ME529 **Homework 5**

1. The process X(t) is stationary with E{X(t)}=1, and $R_{XX}(\tau) = 1 + e^{-2|\tau|}$. Find the mean and variance of the random variable $S = \int_0^1 X(t)dt$

2. If S(w) is the power spectrum of a given real process, show that $\frac{d^2S}{d\omega^2}$ is not a power spectrum.

3. The power spectrum of the process X(t) is given by $S_{XX}(\omega) = \frac{1}{1+\omega^2}$.

   $\frac{dY}{dt} + Y = X(t), \quad -\infty < t < \infty$

   For the stationary response, find the power spectrum and auto-correlation of process Y(t).

4. Given

   $\frac{d^2Y}{dt^2} + 2\varepsilon\omega_0 \frac{dY}{dt} + \omega_0^2 Y = 2\varepsilon\omega_0 \frac{dX}{dt} + \omega_0^2 X$

   relate the power spectrum of Y(t) to that of X(t). For $S_{XX}(\omega) = S_0^{'}$, find $S_{YY}(\omega)$

5. Two random process X(t) and Y(t) are independent, and both are weakly stationary.

   (a) Find the general expression for the spectral density of Z(t)=X(t) Y(t) in terms of $S_{XX}(\omega)$ and $S_{YY}(\omega)$

   (b) Apply the general expression to the special case when $R_{XX}(\tau) = A^2 e^{-a|\tau|}, R_{YY}(\tau) = B^2 Cos(b\tau)$

6. X(t) is a stationary process. Given that

   $$\frac{d^4Y}{dt^4} - a^4 Y = \frac{d^2X}{dt^2} + \alpha^2 X, \alpha = const. \quad -\infty < t < \infty$$

   Determine the power spectrum of Y.

7. W(t) is a stationary process with $S_{WW}(\omega)=\dfrac{\beta}{\left(\omega^2+\beta^2\right)^2}$, Y(t) satisfies the following equation:

$$\int_{-\infty}^{\infty} Y(t-\tau)e^{-\beta|\tau|}d\tau = W(t), \quad -\infty < t < \infty$$

Find $S_{YY}(\omega), R_{YY}(\tau), E\{Y\}$ and σ_Y^2, Assume E{W} is given.

8. Consider $\dfrac{dZ}{dt}=W(t), \quad Z(0)=0$ where W(t) is a Wiener process with

$$R_{WW}(t_1,t_2)=Min(t_1,t_2)$$

Determine the cross correlations of Z and W, $R_{ZW}(t_1,t_2)$ *and* $R_{WZ}(t_1,t_2)$

9. Random variable A is uniform in the interval (0,T) i.e.

$$f_A(a)=\begin{cases}\dfrac{1}{T} & 0\le a\le T\\ 0 & otherwise\end{cases}$$

We define two random processes

$$X(t)=U(t-A), \quad Y(t)=\delta(t-A)$$

Assume all t's are in the interval (0,T), find

$$E\{X\}, E\{Y\}, R_{XX}\{t_1,t_2\}, R_{YY}\{t_1,t_2\}, R_{XY}\{t_1,t_2\}$$