ME 529 - Stochastics

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Definitions of Probability

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Definitions of Probability

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Outline

- Axiomatic Definition
- Relative Frequency Definition
- Classical Definition
- Independent Experiments
- Permutations and Combinations

Axiomatic (Kolmogorov 1933)

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The probability of event *a*, *P(a)* is defined subject to the following axioms

$$i)\ P(a) \geq 0$$

$$ii)\ P(S) = 1$$

$$iii)\ P(a+b) = P(a) + P(b) \quad if \quad a \cap b = o$$

Relative Frequency (Von Mises 1915)

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If event *a*, occurs n_a time in *n* trials, *P(a)* is defined as

$$P(a) = \lim_{n\to\infty} \frac{n_a}{n}$$

For mutually exclusive events *a*, and *b*,

$$P(a+b) = \lim_{n\to\infty} \frac{n_a + n_b}{n}$$

$$= \lim_{n\to\infty} \left(\frac{n_a}{n} + \frac{n_b}{n} \right) = P(a) + P(b)$$

Classical Definition

If N_a is the number of outcomes favorable to event *a*, and *N* is the total number of outcomes, *P(a)* is defined as

$$P(a) = \frac{N_a}{N}$$

For mutually exclusive events *a*, and *b*,

$$P(a+b) = \frac{N_{a+b}}{N} = \frac{N_a + N_b}{N} = \frac{N_a}{N} + \frac{N_b}{N} = P(a) + P(b)$$

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Independent Experiments

If F_1 and F_2 are independent experiments, then the probability that event a_1 occurs in F_1 and event a_2 occurs in F_2 is

$$P(a_1 \text{ and } a_2) = P(a_1)P(a_2)$$

Permutations/Cominations

Permutations

$$P^n = n! = n(n-1)\ldots\times 2\times 1$$

$$P_k^n = \frac{n!}{(n-k)!} = n\times(n-1)\times\ldots(n-k+1)$$

Combinations

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

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Concluding Remarks

- Definition of Probablity
  - Axiomatic
  - Relative Frequency
  - Classical Definition
- Independent Experiments
- Permutations and Combinations