Normal Processes

A stochastic process $X(t)$ is said to be normal if $X(t_1)$, $X(t_2),\ldots,$ $X(t_n)$ are jointly normal for any n and any t_1, $t_2,\ldots,$ t_n. The statistics of a normal are fully determined in terms of its mean $\eta(t)$ and its autocorrelation $R(t_1,t_2)$ (or autocovariance $C(t_1,t_2)$). The first-order density is given as

$$f(x;t)=\frac{1}{\sqrt{2\pi C(t,t)}}e^{-\frac{[x-\eta(t)]^2}{2C(t,t)}}.$$

The nth order joint density is given by

$$f(x_1,\ldots,x_n;t_1,\ldots,t_n)=\frac{1}{(2\pi)^{\frac{n}{2}}|\Lambda|^{\frac{1}{2}}}\exp\left\{-\frac{1}{2}\sum_i\sum_j\Lambda_{ij}^{-1}(x_i-\eta(t_i))(x_j-\eta(t_j))\right\},$$

where $\mathbf{\Lambda}$ is the matrix of covariance function defined as

$$\mathbf{\Lambda}=\left[C(t_i,t_j)\right] \text{ and } |\Lambda|=\det|\Lambda|.$$

Note that the linear combinations of normal processes (or random variables) are also normal processes.