

1. Express the following equations in vector (Gibbs) notation:

$$\mathbf{a}_i = 5\mathbf{b}_i, \quad \mathbf{a} = b_i \mathbf{c}_i, \quad a_i = \epsilon_{ijk} b_j c_k.$$

2. Suppose v_i is the velocity at a point in a fluid. Show that $T_{ij} = v_i v_j$ is a tensor.

3. Acceleration is given as

$$\mathbf{a} = \frac{D\mathbf{v}}{Dt} = \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v}.$$

Express the acceleration in indicial notation.

4. Vorticity is given as

$$\boldsymbol{\omega} = \nabla \times \mathbf{v}.$$

Compute $\boldsymbol{\omega}$ for a rotating fluid with

$$\mathbf{v} = \boldsymbol{\Omega} \times \mathbf{r},$$

where $\boldsymbol{\Omega}$ is a constant angular velocity.

5. Using indicial notation show that

$$\nabla \times \nabla \times \mathbf{u} = \nabla \nabla \cdot \mathbf{u} - \nabla^2 \mathbf{u}.$$

6. Show that

$$\epsilon_{ijk} \epsilon_{njk} u_n = 2u_i.$$