Electrodynamics

Most natural and manmade aerosols carry charges, and their behavior is strongly affected by their charge. In this section the consequences of charges on dynamic of aerosols are discussed. A brief review of electrodynamics is first presented. The Maxell equations governing electrodynamics are listed in Table 1.

Table 1. Electrodynamics equations.

Maxwell's Equations	Gaussian Units	MKS Units
Coulomb's Law	$\nabla \cdot \mathbf{D} = 4\pi\rho_e$	$\nabla \cdot \mathbf{D} = \rho_e$
Ampere's Law	$\nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}$	$\nabla \times \mathbf{H} = \mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}$
Faraday's Law	$\nabla \times \mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = 0$	$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$
Absence of Free Magnetic Poles	$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \mathbf{B} = 0$
Continuity Equation	$\nabla \mathbf{J} + \frac{\partial \rho_e}{\partial t} = 0$	

Constitutive equations for free space are given as:

$$\mathbf{D} = \varepsilon_0 \mathbf{E} \qquad \varepsilon_0 = 1 \qquad \varepsilon_0 = \frac{10^7}{4\pi c^2} = 8.854 \times 10^{-12}\ \text{Coul/Volt} \cdot \text{m} \tag{1}$$

$$\mathbf{B} = \mu_0 \mathbf{H} \qquad \mu_0 = 1 \qquad \mu_0 = 4\pi \times 10^{-7} \tag{2}$$

Ohm's Law is given by

$$\mathbf{J} = \sigma \mathbf{E} \qquad c = (\varepsilon_0 \mu_0)^{-1/2} \tag{3}$$

In these equations:

D = Electric Displacement c = Speed of Light

B = Magnetic Induction ε = Dielectric Constant

E = Electric Field μ = Permeability

H = Magnetic Field σ = Conductivity

J = Current ρ_e = Charge Density

Note that 1 electronic unit of charge = 4.8×10^{-10} Statcoulombs = 1.59×10^{-19} Coulombs (MKS). A conversion table for different physical quantities are given in Table 1.

Table 1. Conversion table.

Physical Quantities	**Symbol**	**MKS**	**Gaussian**
Length	ℓ	1 meter (m)	10^2 centimeter (cm)
Mass	m	1 kilogram (kg)	10^3 gram (gm)
Time	t	1 second (s)	1 second (s)
Force	F	1 newton (N)	10^5 dynes
Work, Energy	W, U	1 joule (J)	10^7 ergs
Power	P	1 watt (W)	10^7 ergs/s
Charge	q	1 coulomb (coul)	3×10^9 statcoulomb
Charge Density	ρ	1 coul/m^3	3×10^3 statcoul/cm^3
Current	I	1 ampere (coul/s)	3×10^9 statampere
Current Density	J	1 amp/m^2	3×10^5 statamp/cm^2
Electric Field	E	1 volt/m	$\frac{1}{3}\times10^{-4}$ statvolt/cm
Electric Potential	V	1 volt	$\frac{1}{300}$ statvolt
Polarization	P	1 coul/m^2	3×10^5 dipole moment/cm^3
Displacement	D	1 coul/m^2	$12\pi\times10^5$ statcoul/cm^2 (statvolt/cm)
Conductivity	σ	1 mho/m	9×10^9 1/s
Resistance	R	1 ohm	$\frac{1}{9}\times10^{-11}$ s/cm
Capacitance	C	1 farad	9×10^{11} cm
Magnetic flux	F	1 weber	10^8 gauss cm^2 (maxwell)
Magnetic induction	B	1 weber/m^2	10^4 gauss
Magnetic field	H	1 amp-turn/m	$4\pi\times10^3$ oersted
Magnetic Induction	M	1 amp/m	$\frac{1}{4\pi}\times10^{-3}$ magnetic moment/ cm^3
Inductance	L	1 henry	$\frac{1}{9}\times10^{-11}$
For accurate works, all factors of 3 in the coefficients should be replaced by 2.99793.			

Aerosols Charging and Their Kinetics

Most aerosol particles carry some electrical charges. In an electric field of strength **E**, a charged particle experiences a force, which is given by

$$\mathbf{F}_\mathbf{E} = q\mathbf{E} . \tag{4}$$

Here

$$q = ne \tag{5}$$

where e is the elementary unit of charge and n is the number of elementary units carried by the particle. ($e = 1.6 \times 10^{-19}$ coulombs (MKS)= 4.8×10^{-10} Statcoulombs (electrostatic cgs unit).)

Particle Mobility

Particle mobility is defined as the velocity that it will acquire when subjected to an electric field of unit strength. Equating the drag and Columbic forces, it follows that

$$qE = 3\pi\mu Ud / C_c \tag{6}$$

For $E = 1$, the mobility becomes

$$u = Z^p = \frac{qC_c}{3\pi\mu d} \tag{7}$$

Particle Charging

Aerosol particle can be electrified by a number of different processes. Among the important ones are: Direct ionization, static electrification, electrolytic effects (liquids of high dielectric constant exchange ions with metals), contact electrification, spray electrification, frictional electrification, collisions with ion, diffusion, and field charging.

Boltzmann Equilibrium Charge Distribution

Under equilibrium conditions, the aerosols approach the Boltzmann charge distribution. Accordingly, the fraction of particles having n unites of charge of one sign, $f(n)$, is given by

$$f(n) = \frac{\exp\{-n^2 e^2 / dkT\}}{\sum_{n=-\infty}^{\infty} \exp\{-n^2 e^2 / dkT\}} . \tag{8}$$

For $d > 0.02\mu m$, the approximate expression for the charge distribution becomes

$$f(n) = \sqrt{\frac{e^2}{dkT\pi}} \exp\{-\frac{n^2 e^2}{dkT}\}. \tag{9}$$

For a room temperature of $20^o C$, (9) reduces to

$$f(n) = \frac{0.24}{\sqrt{d\pi}} \exp\{-\frac{0.05 n^2}{d}\} \quad d \text{ in } \mu m \quad \text{for} \quad d > 0.02\mu m. \tag{10}$$

The average number of charge per particle is given by

$$\bar{n} = \sum_{-\infty}^{\infty} |n| f(n) \approx \int_{-\infty}^{\infty} |n| f(n) dn \approx \sqrt{\frac{dkT}{\pi e^2}} \quad \text{for} \quad d > 0.02\mu m. \tag{11}$$

Electric Field of a Point Charge

The electric field strength of a point charge at a distance r is given by

$$E = \frac{\gamma q}{4\pi r^2}, \tag{12}$$

where $\gamma = 4\pi/\varepsilon$ for cgs or electrostatic unit with ε being the dielectric constant of the medium. For air, $\varepsilon = 1$, $\gamma = 4\pi$, and $\gamma = \frac{1}{\varepsilon_o \varepsilon}$ (for MKS).

Coulomb's Law

Combining (5) and (12), we find

$$F = q' E = \frac{\gamma q' q}{4\pi r^2}, \quad (\text{cgs}) \tag{13}$$

which is Coulomb's law for forces between two charge particles. Note that in the MKS units, $\gamma = \frac{1}{\varepsilon_o \varepsilon}$, and the permittivity (dielectic constant) of free space, $\varepsilon_o = 8.859 \times 10^{-12} \frac{\text{amp} - \text{sec}}{\text{volt} - \text{meter}}$.

$$F = \frac{q'q}{\varepsilon r^2}(9 \times 10^9) \ , \qquad \text{(MKS)}. \tag{14}$$

Field Charging

When a particle is in an electric field, the particle acquires charges due to collisions with ions, which are moving along the lines of force that intersect the particle surface. This process is known as field charging. The number of charges accumulated by the particle is given by

$$n = [\frac{\pi e Z_i n_{i\infty} t}{\pi e Z_i n_{i\infty} t + 1}](1 + \frac{2(\varepsilon_p - 1)}{\varepsilon_p + 2})\frac{Ed^2}{4e} \ , \ \text{(cgs)}, \tag{15}$$

where t is time, Z_i is the mobility of ions, $n_{i\infty}$ is the ions concentration far from the particle, e is the electronic charge, ε_p is the dielectric constant of the particle, and E is the electric field intensity.

For sufficiently large time,

$$n_\infty = [1 + \frac{2(\varepsilon_p - 1)}{\varepsilon_p + 2}]\frac{Ed^2}{4e} \qquad \text{as } t \to \infty. \tag{16}$$

The factor $[1 + \frac{2(\varepsilon_p - 1)}{\varepsilon_p + 2}]$ is a measure of distortion of the electrostatic field by the particle. The factor varies between 1 and 3 for $\varepsilon_p = 1 \text{ to } \infty$. For dielectric materials, the values of ε_p usually are less than 10. ε_p is 4.3 for quartz and 2.3 for benzene.

Diffusion Charging

Even in the absence of an external electric field, particles exposed to an ion cloud become charged. Ions will collide with the particle due to their thermal motions. As the particle becomes charged, it will repel ions of the same sign and leads to a non-homogenous distribution of ions in its neighborhood. After time t, the number of charge acquired by the particle is given by

$$n = \frac{dkT}{2e^2}\ln[1 + (\frac{2\pi}{m_i kT})^{1/2} n_{i\infty} d e^2 t] \ , \qquad \text{(cgs)} \tag{17}$$

where m_i is the ionic mass and $(\frac{kT}{2\pi m_i})^{1/2}$ is the mean velocity with which the ions strike the particle surface.

As $t \to \infty$, the number of charge (according to the formula) also approaches infinity, which is not correct. However, for typical values of $n_{i\infty} t \simeq 10^8$ ion sec/cm^3, the equation gives reasonable results, especially for particles, which are smaller than the mean free path.

Electrical Precipitation

Electrical precipitation is used widely in power plants for removing particles from discharging smokestacks. In the most common type, the dusty gas flows between parallel plate electrodes. The particles are charged by ions generated in a corona discharge surrounding rods or wires suspended between the plates. Figure 1 shows schematics of electrostatic precipitators.

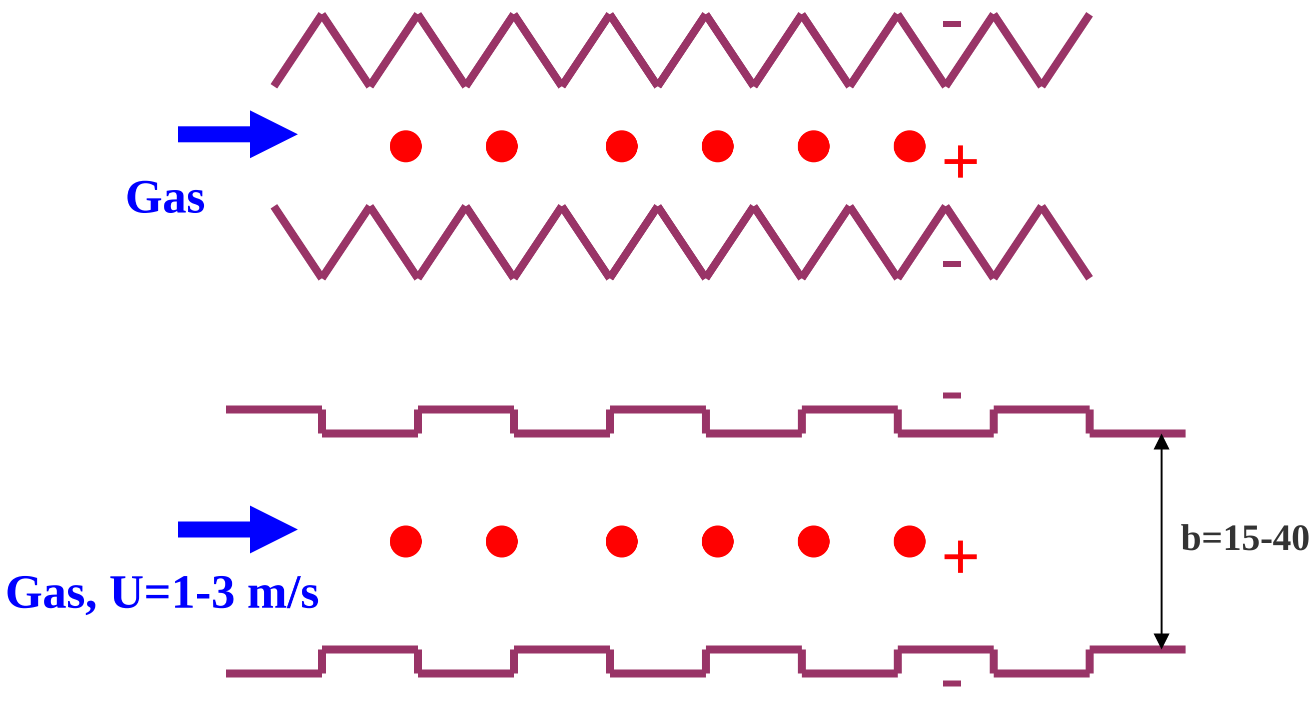

Figure 1. Schematics of electrostatics precipitators.

In the precipitators the cloud of electrons and negative ions moves towards the collecting electrodes. Particles are charged by field or diffusion charging, depending on their size. The Reynolds number is of the order of 10^4 or greater and hence the flow is turbulent. The flux normal to the collecting plate is given by

$$\bar{J} = -(D + \nu^T)\frac{\partial \bar{C}}{\partial y} - u_e \bar{C}, \tag{18}$$

where U_e is the particle migration velocity towards the plate given by

$$u_e = \frac{EqC_c}{3\pi\mu d}. \tag{19}$$

Here, q is the charge on the particle and E is the intensity of the electric field.

For u_e being a constant, assuming $\bar{J}$ remains constant, it follows that

$$\frac{\bar{C}}{\bar{C}_\infty} = \frac{1 - \exp\{-\int_0^y \frac{u_e dy}{D + \nu^T}\}}{1 - \exp\{-\int_0^\infty \frac{u_e dy}{D + \nu^T}\}} \tag{20}$$

and

$$\bar{J}(x) = \frac{-u_e \bar{C}_\infty}{1 - \exp\{-u_e \int_0^\infty \frac{dy}{D + \nu^T}\}} = -\frac{u_e \bar{C}_\infty}{1 - \exp\{-\frac{u_e}{u_D}\}} \tag{21}$$

where

$$u_D = \frac{1}{\int_0^\infty \frac{dy}{D + \nu^T}}, \tag{22}$$

is the deposition velocity due to combined Brownian and turbulent diffusion.

For $u_e << u_D$, it follows that $|J| = u_D \bar{C}_\infty$ and deposition is controlled by diffusion. When $u_D << u_e$ as is the case for electrical precipitators, $|J| = u_e \bar{C}_\infty$. For a length dx length,

$$bUd\bar{C}_\infty = 2Jdx = -2u_e \bar{C}_\infty dx. \tag{23}$$

Hence,

$$\bar{C}_\infty = \bar{C}_{\infty 0} \exp\{-\frac{2u_e x}{bU}\}. \tag{24}$$

For a length L,

$$\frac{\overline{C}_{\infty L}}{\overline{C}_{\infty 0}} = \exp\{-\frac{2u_e L}{bU}\}. \tag{25}$$

Figure 2 shows typical collection efficiencies of electrostatic precipitators.

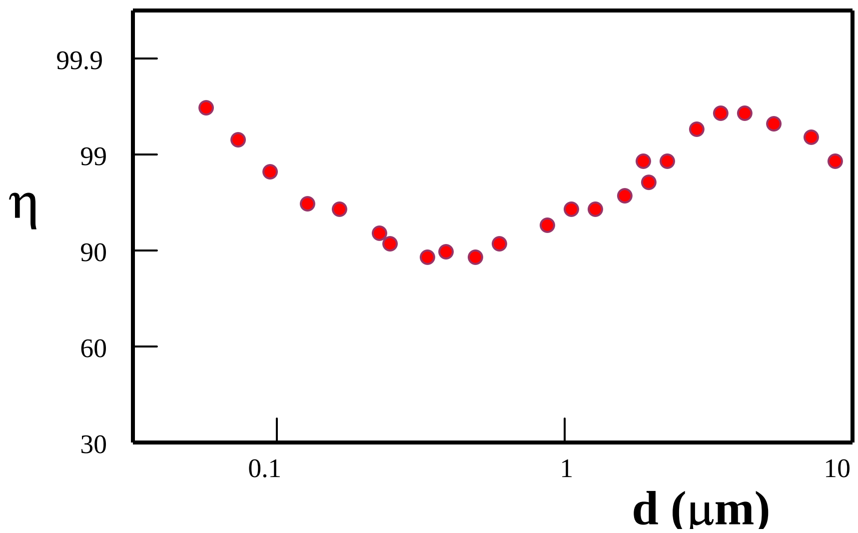

Figure 2. Variation of collection efficiency of an electrostatic precipitator.

Field Strength

The electrostatic potential V satisfies a Poisson equation given as

$$\nabla^2 V = -\gamma \rho_s, \tag{26}$$

where ρ_s is the space charge per unit volume. Then electric field is given as

$$\mathbf{E} = -\nabla V \tag{27}$$

Particle Drift in an Electric Field

The equation of motion of a particle in an electric field is given by

$$m\frac{d\mathbf{u}^p}{dt} = \mathbf{F_D} + \mathbf{F_G} + \mathbf{F_E}, \tag{28}$$

where the forces are due to drag, gravity, and electric field. Equation (28) may be restated as

$$\tau\frac{d\mathbf{u^p}}{dt} = \mathbf{u^f} - \mathbf{u^p} + \tau\mathbf{g} - \mathbf{E}q\frac{\tau}{m}. \tag{29}$$

Equation (29) may be restated as

$$\tau\frac{d\mathbf{u^p}}{dt} + \mathbf{u^p} = \mathbf{u_o} - \mathbf{E}q\frac{\tau}{m}, \tag{30}$$

where $\mathbf{u_o} = \mathbf{u}^f + \tau\mathbf{g}$. Suppose $\mathbf{E}$ is in the same direction as $\mathbf{u_o}$, non-dimensionalizing with respect to u_o, we find

$$\tau\frac{d\hat{u}^p}{dt} + \hat{u}^p = 1 - \Gamma, \tag{31}$$

where

$$\hat{u}^p = \frac{u^p}{u_0}, \qquad \Gamma = \frac{Eq\tau}{mu_0} \tag{32}$$

If $|\Gamma| >> 1$, then the electrical force dominates. In this case, if the inertia is also neglected, we find

$$\hat{U}^p = -\Gamma \text{ or } u^p = -Eq\frac{\tau}{m}. \tag{33}$$

In addition to the gravitational and electrical forces, particles could experience forces induced by temperature gradient, concentration gradient, and electromagnetic radiation. These are discussed in the next section.

Thermophoretic Force

The presence of temperature gradient imposes thermophoretic force on the particle, which is given as

$$\underline{F}_t = -\frac{8}{15} d^2 \frac{\kappa^f}{\overline{|v^{f\,\prime}|}} \nabla T \exp\{-\frac{\hat{\theta} d}{2\lambda}\}, \quad \text{for} \quad 0.25 \le K_n \le \infty, \; M << 1. \quad (34)$$

where $\bar{c}^f = \overline{|v^{f\,\prime}|}$ is the mean thermal speed of the gas and

$$\hat{\theta} = 0.9 + 0.12\alpha_m + 0.21\alpha_m (1 - \frac{\alpha_t \kappa^f}{2\kappa^p}), \quad (35)$$

$$\bar{c}^f = \overline{|v^{f\,\prime}|} = (\frac{8kT}{\pi m^f})^{1/2}, \quad (36)$$

for monatomic gases. Here α_m and α_t are the momentum and thermal accommodation coefficients and κ^f and κ^p are the thermal conductivity of gas and the particle.

The accommodation coefficients vary between zero and infinity. For mono-atomic gases, $\alpha_t \approx \alpha_m$. Typical values are listed in Table 2,

Table 2. Variations of momentum and thermal accommodation factors.

System	α_m	α_t
Air on Brass	1.00	0.91-0.94
Air on Oil	0.895	
Air on Glass	0.89	
Air on Ag_2O	0.92	

For the continuum limit, the thermophoretic force is given as

$$\mathbf{F_t} = \frac{-3\pi\mu d^2 C_{tm} K_n [(\frac{\kappa^f}{\kappa^p} + C_t K_n)(1 + 1.33 C_m \kappa_n) - 1.33 C_m \kappa_n] \nabla T}{(1 + 3C_m \kappa_n)(1 + 2\frac{\kappa^f}{\kappa^p} + 2C_t K_n)}, \quad 0 \le K_n \le 0.2 \quad (37)$$

where

$$C_{tm} = \frac{3\mu}{4\rho^f T\lambda}$$

$$C_t = \hat{P}_t \frac{(2-\alpha_t)}{\alpha_t}, \qquad 1.87 \le \hat{P}_t \le 2.48 \tag{38}$$

$$C_m = \hat{P}_m \frac{(2-\alpha_m)}{\alpha_m} \approx 1.2 \qquad 1 \le \hat{P}_m \le 1.274$$

A simpler expression for thermophoretic force given by

$$\mathbf{F_t} = -\frac{9\pi\mu\nu d}{T_0}\nabla T\left[\frac{1}{2+\frac{\kappa^p}{\kappa^f}}\right] \quad \text{for} \quad K_n \ge 1 \tag{39}$$

is more commonly used.

Photophoretic Force

The force generated by the electromagnetic radiation is referred to as the photophoretic force. For large K_n (free molecular) flow regimes, the photophoretic force is given by

$$\mathbf{F}_p = \frac{-\pi d^3 p\mathbf{I}}{48(\frac{1}{2\rho^f\sqrt{\overline{v^{f\,\prime 2}}}R + \kappa_p T})}, \; K_n \to \infty, \tag{40}$$

where p is the gas pressure, $\mathbf{I}$ is the radiation flux, and R is the gas constant.

Diffusiophoretic Force

Non-uniformity in the composition of a gas mixture results in a diffusion (diffusiophoretic) force acting on the suspended particle. This force is proportional to the negative of concentration gradient and has a similar form as the thermophoretic force described earlier.