ME 326 - Intermediate Fluid Mechanics

Compressible Flows

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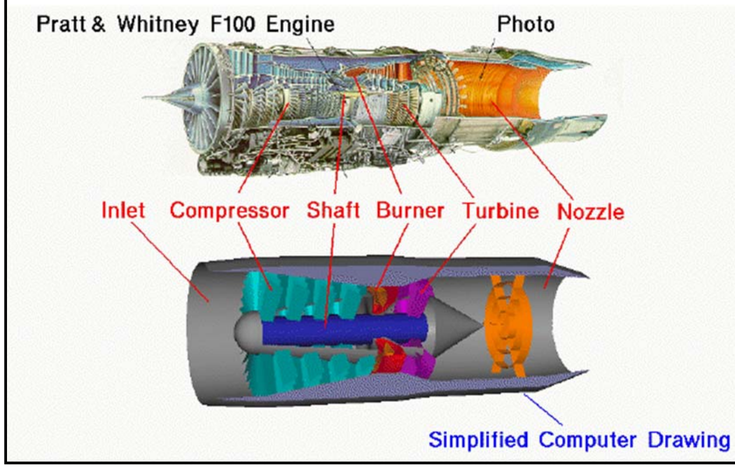
Gas Turbine Engines



Compressible Flows

Outline

- Compressible Flow Regimes
  - Thermodynamics
  - Speed of Sound & Mach Number
- Compressible Flows with Area Change
  - Variations with Mach number
- Shock Waves
  - Nozzle and Diffusers
- Flows with Friction
- Flows with Heat Transfer

Compressible Flows

Mach Number

$$M = \frac{V}{c}$$

- Compressible Flow Regimes
  - Incompressible Flow, M<0.3
  - Subsonic Flow, 0.3<M<0.8
  - Transonic Flow, 0.8<M< 1.2
  - Supersonic Flow, 1.2<M< 3
  - Hypersonic Flow, 3<M

Thermodynamics

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Enthalpy

$$h = \hat{u} + P/\rho = \hat{u} + P\upsilon$$

Heat Capacity

$$c_\upsilon = \left.\frac{\partial \hat{u}}{\partial T}\right|_\upsilon \qquad c_p = \left.\frac{\partial h}{\partial T}\right|_p$$

First Law

$$\delta Q = \delta W + d\hat{U}$$

First Law (reversible)

$$TdS = Pd\upsilon + d\hat{u}$$

$$TdS = dh - \upsilon dP$$

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Idea Gas

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Equation of State →

$$P = \rho RT \qquad \hat{u} = \hat{u}(T)$$

$$h = \hat{u} + RT \qquad R = \Lambda / M_{gas}, \quad \Lambda = 8314\, m^2/s^2K$$

Heat Capacity

$$c_\upsilon = \frac{d\hat{u}}{dT} \qquad c_p = \frac{dh}{dT}$$

$$c_\upsilon = \frac{R}{k-1} \qquad c_p = \frac{Rk}{k-1}$$

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Idea Gas

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Gas Constant

$$R = c_p - c_v$$

Air

$$R_{air} = 287 m^2/s^2K$$

$$c_v = 718\, m^2/s^2K$$

$$c_p = 1005\, m^2/s^2K$$

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Combined First and Second Laws

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$$dS = \frac{Pd\upsilon}{T} + \frac{d\hat{u}}{T} = R\frac{d\upsilon}{\upsilon} + c_v\frac{dT}{T}$$

$$dS = \frac{dh}{T} - \frac{\upsilon dP}{T} = c_p\frac{dT}{T} - R\frac{dP}{P}$$

$$S_2 - S_1 = c_v \ln(\frac{T_2}{T_1}) - R\ln(\frac{\rho_2}{\rho_1})$$

$$S_2 - S_1 = c_p \ln(\frac{T_2}{T_1}) - R\ln(\frac{P_2}{P_1})$$

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Isoentropic Flows

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Pressure Ratio

$$\frac{P_2}{P_1} = \left[\frac{T_2}{T_1}\right]^{\frac{k}{k-1}} = \left[\frac{\rho_2}{\rho_1}\right]^{k}$$

Temperature Ratio

$$\frac{T_2}{T_1} = \left[\frac{P_2}{P_1}\right]^{\frac{k-1}{k}}$$

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Energy Equation

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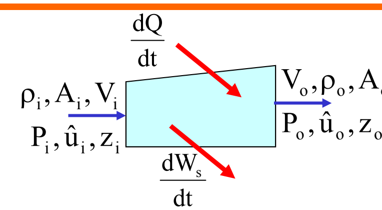
$$\sum \text{Energy Out} = \sum \text{Energy In}$$



$$\frac{dQ}{dt} + \sum_i \dot{m}_i \left(\frac{P_i}{\rho_i} + \hat{u}_i + \frac{V_i^2}{2} + gz_i\right) =$$

$$\frac{dW_s}{dt} + \sum_o \dot{m}_o \left(\frac{P_o}{\rho_o} + \hat{u}_o + \frac{V_o^2}{2} + gz_o\right)$$

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Energy Equation

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Simplified Energy Equation

$$\frac{dQ}{dm} + \left(h_i + \frac{V_i^2}{2}\right) = \left(h_o + \frac{V_o^2}{2}\right)$$

Isoentropic Flows

$$\left(h_i + \frac{V_i^2}{2}\right) = \left(h_o + \frac{V_o^2}{2}\right)$$

Stagnation Enthalpy

$$h_o = \left(h_i + \frac{V_i^2}{2}\right)$$

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Stagnation Properties

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Stagnation Temperature

$$c_p T_o = c_p T + \frac{V^2}{2}$$

$$T_o = T + \frac{V^2}{2c_p}$$

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Speed of Sound

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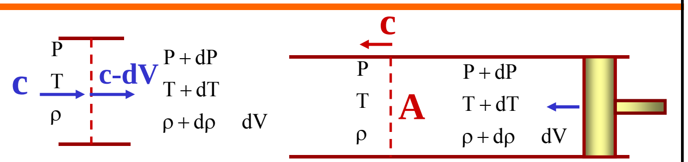

Continuity

$$\rho c A = (c - dV)(\rho + d\rho)$$

$$dV = c\frac{d\rho}{\rho}$$

Momentum

$$PA - (P + dP)A = \rho c A(c - dV - c)$$

$$dP = \rho c dV$$

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Speed of Sound

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Eliminating dV

$$c^2 = \frac{dP}{d\rho} = \left.\frac{\partial P}{\partial \rho}\right|_s$$

Speed of Sound

$$c^2 = kRT = k\frac{P}{\rho}$$

Bulk Modulus

$$K = -\upsilon\left.\frac{\partial P}{\partial \upsilon}\right|_s = \rho\left.\frac{\partial P}{\partial \rho}\right|_s = \rho c^2$$

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Concluding Remarks

- Compressible Flow Regimes
- Review of Thermodynamics
- Speed of Sound
- Mach Number

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Thank you!

Questions?

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