ME 637 - Particle -II

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Aerosols Charging and Their Kinetics

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Aerosols Charging

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Outline

- Electrostatics
- Particle Charging
- Charged Particle Kinetic
- Thermophoretic Force
- Photophoretic Force

Electrodynamics, Maxwell's Equations

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	Gaussian Units	MKS Units
Coulomb's Law	$\nabla \cdot \mathbf{D} = 4\pi\rho_e$	$\nabla \cdot \mathbf{D} = \rho_e$
Ampere's Law	$\nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}$	$\nabla \times \mathbf{H} = \mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}$
Faraday's Law	$\nabla \times \mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = 0$	$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$
Absence of Free Magnetic Poles	$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \mathbf{B} = 0$

Electrodynamics

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Continuity Equation → $\nabla \mathbf{J} + \frac{\partial \rho_e}{\partial t} = 0$

Constitutive Equations, Free space

$\mathbf{D} = \varepsilon_0 \mathbf{E}$ $\varepsilon_0 = 1$ $\varepsilon_0 = \frac{10^7}{4\pi c^2} = 8.854 \times 10^{-12}\ \mathrm{Coul/Volt \cdot m}$

$\mathbf{B} = \mu_0 \mathbf{H}$ $\mu_0 = 1$ $\mu_0 = 4\pi \times 10^{-7}$

Ohm's Law → $\mathbf{J} = \sigma \mathbf{E}$ $c = (\varepsilon_0 \mu_0)^{-1/2}$

Electrodynamics

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Physical Quantities	Symbol	MKS	Gaussian
Length	ℓ	1 meter (m)	10^2 centimeter (cm)
Mass	m	1 kilogram (kg)	10^3 gram (gm)
Time	t	1 second (s)	1 second (s)
Force	F	1 newton (N)	10^5 dynes
Work, Energy	W, U	1 joule (J)	10^7 ergs
Power	P	1 watt (W)	10^7 ergs/s
Charge	q	1 coulomb (coul)	3×10^9 statcoulomb
Charge Density	ρ	1 coul/m^3	3×10^3 statcoul/cm^3
Current	I	1 ampere(coul/s)	3×10^9 statampere
Current Density	J	1 amp/m^2	3×10^5 statamp/cm^2
Electric Field	E	1 volt/m	$\frac{1}{3}\times10^{-4}$ statvolt/cm
Electric Potential	V	1 volt	$1/300$ statvolt

Electrodynamics

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Physical Quantities	Symbol	MKS	Gaussian
Polarization	P	1 coul/m^2	3×10^5 dipole moment/cm^3
Displacement	D	1 coul/m^2	$12\pi\times10^5$ statcoul/cm^2 statvolt/cm)
Conductivity	σ	1 mho/m	9×10^9 1/s
Resistance	R	1 ohm	$(1/9)\times10^{-11}$ s/cm
Capacitance	C	1 farad	9×10^{11} cm
Magnetic flux	F	1 weber	10^8 gauss cm^2 (maxwell)
Magnetic induction	B	1 weber/m^2	10^4 gauss
Magnetic field	H	1 amp-turn/m	$4\pi\times10^3$ oersted
Magnetic Induction	M	1 amp/m	$\frac{1}{4\pi}\times10^{-3}$ mag. moment/cm^3
Inductance	L	1 henry	9×10^{11}

For accurate works, all factors of 3 in the coefficients should be replaced by 2.99793.

Aerosols Charging and Their Kinetics

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Most aerosol particles carry some electrical charges

Coulomb Force → $F_E = qE$ $\quad q = ne$

Electric Charge

$e = 1.6\times10^{-19}$ coul $\qquad e = 4.8\times10^{-10}$ statcoul

Particle Mobility $\quad qE = 3\pi\mu Ud / C_c$ → $u = Z^p = \frac{qC_c}{3\pi\mu d}$

Particle Charging

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Boltzmann Equilibrium Charge Distribution →

$$f(n) = \frac{\exp\{-n^2e^2/dkT\}}{\sum_{n=-\infty}^{\infty}\exp\{-n^2e^2/dkT\}}$$

$$f(n) = \sqrt{\frac{e^2}{dkT\pi}}\exp\{-\frac{n^2e^2}{dkT}\} \qquad d > 0.02\mu$$

$$f(n) = \frac{0.24}{\sqrt{d\pi}}\exp\{-\frac{0.05n^2}{d}\} \qquad d > 0.02\mu$$

Particle Charging

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Average Number of Charge

$$\bar{n} = \sum_{-\infty}^{\infty} |n| f(n) \approx \int_{-\infty}^{\infty} |n| f(n) dn \approx \sqrt{\frac{dkT}{\pi e^2}}$$

$d > 0.02\mu$

$\bar{n} = \approx 2.36\sqrt{d}, \quad d(\mu m)$

Point Charge → $E = \frac{\gamma q}{4\pi r^2}$ $\gamma = 4\pi/\varepsilon$ cgs

Air → $\varepsilon = 1$ $\gamma = 4\pi$ $\gamma = \frac{1}{\varepsilon_o \varepsilon}$ MKS

Coulomb's Law

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$$F = q'E = \frac{\gamma q' q}{4\pi r^2}$$

$$\gamma = \frac{1}{\varepsilon_o \varepsilon}$$

Permittivity → $\varepsilon_o = 8.859 \times 10^{-12} \frac{amp - sec}{volt - meter}$

Coulomb's Law → $F = \frac{q' q}{\varepsilon r^2} (9 \times 10^9)$

Field Charging

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$$n = [\frac{\pi e Z_i n_{i\infty} t}{\pi e Z_i n_{i\infty} t + 1}](1 + \frac{2(\varepsilon_p - 1)}{\varepsilon_p + 2})\frac{Ed^2}{4e}$$ cgs

$$n_\infty = [1 + \frac{2(\varepsilon_p - 1)}{\varepsilon_p + 2}]\frac{Ed^2}{4e} \quad \text{as } t \to \infty$$ cgs

$\varepsilon_p = 4.3$ for Quartz

Diffusion Charging

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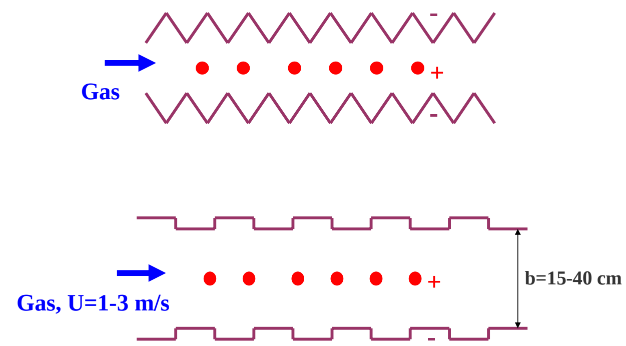
$$n = \frac{dkT}{2e^2} \ln[1 + (\frac{2\pi}{m_i kT})^{1/2} n_{i\infty} d e^2 t]$$

$n_{i\infty} t \simeq 10^8$ ion sec/ cm^3

Field and Diffusion Charging

Diameter	Number of Units of Charge		
μm	Diffusion	Field	Combined
0.01	0.276	0.0007	0.277
0.02	0.672	0.0027	0.675
0.03	1	0.0062	1.12
0.05	2.1	0.02	2.12
0.1	5	0.07	5.1
0.5	32	2	34
1	69	7	76
2	149	28	177
3	234	63	297
5	414	174	588
10	889	694	1583
20	1901	2778	4679
50	5162	17361	22523
100	10954	69583	80537
200	23121	277778	300899
300	35767	625000	660767

Electrical Precipitation



Electrical Precipitation

$$\bar{J} = -(D+\nu^T)\frac{\partial \bar{C}}{\partial y} - u_e \bar{C}$$

$$\frac{\bar{C}}{\bar{C}_\infty} = \frac{1-\exp\{-\int_0^y \frac{u_e dy}{D+\nu^T}\}}{1-\exp\{-\int_0^\infty \frac{u_e dy}{D+\nu^T}\}}$$

$$u_e = \frac{EqC_c}{3\pi\mu d}$$

$$\bar{J}(x) = \frac{-u_e \bar{C}_\infty}{1-\exp\{-u_e\int_0^\infty \frac{dy}{D+\nu^T}\}} = -\frac{u_e \bar{C}_\infty}{1-\exp\{-\frac{u_e}{u_D}\}}$$

$$u_D = \frac{1}{\int_0^\infty \frac{dy}{D+\nu^T}}$$

Electrical Precipitation

$u_e << u_D$ → $|J| = u_D \bar{C}_\infty$

$u_D << u_e$ → $|J| = u_e \bar{C}_\infty$

$$bUd\bar{C}_\infty = 2Jdx = -2u_e \bar{C}_\infty dx$$

$$\bar{C}_\infty = \bar{C}_{\infty 0}\exp\{-\frac{2u_e x}{bU}\}$$

$$\frac{\bar{C}_{\infty L}}{\bar{C}_{\infty 0}} = \exp\{-\frac{2u_e L}{bU}\}$$

Collection Efficiency

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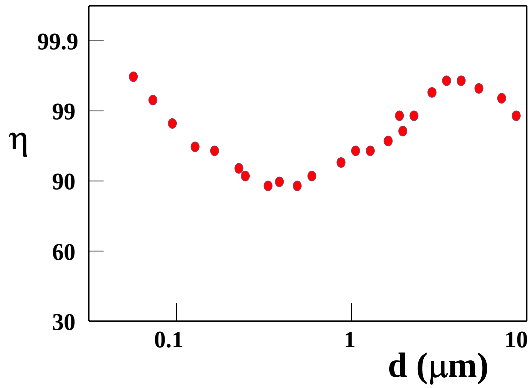

Particle Drift in an Electric Field

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Equation of Motion

$$m\frac{d\mathbf{u}^p}{dt} = \mathbf{F}_D + \mathbf{F}_G + \mathbf{F}_E$$

$$\tau\frac{d\mathbf{u}^p}{dt} = \mathbf{u}^f - \mathbf{u}^p + \tau\mathbf{g} - \mathbf{E}q\frac{\tau}{m}$$

$$\tau\frac{d\mathbf{u}^p}{dt} + \mathbf{u}^p = \mathbf{u}_o - \mathbf{E}q\frac{\tau}{m}$$

$$\mathbf{u}_o = \mathbf{u}^f + \tau\mathbf{g}$$

Particle Drift in an Electric Field

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For $\mathbf{u}_o \parallel \mathbf{E}$

$$\tau\frac{d\hat{u}^p}{dt} + \hat{u}^p = 1 - \Gamma$$

$$\hat{u}^p = \frac{u^p}{u_0} \qquad \Gamma = \frac{Eq\tau}{mu_0}$$

For $|\Gamma| >> 1$, neglecting inertia

$$\hat{U}^p = -\Gamma \qquad u^p = -Eq\frac{\tau}{m}$$

Thermophoretic Force

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$$\underline{F}_t = -\frac{8}{15}d^2\frac{\kappa^f}{\overline{|v^{f\prime}|}}\nabla T\exp\{-\frac{\hat{\theta}d}{2\lambda}\}$$

$$0.25 \le K_n \le \infty$$

$$M << 1$$

$$\bar{c}^f = \overline{|v^{f\prime}|} = (\frac{8kT}{\pi m^f})^{1/2}$$

$$\hat{\theta} = 0.9 + 0.12\alpha_m + 0.21\alpha_m(1 - \frac{\alpha_t\kappa^f}{2\kappa^p})$$

α_m = Momentum Accommodation

α_t = Thermal Accommodation

Thermophoretic Force

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Continuum Regime $0 \le K_n \le 0.2$

$$\mathbf{F}_t = \frac{-3\pi\mu d^2 C_{tm} K_n [(\frac{\kappa^f}{\kappa^p} + C_t K_n)(1 + 1.33 C_m \kappa_n) - 1.33 C_m \kappa_n]\nabla T}{(1 + 3C_m \kappa_n)(1 + 2\frac{\kappa^f}{\kappa^p} + 2C_t K_n)}$$

$$\mathbf{F}_t = -\frac{9\pi\mu\nu d}{T_0}\nabla T\left[\frac{1}{2 + \frac{\kappa^p}{\kappa^f}}\right]$$

$\kappa_n \ge 1$

Photophoretic Force

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$$\mathbf{F}_p = \frac{-\pi d^3 p \mathbf{I}}{48(\frac{1}{2\rho^f\sqrt{\overline{v^{f\,\prime\,2}}}R + \kappa_p T})}$$

$K_n \to \infty$

Diffusiophoretic Force

Electrical Forces

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Coulomb Force — Image Force — Polarization Force

$$F_e = qE - \frac{q^2}{16\pi\varepsilon_o y^2} + \frac{qEd^3}{16y^3} - \frac{3\pi\varepsilon_o E^2 d^6}{128 y^4}$$

Dipole Interactions

Deposition Velocity

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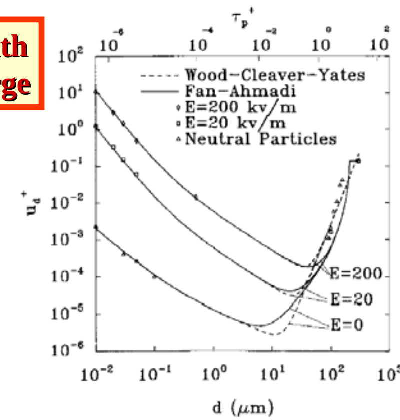
Particles with Single Charge



Deposition Velocity

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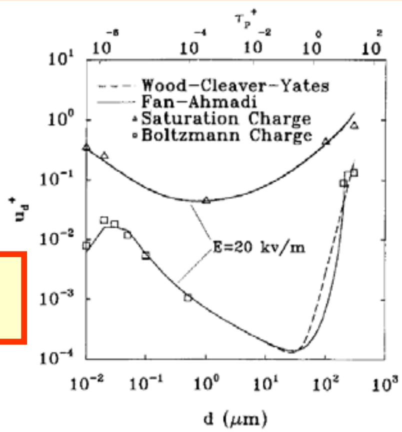
Saturation Charge

Boltzmann Charge



Concluding Remarks

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- **Electrostatics**
- **Particle Charging**
- **Charged Particle Kinetic**
- **Thermophoretic Force**
- **Photophoretic Force**

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Thank you!

Questions?