Problem 1. (35 points) Consider a one-dimensional diffusion process with a time dependent diffusivity in the absence of a flow field. The governing equation is given by

\[ \frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial y^2} \]

Assume that \( D = \alpha t^{1/2} \), and an initially uniform concentration of aerosols in the neighborhood of an absorbing wall. The initial and boundary conditions are:

\[ C(y,0) = C_0 \quad , \quad C(0,t) = 0 \quad , \quad \text{and} \quad C(\infty,t) = C_0. \]

a) Find the appropriate similarity variable, \( \eta \), and reduce the governing equation and boundary conditions to the similarity form.

b) Does this equation accept a similarity solution?

c) Evaluate the solution.

d) Find the deposition velocity.
\[ \frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial y^2} = \alpha + \frac{1}{2} \frac{\partial^2 C}{\partial y^2} \]

(1) \[ t - t', \quad y - t' \quad 1 = 2a - \frac{1}{2} \Rightarrow a = \frac{3}{4} \]

\[ \eta = \frac{y}{\beta t^{3/4}} \]

\[ \frac{\partial \eta}{\partial t} = -\frac{3}{4} \frac{y}{\beta t^{3/4}} \eta = -\frac{3}{4} \frac{\eta}{t} \]

b) \[ \frac{\partial C}{\partial t} = \frac{\partial C}{\partial \eta} \frac{\partial \eta}{\partial t} = \frac{\partial C}{\partial \eta} (-\frac{3}{4} \frac{\eta}{t}) \]

\[ \frac{\partial C}{\partial \eta} = \frac{\partial C}{\partial y} \frac{\partial y}{\partial \eta} = \frac{\partial C}{\partial y} \frac{1}{\eta} \beta t^{3/4} \quad \frac{\partial^2 C}{\partial \eta^2} = \frac{\partial^2 C}{\partial y^2} \frac{1}{\eta^2} \beta^2 t^{3/2} \]

\[ -\frac{3}{4} \frac{\eta}{t} \frac{\partial C}{\partial \eta} = \lambda t^{1/3} \frac{\partial^2 C}{\partial y^2} \frac{1}{\eta} \beta t^{3/2} \]

\[ \frac{d^2 C}{d \eta^2} + \left( \frac{3}{4} \frac{\beta^2}{\alpha} \right) \eta \frac{dc}{dt} = 0 \quad C \big|_{\eta = 0} = 0 \quad C \big|_{\eta = \infty} = C_0 \]

\[ \rho = \sqrt{\frac{8 \alpha}{3}} \quad \eta = \sqrt{\frac{8 \alpha}{3}} \frac{t^{3/4}}{c} \quad \text{YES} \]
\[ c) \quad \frac{d^2 c}{d \eta^2} + 2 \eta \frac{dc}{d \eta} = 0 \]

\[ c'' / c' = -2 \eta \rightarrow \quad \lambda c' = -\eta^2 + \ln A, \quad c' = A e^{-\eta^2} \]

\[ c = A \int_0^\eta e^{-\eta_1^2} \, d\eta_1 + B \rightarrow \quad A = \frac{c_0}{\int_0^\infty e^{-\eta_1^2} \, d\eta_1} \]

\[ C = c \text{erf}(\eta) = c_0 \text{erf}\left( \frac{y}{\sqrt{8} \alpha^2 \tau^{3/4}} \right) = \frac{2}{\sqrt{\pi}} \int_0^\eta e^{-\eta_1^2} \, d\eta_1 \quad (10) \]

\[ \eta' = D \frac{dc}{d \eta} \bigg|_{\eta = 0} = \frac{2 \alpha}{\sqrt{\pi}} \frac{\alpha^{1/2}}{e^{3/4}} = \frac{2 c_0 \alpha}{\sqrt{\pi} \beta^{1/4}} \]

\[ u_d = \frac{J}{c_0} = \frac{2 \alpha}{\sqrt{\pi} \beta^{1/4}} = \sqrt{\frac{3 \alpha}{2 \pi}} \frac{1}{\beta^{1/4}} \quad (5) \]
Problem 2. (30 points) Consider steady particle diffusion process between two cylinders as shown. At radius \( a \), particles are being emitted with a concentration of \( C_0 \). At \( r=b \), the surface is absorbing with \( C(b)=0 \).

a) evaluate the steady concentration profile \( C(r) \) in terms of particle diffusivity and radii \( a \) and \( b \).

b) Evaluate the expression for particle deposition velocity and concentration boundary layer thickness at \( r=b \).

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial C}{\partial r} \right) = 0
\]

\[
r \frac{\partial C}{\partial r} = A, \quad \frac{\partial C}{\partial r} = \frac{A}{r}
\]

\[
C = A \ln r + B, \quad C(a) = C_0, \quad C(b) = 0
\]

\[
0 = A \ln b + B, \quad B = -A \ln b
\]

\[
C = A \ln \frac{r}{b}, \quad C_0 = A \ln \frac{a}{b}, \quad C = C_0 \frac{\ln \frac{r}{b}}{\ln \frac{a}{b}}
\]

b) \[
J = \left. \frac{\partial C}{\partial r} \right|_{r=b} = \frac{DC_0}{b \ln \frac{a}{b}}, \quad \frac{\delta}{C_0} = \frac{C}{C_0} \frac{\ln \frac{r}{b}}{\ln \frac{a}{b}}
\]

\[
S = \frac{D \frac{\delta}{C_0}}{u_b} > b \ln \frac{a}{b}
\]
Problem 3. (35 points) Consider a collection of three 0.02 \( \mu \text{m} \) particles of density of 2000 \( \text{kg/m}^3 \) in air under normal condition as shown.

i. Find the terminal velocity with and without Cunningham correction.

ii. Determine the aerodynamic diameter of the particle.

iii. Find the intensity of Brownian excitation for a \( \Delta t \) of 10-6 s.

iv. When the particle is falling in a shear field of 500 s-1, find the Saffman lift force.

(Assume a kinematic viscosity of \( 1.5 \times 10^{-5} \) m\(^2\)/s, a temperature of 300 K, and \( \lambda = 0.07 \mu \text{m} \). For other needed parameters assume a typical value.)

\[
U_t = \frac{n \beta d^2 g C_c}{18 \nu K}
\]

\[
d = 0.02 \mu \text{m}
\]

\[
C_c = 1 + \frac{2 \lambda}{d} (1.257 + 0.4 \frac{d}{12 \lambda}) = 12.19
\]

\[
U_t = \frac{3 \pi}{18} \frac{(2000)(2 \times 10^{-6})^2}{1.8 \times 10^{-5}(1.25)} \times 9.81(12.19)
\]

\[
U_t = 3.97 \times 10^{-8} \text{m/s}
\]

Without \( C_c \)}
b) \( V_t = \bar{V}_A \frac{n^2 \rho d^2 g c_c}{18 \eta \kappa} = \frac{\rho d^2 g c_c}{18 \mu} \quad c_c = c \)  

\[ \frac{d_A}{\sqrt{K}} = \frac{0.342 \mu m}{n} \quad 10 \]

\[ d_A = \frac{n^{1/3} (\rho \kappa_{m})^{1/2}}{\sqrt{K}} \]

C) Equivalent volume. 

\[ d_n = 3 \frac{d}{4} \]

\[ \frac{\pi d_n^3}{6} = 3 \frac{\pi d^3}{6} \]

\[ d_n = 0.0288 \mu m \]

\[ S_m = \frac{6 kT \rho d_n}{n^2} = \frac{216 T \rho d_n}{n^2} \]

\[ S_m = \frac{216 \times 1.38 \times 10^{-23} \times 1.6 \times 10^{-5}}{8.66 \times 10^{-2}} \]

\[ = 1.58 \times 10^6 \frac{m^{1/3}}{5^3} \]

\[ \sqrt{S_m} = \frac{2.37 \times 10^6}{m^{1/3}} \]

d) \[ F_c = 1.615 \rho \sqrt{d^3} (u_t - u^9) \left| \frac{du}{dt} \right| \]

\[ = 3.25 \times 10^{-23} N \quad 7 \]

\[ (300)^{1/2} \]