Set is a collection of elements
Subset of a (all elements of b are element of a)
Space S – largest set
Null Set Ø – empty set

Subset: b is a subset of a if all elements of b are element of a)

Equality: a = b iff a ∈ b & b ∈ a
**Union (Sum)**

Elements of union of sets $a$ and $b$ are elements of $a$ or $b$ or both

$$a \cup b = b \cup a$$

$$a \cup a = a$$

$$a \cup 0 = a$$

$$a \cup S = S$$

$$(a \cup b) \cup c = a \cup (b \cup c) = a \cup b \cup c$$

---

**Intersection**

Elements of intersection of sets $a$ and $b$ are elements of both $a$ and $b$.

$$a \cap b = b \cap a$$

$$a \cap 0 = 0$$

$$a \cap S = S$$

$$(a \cap b) \cap c = a \cap (b \cap c) = a \cap b \cap c$$

$$a \cap (b \cup c) = (a \cap b) \cup (a \cap c)$$

If $b \subseteq a$ then $b \cap a = b$.

---

**Mutually Exclusive Sets**

Mutually exclusive sets have no common element.

$$a \cap b = 0$$

Sets $a_1, a_2, ...$ are mutually exclusive if

$$a_i \cap a_j = 0 \text{ for } i \neq j$$

---

**Complements**

Elements of complement of set $a$ are elements of $S$ which are not in $a$.

$$a \cup \bar{a} = S$$

$$a \cap \bar{a} = 0$$

$$0 = \bar{S}$$

$$\bar{S} = 0$$

If $b \subseteq a$ then $b \supseteq \bar{a}$.
**De Morgan Law**

\[ a \cup b = a \cap b \]

\[ a \cap b = a \cup b \]

---

**Difference of Two Sets**

Elements of \(a - b\) are elements of \(a\) that are not in \(b\).

\[ a - b = a \cap \overline{b} = a - a \cap b \]

\[ a = (a - b) \cup (a \cap b) \]

\[ (a - a) \cup a = a \quad \overline{a} = S - a \]

\[ a \cup a - a = 0 \]

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**Review of Set Theory**

**Summary**

- Mutually exclusive sets
- Complements
- Difference of Sets

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**Probability Space**

**Random Experiment \(\mathcal{I}\)**

By an experiment \(\mathcal{I}\) we mean a set (space) \(S\) of outcomes \(\xi\). Elements of \(S\) are outcomes or elementary events. \(S\) is a probability (sample) space. Subsets of \(S\) are called events. Space \(S\) is the sure (certain) event. Empty set \(O\) is the impossible event.

**Mutually Exclusive Events**

\[ a \cap b = 0 \]
Probability Space

Axioms of Probability

To each event $a$, a measure $P(a)$ is assigned subject to the following axioms

\[
\begin{align*}
  i) & \quad P(a) \geq 0 \\
  ii) & \quad P(S) = 1 \\
  iii) & \quad P(a \cup b) = P(a) + P(b) \quad \text{if} \quad a \cap b = 0
\end{align*}
\]

Field

Field $F$ is a nonempty class of sets such that

\[
\begin{align*}
  \text{If} & \quad a \in F \quad \Rightarrow \quad \overline{a} \in F \\
  \text{If} & \quad a \in F \quad \& \quad b \in F \quad \Rightarrow \quad a \cup b \in F
\end{align*}
\]

Borel Field

If a field has the property that if the sets $a_1, a_2, \ldots, a_n, \ldots$ belong to it then so does the set $a_1 \cup a_2 \cup a_3 \cup \ldots \cup a_n \cup \ldots$, then the field is called a Borel field. Note that the class of all subsets of $S$ is a Borel field.

Corollaries

\[
\begin{align*}
  P(0) &= 0 \\
  P(a) &= 1 - P(\overline{a}) \leq 1 \\
  \text{If} & \quad a \cap b \neq 0 \quad \Rightarrow \quad P(a \cup b) = P(a) + P(b) - P(a \cap b) \\
  \text{If} & \quad b \subset a \quad \Rightarrow \quad P(a) = P(b) + P(a \cap \overline{b}) \geq P(b)
\end{align*}
\]
1. Set $S$ of outcomes $\xi$; this set is called space or sure (certain) event
2. Borel field $F$ consisting of certain subsets of $S$ called events
3. Measure $P(a)$ assigned to every event $a$; this measure is called probability of event $a$, satisfies axioms 1 to 3

Example. Probability Experiment of Tossing a Coin, $\mathcal{F}$: $(S, F, P)$

- $S = \{h, t\}$
- $F : 0, \{h\}, \{t\}, \{h, t\}$
- $P(h) = p$, $P(\{t\}) = q$, $p + q = 1$