**Diffusion to a Cylinder**

**Stream Function**

\[ \psi = AU \sin \theta \left[ \frac{r}{a} - (2 \ln \frac{r}{a} - 1) + \frac{1}{r} \right] \]

\[ \psi = \frac{1}{\sqrt{2 \ln R}} \quad \text{Re} \ll 1 \]

**Outline**

- Diffusion to a Cylinder in Cross Flow
- Deposition Velocity
- Interception
- Filtration

**Diffusion Equation**

\[ v_0 \frac{\partial c}{\partial \theta} + v_r \frac{\partial c}{\partial r} = \frac{D}{\partial^2 c} + \frac{1}{r} \frac{\partial c}{\partial r} \]

**Boundary Conditions**

- \[ r = a + \frac{d}{2}, \quad c = 0 \]
- \[ r = \infty, \quad c = c_o \]
**Diffusion to a Cylinder**

**Diffusion Equation**
\[ u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} = D \frac{\partial^2 c}{\partial y^2} \]

**Boundary Conditions**
- \( y = 0, \quad c = 0 \)
- \( y = \infty, \quad c = c_\infty \)

**Stream Function**
\[ u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \]

**Using \( x \) and \( \psi \)**
\[ \frac{\partial c}{\partial x} = D \left( \frac{\partial}{\partial y} \left[ u \frac{\partial c}{\partial \psi} \right] \right) \]
\[ \psi \approx 2AaUy^2 \sin x \]

**Similarity Equation**
\[ \psi = \left( \frac{\psi_1}{\chi} \right)^{2/3} \]
\[ c = \frac{c_\infty (AP_e)^{1/3}}{1.45} \int_0^\xi \exp\left\{ -\frac{2}{9}AP_e z^3 \right\} dz \]
\[ P_e = \frac{2Ua}{D} = R_e \cdot S \]
**Diffusion to a Cylinder**

**Sherwood Number**

\[ \text{Sh} = \frac{\bar{h}(2a)}{D} = 1.17(APe)^{1/2} \]

**Collection Efficiency**

\[ \eta_R = \frac{\bar{h}(2a)c_{\infty}}{(2a)Uc_{\infty}} = 3.68A^{1/3}Pe^{-2/3} \]

\[ \eta_R \sim d^{-2/3} \]

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**Direct Interception Limit**

**No Diffusion**

\[ R = \frac{d}{2a} \]

\[ \int U \left| \frac{\partial y}{\partial x} \right| dx = \frac{2}{3} \pi a^2 U \]

\[ \eta_R = \int \left[ \eta_R(2a)c \right] \left( \frac{1}{\pi a} \right) \frac{dC}{C} = 2AR^2 \]

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**Fiber Efficiency**

\[ dc = - \left[ \eta_R(2a)c \right] \left( \frac{1}{\pi a} \right) \frac{dC}{C} \]

\[ \eta_R = \frac{\pi a}{2UL} \ln \left( \frac{c_1}{c_2} \right) \]

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Variations of Sherwood number and collection efficiency with Peclet number.
Empirical Equation

\[ \eta_r (R_P e) = 1.3R_P e^{1/3} + 0.7(R_P e^{1/3})^3 \]

- \( P_e \to \infty \) \( \eta_R \propto R^2 \)
- \( P_e \to 0 \) \( \eta_R \propto P_e^{-2/3} \)

Conclusions

- Deposition by Diffusion to a Cylinder
- Deposition by Interception to a Cylinder
- Fiber Filter Efficiency

Variation of filter collection efficiency.