1. Given a velocity vector field $\underset{\sim}{v} = x_2 \underset{\sim}{i} + x_1 \underset{\sim}{j}$.
a) Find the path lines, the stream lines and the streak lines. b) Determine the regions for which the inverse motion does not exist.
c) Find the deformation gradient, $\nabla \underset{\sim}{v}$ and the deformation rate tensors.

2. Prove that $\frac{dJ}{dt} = J\, v_{k,k}$

3. Based on a thermodynamical argument derive the constitutive equation for the stress tensor for a viscoelastic fluid. Obtain the equations of motion for such a fluid.
Hint: Assume $\psi = \psi(T, \rho, d_{kl}, e_{kl})$, assume also $e_{kl} = \frac{1}{2}(u_{k,l} + u_{l,k})$ = linearized strain tensor is such that $\dot{e}_{kl} = d_{kl}$ within the limits of the linear theory

4. Starting with the basic equations of motion:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underset{\sim}{v}) = 0,$$

$$\rho \frac{d\underset{\sim}{v}}{dt} = -\nabla \hat{p} + \mu \nabla^2 \underset{\sim}{v} - \rho_0 \beta (T - T_0) \underset{\sim}{g},$$

$$\rho c_p \frac{dT}{dt} = \frac{dp}{dt} + k \nabla^2 T + \Phi,$$

and using the dimensionless variables defined in the class

$$R_e = \frac{\rho_0 U_0 L}{\mu}, \quad P_r = \frac{\mu c_p}{k}, \quad E_c = \frac{U_0^2}{c_p \Delta T_0}, \quad G_r = \frac{g \beta \rho_0^2 L^3 \Delta T_0}{\mu^2}$$