216\, kT \mu}{C_c\, \rho_p^2 \pi^2 d^5}$, $m = \dfrac{\pi}{6} d^3 \rho_p = 2.83 \times 10^{-20}$ kg

$S_{nn} = 2.01 \times 10^{6}\ m^2/s^3$

$\sqrt{\dfrac{\pi S_{nn}}{\Delta t}} = 2.57 \times 10^{6}\ m/s^2$

iv) $F_L = 1.615\, \rho\, \nu^{1/2} d^2 (v^f - v^p) \left| \dfrac{du^f}{dy} \right|^{1/2}$

$= 1.615 (1.2)(1.5 \times 10^{-5})^{1/2} (3 \times 10^{-8})^2 (4.5 \times 10^{-7}) (1000)^{1/2}$

$= 9.61 \times 10^{-23}$ N