

1. (30 points) Suppose that the turbulent diffusivity near the wall is given as

$$v_T^+ = 0.02y^+$$

Assume that the concentration is C_0 at $y^+ = 30$ and vanishes at the stopping distance of particle at $y^+ = \tau^+$.

- i) Evaluate the expression for the deposition velocity and the corresponding concentration profile. Use your formula and evaluate the deposition velocity for 10 μm particles.
 - ii) Compare your solution for the deposition velocity with that given by the empirical equation of Wood for 10 μm particles. (Assume a kinematic viscosity of $1.5 \times 10^{-5} \text{ m}^2/\text{s}$, $S=2000$, a shear velocity of 0.3 m/s and a temperature of 300 K.) ($\lambda=0.07\mu\text{m}$)
3. (30 points) Consider a cloud of 9 μm quartz particles with a concentration of 10^5 particles per cm^3 .
- i) Find the average absolute number of charge for the equilibrium Boltzmann distribution.
 - ii) Determine the number of particles that will carry 5 positive charges. How many will carry no charges in this case?
 - iii) Find the number of charges on the particle for the saturation condition in a field of 1000 volt/cm.
 - iv) Find the electrostatic precipitation velocity for a field of 1000 volt/cm for the mixtures with the average absolute charge distribution, and for the quartz particles that carry the saturation field charge. (The density of air is 1.2 kg/m^3 , the density ratio of quartz particle to air is 2000, and for quartz $\epsilon_p = 4.3$.)

4. (40 points) The instantaneous velocity and vorticity in turbulent flows satisfy the following equations:

$$\frac{\partial u'_i}{\partial t} + U_j \frac{\partial u'_i}{\partial x_j} + u'_j \frac{\partial U_i}{\partial x_j} + u'_j \frac{\partial u'_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p'}{\partial x_j} + \nu \frac{\partial^2 u'_i}{\partial x_j \partial x_j} + \overline{\frac{\partial u'_i u'_j}{\partial x_j}}$$

$$\frac{\partial \omega'_i}{\partial t} + U_j \frac{\partial \omega'_i}{\partial x_j} = -u'_j \frac{\partial \Omega_i}{\partial x_j} - u'_j \frac{\partial \omega'_i}{\partial x_j} + \Omega_j d'_{ij} + \omega'_j D_{ij} + \omega'_j d'_{ij} + \nu \frac{\partial^2 \omega'_i}{\partial x_j \partial x_j} + \overline{\frac{\partial \omega'_i u'_j}{\partial x_j}} - \overline{\omega'_j d'_{ij}}$$

- i) Develop a transport equation for velocity-vorticity correlations ($\overline{u'_i \omega'_j}$).
- ii) Identify various terms.
- iii) Evaluate the order of terms in terms of u, λ, Λ .
- iv) Propose a closure model.