ME 637 - Particle II

Features of Turbulence

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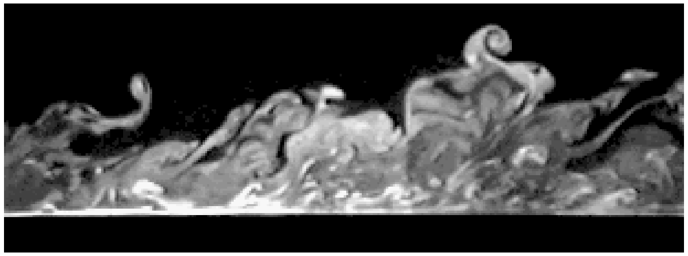
Turbulence

Outline

- Definition of Turbulence
- Features of Turbulence
- Kolmogorov Scales
- Energy Cascade

Turbulence



Turbulence

G.I. Taylor & von Karman (1937)

> "Turbulence is an irregular motion which in general makes its appearance in fluids, gaseous or liquid, when they flow past solid surfaces or even when neighboring streams of the same fluid flow past or over one another."

Turbulence

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Hinze (1959)

> "Turbulent fluid motion is an irregular condition of flow in which the various quantities show a random variation with time and space coordinates, so that statistically distinct average values can be discerned."

Features of Turbulence

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- Random, Irregular, Chaotic → Statistical Approach
- Highly Diffusive → Rapid Mixing
- Highly Dissipative → Needs a Source of Energy

Features of Turbulence

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- High Vorticity Fluctuation → Vortex Stretching
- Continuum Phenomena → Kolmogrov Scale >> λ
- Rotational and Three-D

Features of Turbulence

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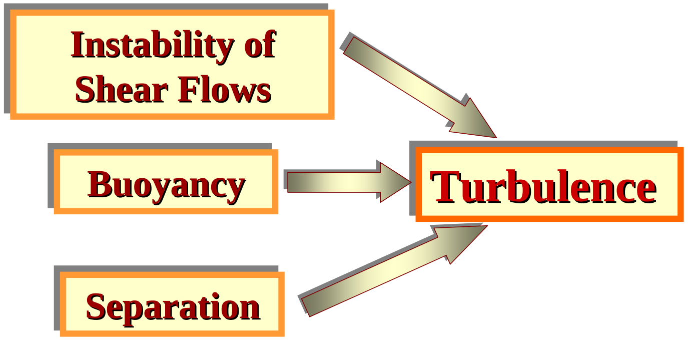
Manifestation of the Flow

Mean Field Fluid is non-Newtonian, viscoelastic, memory-dependent, multi-temperature, nonlocal, and contains several internal variables

Origin of Turbulence



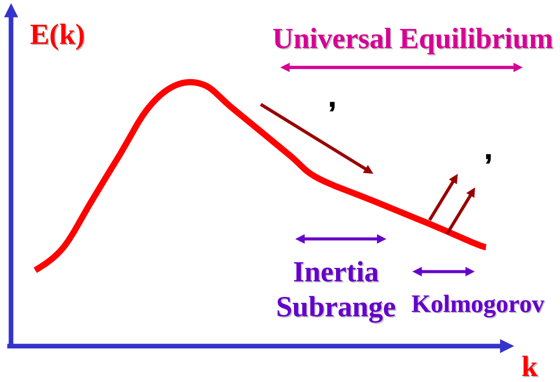
Instability of Shear Flows

Buoyancy

Separation

Turbulence

Energy Spectrum of Turbulence



Turbulence Dissipation

Dissipation ≅ Production → $\varepsilon = \frac{u^3}{\Lambda}$

Direct Viscous Dissipation → $\upsilon\left(\frac{\partial U}{\partial y}\right)^2 \sim \upsilon\frac{u^2}{\Lambda^2}$

$$\frac{\upsilon\left(\frac{\partial U}{\partial y}\right)^2}{\varepsilon} = \frac{\text{Large Eddy Direct Viscous Dissip.}}{\text{Turbulence Dissipation Rate}} = \frac{\upsilon\frac{u^2}{\Lambda^2}}{\frac{u^3}{\Lambda}} = \frac{1}{\mathrm{Re}_\Lambda}$$

Reynolds Number $\mathrm{Re}_\Lambda = \frac{u\Lambda}{\nu}$

Kolmogorov Scales

Turbulent Motions

- Large-scale turbulent motion is roughly independent of viscosity.
- The small-scale motion is controlled by viscosity.
- Small-scale motions are statistically independent of large-scale turbulent fluctuations (and/or mean motions).

Kolmogorov Scales

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Kolmogorov (Universal Equilibrium Theory)

Small-scale turbulence is in equilibrium (independent of large-scale) and is controlled solely with dissipation rate, ε, and viscosity, ν.

Length Scale

$$\eta \equiv \left(\frac{\nu^3}{\varepsilon}\right)^{\frac{1}{4}}$$

Time Scale

$$\tau \equiv \left(\frac{\nu}{\varepsilon}\right)^{\frac{1}{2}}$$

Velocity Scale

$$\upsilon \equiv (\nu\varepsilon)^{\frac{1}{4}}$$

Kolmogorov Scales

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Length Scale

$$\frac{\eta}{\Lambda} \sim \mathrm{Re}_\Lambda^{-\frac{3}{4}}$$

Time Scale

$$\frac{\tau U}{\Lambda} \sim \mathrm{Re}_\Lambda^{-\frac{1}{2}}$$

Velocity Scale

$$\frac{\upsilon}{U} \sim \mathrm{Re}_\Lambda^{-\frac{1}{4}}$$

For Water

$$\varepsilon = 1\frac{\mathrm{W}}{\mathrm{kg}} \longrightarrow \eta \approx 30\,\mu\mathrm{m}$$

Kolmogorov Inertia Subrange Spectrum

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Kolmogorov -5/3 Law

For eddies much smaller than the energy containing eddies and much larger than η, turbulence is controlled by the dissipation rate, ε, and wave number k.

$$E(k) \sim \frac{\upsilon_k^2}{k} \sim \frac{\left[\left(\frac{\varepsilon}{k}\right)^{\frac{1}{3}}\right]^2}{k} \sim \varepsilon^{\frac{2}{3}} k^{-\frac{5}{3}}$$

Energy Cascade

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Richardson Verse

"Big whirls have little whirls that feed on their velocity. Little whirls have lesser whirls, and so on to viscosity."

Estimates of Turbulence Scales

For a Pipe of Diameter d

Energy containing Eddies

$$\ell_e = 0.05 d \, \mathrm{Re}^{-\frac{1}{8}}$$

$$f_e \simeq \frac{u^*}{\ell_e} = 20 \frac{u^*}{d} \mathrm{Re}^{\frac{1}{8}} \approx 4 \frac{\overline{U}}{d}$$

Kolmogorov Eddies

$$\eta = 4 d \, \mathrm{Re}^{-0.78}$$

$$f_k = \frac{\upsilon}{\eta} = 0.06 \frac{\overline{U}}{d} \mathrm{Re}^{0.56} = \frac{17 u^{*2} / \nu}{\mathrm{Re}^{0.44}}$$

Estimates of Turbulence Scales

Most dissipative Eddies (5η)

$$\ell_d = 20 d \, \mathrm{Re}^{-0.78}$$

$$f_d = 0.02 \frac{\overline{U}}{d} \mathrm{Re}^{0.56} = \frac{6 u^{*2} / \nu}{\mathrm{Re}^{0.44}}$$

Largest Eddies

$$d/2$$

$$f_L = \frac{2u^*}{d} = 0.4 \frac{\overline{U}}{d} \mathrm{Re}^{-\frac{1}{8}}$$

Example

For a pipe with d= 5cm, V= 1.8m/s and Re=10^5

Table of eddy size and frequencies.

Eddies	Size	Frequency
Largest Eddies	25 mm	3.5 Hz
Energy Containing Eddies	0.6 mm	140 Hz
Most Dissipative Eddies	0.125 mm	450 Hz
Kolmogorov Eddies	0.025 mm	1300 Hz

Example- Kolmogorov Scale in Inertial Sublayer

$$-\overline{u'v'} \approx u^{*2}$$

$$\frac{\partial U}{\partial y} \approx \frac{u^*}{\kappa y}$$

$$\mathrm{Pr\,oduction} = -\overline{u'v'} \frac{\partial U}{\partial y} = \frac{u^{*3}}{\kappa y}$$

$$\varepsilon = \frac{u^{*3}}{\kappa y}$$

Kolmogorov Length Scale

$$\eta = \left(\frac{\nu^3}{\varepsilon} \right)^{\frac{1}{4}}$$

$$\eta^+ = \frac{\eta u^*}{\nu}$$

$$\eta^+ = (\kappa y^+)^{\frac{1}{4}}$$

Kolmogorov Scale in Inertial Sublayer

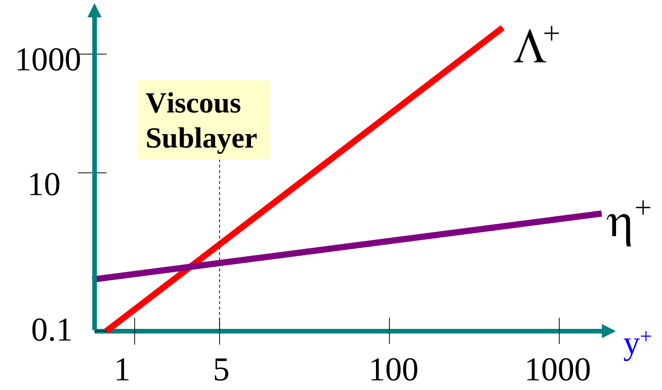
Turbulence Macroscale

$$\Lambda = \kappa y$$

$$\Lambda^+ = \frac{\Lambda u^*}{\nu} = \kappa y^+$$

y^+	$\eta^+ = (\kappa y^+)^{\frac{1}{4}}$	$\Lambda^+ = \kappa y^+$
5	1.2	2
12	1.5	4
40	2	16
200	3	80
1000	4.5	400

Turbulence Scales



Turbulence

Concluding Remarks

- Definition of Turbulence
- Features of Turbulence
- Kolmogorov Scales
- Energy Cascade

Turbulence

Thank you!

Questions?