1) (Problem 1.3, Tennekes and Lumley) Large eddies in turbulent flows have a length scale $\ell$ and a time scale $t(\ell) = \ell/u$. The smallest eddies have a length scale of $\eta$, a velocity scale of $L$, and time scale $\tau$. Estimate the characteristic velocity $U(r)$ and characteristic time $t(r)$ of eddies of size $r$, where $r$ is in the range of $\eta < r < \ell$. (Note that in this range $U(r)$ and $t(r)$ are determined by $\varepsilon$ and $r$.) Show that your results agree with the known results at $r = \eta$ and $r = \ell$.

Find an express for the energy spectrum of turbulence, $E(\kappa) = \frac{v^2(\kappa)}{\kappa}$.

2) (Problem 3.1, Tennekes and Lumley) Estimate the characteristic velocity of eddies whose size is equal to the Taylor microscale $\lambda$. (See problem 1) Show that eddies of this size dissipates little energy.

3) Derive the energy equation for the Burger model

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = \frac{\partial^2 u}{\partial x^2}$$

Assume $u = U + u'$. Discuss the meaning of the terms in the energy equation.

4) Consider a turbulent flow between two parallel plates. Derive the expression for the velocity in the viscous sublayer and in the log region. Assume the two solutions should match at $y^* = 10$. Assuming that the log profile is valid up to the channel centerline, find the expression for the friction coefficient

$$C_f = \frac{\tau_o}{\frac{1}{2} \rho U_c^2} = 2 \left( \frac{u^*}{U_c} \right)^2$$