ME 326 - Intermediate Fluid Mechanics

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Flows With Friction

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Compressible Flows

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Outline

- Compressible Flow Regimes
  - Thermodynamics
  - Speed of Sound & Mach Number
- Isentropic Flows with Area Change
  - Variations with Mach number
- Shock Waves
  - Nozzle and Diffusers
- Flows with Heat Transfer
- **Flows with Friction**

Fanno Line

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Flows with Friction (No Heat Transfer)

Energy Equation

$$(h_1 + \frac{V_1^2}{2}) = (h_2 + \frac{V_2^2}{2}) = h_o$$

Mass

$$\rho_1 V_1 = \rho_2 V_2$$

Momentum

$$P_1 - P_2 + R/A = \rho V(V_2 - V_1)$$

Fanno Line

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Equation of State

$$h = h(S, \rho)$$

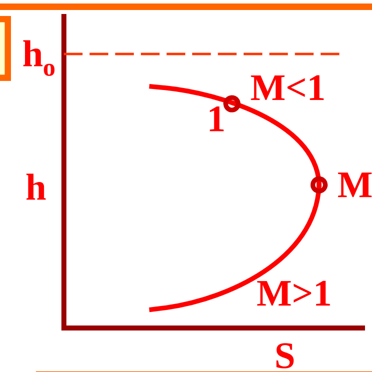
$$\rho = \rho(S, P)$$

Select a v_2

Mass $\Rightarrow \rho_2$

Energy $\Rightarrow h_2$

State $\Rightarrow S_2$



Point 2 could be any point on Fanno Line

Adiabatic Flows with Friction

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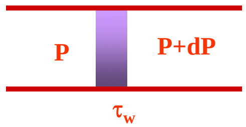
Energy Equation



1 → $C_p dT + VdV = 0$

Continuity Equation

$\rho V = \text{Const.}$ → 2 → $\dfrac{d\rho}{\rho} + \dfrac{dV}{V} = 0$

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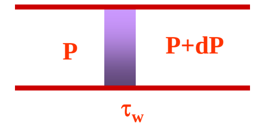
Momentum Equation



$$PA - (P + dP)A - \tau_w \pi dx = \rho VA(V + dV - V)$$

3 → $dP + 4\tau_w \pi \dfrac{dx}{D} + \rho VdV = 0$

$$\tau_w = \frac{f}{4}\frac{\rho V^2}{2}$$

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Equation of State

$$P = \rho RT$$

4 → $\dfrac{dP}{P} = \dfrac{d\rho}{\rho} + \dfrac{dT}{T}$

Mach Number

$$M^2 = \frac{V^2}{kRT}$$

5 → $\dfrac{2dM}{M} = \dfrac{2dV}{V} - \dfrac{dT}{T}$

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5 Equations for 5 Unknowns

$$\frac{dM}{M}, \frac{dV}{V}, \frac{dT}{T}, \frac{dP}{P}, \frac{d\rho}{\rho}$$

→ $\dfrac{dP}{P} = -kM^2 \dfrac{1 + (k-1)M^2}{2(1 - M^2)} \dfrac{fdx}{D}$

→ $\dfrac{dM^2}{M^2} = kM^2 \dfrac{1 + \dfrac{k-1}{2}M^2}{(1 - M^2)} \dfrac{fdx}{D}$

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$$\frac{dT}{T} = -\frac{k(k-1)M^4}{2(1-M^2)}\frac{fdx}{D}$$

$$\frac{d\rho}{\rho} = -\frac{dV}{V} = -\frac{kM^2}{2(1-M^2)}\frac{fdx}{D}$$

$$\frac{dP_o}{P_o} = \frac{d\rho_o}{\rho_o} = -\frac{kM^2}{2}\frac{fdx}{D}$$

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The effects of friction on the properties of Fanno flow

Property	Subsonic	Supersonic
Velocity, V	Increase	Decrease
Mach number, Ma	Increase	Decrease
Stagnation temperature, T_0	Constant	Constant
Temperature, T	Decrease	Increase
Density, ρ	Decrease	Increase
Stagnation pressure, P_0	Decrease	Decrease
Pressure, P	Decrease	Increase
Entropy, s	Increase	Increase

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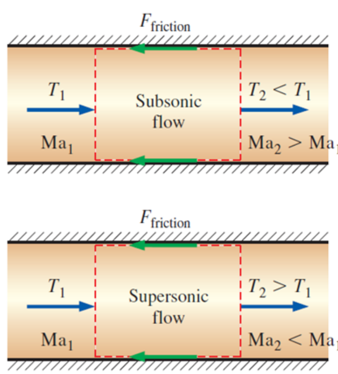
$$\frac{dT}{T} = -\frac{k(k-1)M^4}{2(1-M^2)}\frac{fdx}{D}$$



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Mach Number-Distance Relationship

$$\frac{\bar{f}L^*}{D_h} = \frac{1-M^2}{kM^2} + \frac{k+1}{2k}\ln\frac{(k+1)M^2}{(2+(k-1)M^2)}$$

f is found from Moody Diagram or Haaland Equation

$$\frac{1}{\sqrt{f}} = 1.8\log\left(\frac{6.9}{R_e} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right)$$

Full Rough, high Re

$$1/\sqrt{f} = 2\log(\varepsilon/3.7D)$$

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Property Ratios

$$\frac{P}{P^*}=\frac{1}{M}\left(\frac{k+1}{2+(k-1)M^2}\right)^{1/2}$$

$$\frac{T}{T^*}=\frac{(k+1)}{2+(k-1)M^2}$$

$$\frac{V}{V^*}=\frac{\rho^*}{\rho}=M\left(\frac{k+1}{2+(k-1)M^2}\right)^{1/2}$$

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Property Ratios

$$\frac{P_o}{P_o^*}=\frac{\rho_o}{\rho_o^*}=\frac{1}{M}\left(\frac{2+(k-1)M^2}{k+1}\right)^{(k+1)/[2(k-1)]}$$

$$\frac{T_o}{T_o^*}=1$$

Flow with Friction

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Table 3 To2/To1=1

M	T/T*	P/P_*	P_o/P_{o*}	V/V*	fL*/D
0	1.2	∞	∞	0	∞
0.02	1.199	54.77	28.9	0.022	1773.4
0.5	1.143	2.14	1.34	0.535	1.069
1	1	1	1	1	0
3	0.428	0.22	4.23	1.964	0.522

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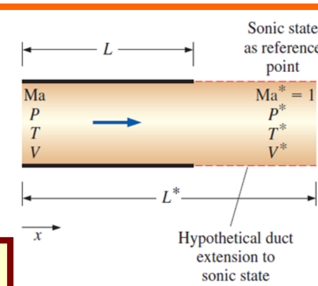
Length to sonic state



$$\frac{\bar{f}L}{D_h}=\left(\frac{\bar{f}L^*}{D_h}\right)_1-\left(\frac{\bar{f}L^*}{D_h}\right)_2$$

Isothermal Flows with Friction

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Mach Number-Distance Relationship

$$\frac{\bar{f}L^*}{D} = \frac{1-kM^2}{kM^2} + \ln(kM^2)$$

Mass Flow Rate

$$\dot{m} = \left(\frac{P_1^2 - P_2^2}{RT[fL/D + 2\ln(P_1/P_2)]} \right)^{1/2} A$$

Compressible Flows

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Concluding Remarks

- Compressible Flows with Friction
- Fanno Line
- Mach Number-Distance Relationship
- Mass Flow Rate
- Pipe Lines with Friction

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Thank you!

Questions?