ME529 HW Set 4

1. If *X(t)* is a Possion process, find $E\{[X(t_a)-X(t_b)][X(t_c)-X(t_d)]\}$ when the intervals (t_a, t_b) and (t_c, t_d) are overlapping.

2. In the experiment of tossing a coin, we define a stochastic process *X(t)* such that $X(t)=\begin{cases} S\,int & if \quad \zeta = Head \\ Cost & if \quad \zeta = Tail \end{cases}$ with P{Head}=P{Tail}=1/2.
   Find the density function $f_X(x,t)$ and sketch the distribution function for t= 1, ½ and ¼.

3. Given a Possion process, *X(t)*, show that if $t_1 > t_2$ and k and n are two integer, then
   $$P(X(t_1)=k+n \cap X(t_2)=k\} = e^{-\lambda t_1}\lambda^{k+n}\frac{(t_1-t_2)^n t_2^k}{n!\,k!}.$$

4. Suppose the stochastic process *X(t)* is stationary and differentiable with derivative $\dot{X}(t)$. Show that the random variables *X(t)* and $\dot{X}(t)$ are orthogonal and uncorrelated. That is, $E\{X(t)\dot{X}(t)\} = E\{X(t)\}E\{\dot{X}(t)\} = 0$.

5. The process X(t) has zero mean and orthogonal increment, and $R_{XX}(t_1,t_2) \to 0$ as $t_2 \to \infty$. Show that $R_{XX}(t_1,t_2) = R_{XX}(t_2,t_2) \quad for \quad t_1 > t_2$.

6. Suppose *X(t)* is a zero mean stochastic process with $R_{XX}(t_1,t_2) = e^{-|t_1-t_2|}$. Also suppose *Y(t)* is the solution to $\frac{dY}{dt} + Y(t) = X(t)$, with initial condition Y(0)=0. Find $E\{Y(t)\}$, $R_{XY}(t_1,t_2)$, and $R_{YY}(t_1,t_2)$.

7. Assume that in every 40 years an earthquake of magnitude 5 is expected to occur in a region. Using the Gutenberg-Richter equation, ($log_{10} N = a - bM$), estimate *a* assuming *b=1*. Find the seismic risk for occurrence of earthquakes with magnitude 3, 5 and 7 in time duration of 5 years.

8. For S_o =55.44cm^2/s^3, and $\Delta t = 0.02\,s$ numerically generate a sample white noise processes. Evaluate the mean, the variance and autocorrelation of the white noise process. Also plot the probability density function of the process.