ME 529 **EXAM 2** **Sample Exam-2**

1) Given that n(t) is a Gaussian White noise process with a constant spectrum S_o, $R_{nn}(\tau) = 2\pi S_o \delta(\tau)$, and

$$\frac{d^3Y}{dt^3} + \frac{d^2Y}{dt^2} - \frac{dY}{dt} - Y = n(t) + \frac{dn}{dt}$$

a) Evaluate the power spectrum of Y process, $S_{YY}(\omega)$.
b) Evaluate $R_{YY}(\tau)$, $E\{Y^2\}, E\{Y^6\}$.

2) Suppose V is a uniform random variable with

$$f_V(v) = \begin{cases} 1 & 0 \le v \le 1 \\ 0 & \text{otherwise} \end{cases}$$

We define two stochastic processes, X(t) = u(t-V), Y(t) = δ(t-V). (Assume $0 \le t \le 1$.)

a) Evaluate the expected value of X and Y.
b) Evaluate the $R_{XX}(t_1,t_2)$, $R_{YY}(t_1,t_2)$, $R_{XY}(t_1,t_2)$. (Assume $0 \le t_1 \le 1$ and $0 \le t_2 \le 1$.)

3) Let

$$Z(t) = \int_0^t \tau^2 X(\tau) d\tau$$

with $R_{XX}(t_1,t_2) = |t_1 - t_2|$. Find $E\{Z^2(t)\}$.

4) Given that

$$\frac{dX}{dt} + Y = n(t)$$
$$\frac{dY}{dt} - X = \frac{dn}{dt}$$

a) Find $S_{YY}(\omega)$, $S_{XX}(\omega)$, $R_{XX}(\tau)$ and $R_{YY}(\tau)$.
b) Evaluate $E\{X^2\}$, and $E\{X^{250}\}$.
c) Find the expression for the probability density function for random process X(t).

Table of Fourier Transform

$f(t)$	$\bar{f}(\omega) = \int_{-\infty}^{+\infty} e^{-i\omega t} f(t)dt$
$e^{-\alpha\|t\|}$	$\frac{2\alpha}{\omega^2 + \alpha^2}$
$e^{-\alpha\|t\|}\cos(\omega_o t\}$	$\frac{2\alpha(\alpha^2 + \omega_o^2 - \omega^2)}{(\alpha^2 + \omega_o^2 - \omega^2)^2 + 4\alpha^2\omega^2}$
1	$2\pi\delta(\omega)$
$\delta(t)$	1
$\cos(\omega_o t\}$	$\pi\{\delta(\omega + \omega_o) + \delta(\omega - \omega_o)\}$
$e^{-\alpha\|t\|}\{\cos(\beta t) + \frac{\alpha}{\beta}\sin(\beta\|t\|)\}$	$\frac{4\alpha(\alpha^2 + \beta^2)}{(\omega^2 - \alpha^2 - \beta^2)^2 + 4\alpha^2\omega^2}$
$e^{-\alpha\|t\|}(1 + \alpha\|t\|)$	$\frac{4\alpha^3}{(\omega^2 + \alpha^2)^2}$
$\int_{-\infty}^{+\infty} f(t-\tau)g(\tau)d\tau$	$\bar{f}(\omega)\bar{g}(\omega)$
$\begin{cases} 1 & \|t\| \le T \\ 0 & \|t\| > T \end{cases}$	$2T\frac{\sin(\omega T)}{\omega T}$
$\frac{\sin(\omega_o t)}{\pi t}$	$\begin{cases} 1 & \|\omega\| \le \omega_o \\ 0 & \|\omega\| > \omega_o \end{cases}$
$e^{-\alpha^2 t^2}$	$\frac{\sqrt{\pi}}{\alpha}e^{-\omega^2/4\alpha^2}$
$\begin{cases} 1 - \left\|\frac{t}{T}\right\| & \|t\| \le T \\ 0 & \|t\| > T \end{cases}$	$4\frac{\sin^2(\omega T/2)}{T\omega^2}$