ME 529 - Stochastics

Review of Set Theory and Probability Space

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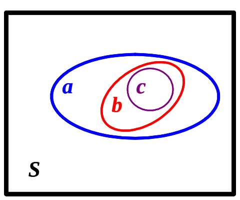
Review of Set Theory

Outline

- Definitions
- Set Operations
- Probability Space
- Borel Field
- Probability Experiment

Review of Set Theory - Definitions

Set is a collection of elements
Subset *b* of *a* (all element of *b* are element of *a*)
Space *S* – largest set
Null Set *O* – empty set



Set Operations

Subset: *b* is a subset of *a* if all element of *b* are element of *a*)

If $b \subset a$ & $c \subset b$ ⇒ $c \subset a$

Equality $a = b$ iff $a \subset b$ & $b \subset a$

Union (Sum)

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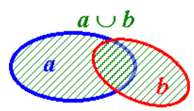
Elements of union of sets *a* and *b* are elements of *a* or *b* or both

$$a \cup b = b \cup a$$

$$a \cup a = a$$

$$a \cup 0 = a$$

$$a \cup S = S$$



$$(a \cup b) \cup c = a \cup (b \cup c) = a \cup b \cup c$$

Intersection

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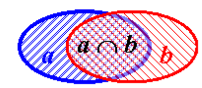
Elements of intersection of sets *a* and *b* are elements of both *a* and *b*.

$$a \cap b = b \cap a$$

$$a \cap a = a$$



$$a \cap 0 = 0$$

$$a \cap S = a$$

$$a \cup S = S$$

$$(a \cap b) \cap c = a \cap (b \cap c) = a \cap b \cap c$$

$$a \cap (b \cup c) = (a \cap b) \cup (a \cap c)$$

If $b \subset a$ ⇒ $b \cap a = b$

Mutually Exclusive Sets

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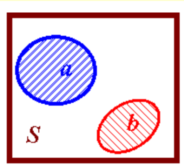
Mutually exclusive sets have no common element.

$$a \cap b = 0$$



Mutually exclusive sets.

Sets a_1, a_2 … are mutually exclusive if

$$a_i \cap a_j = 0 \quad for \quad i \neq j$$

Complements

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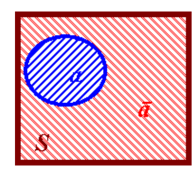
Elements of complement of set *a* are elements of *S* which are not in *a*.

$$a \cup \bar{a} = S$$

$$a \cap \bar{a} = 0$$

$$\bar{0} = S$$

$$\bar{S} = 0$$



Complements

If $b \subset a$ ⇒ $\bar{b} \supset \bar{a}$

De Morgan Law

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$$\overline{a \cup b} = \bar{a} \cap \bar{b}$$

$$\overline{a \cap b} = \bar{a} \cup \bar{b}$$

Difference of Two Sets

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Elements of *a* - *b* are elements of *a* that are not in *b*.

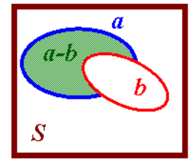
$$a - b = a \cap \bar{b} = a - a \cap b$$

$$a = (a - b) \cup (a \cap b)$$

$$(a - a) \cup a = a$$

$$\bar{a} = S - a$$

$$a \cup a - a = 0$$



Difference of Sets

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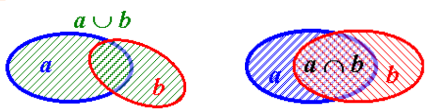
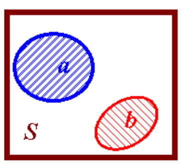
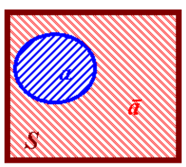
Summary

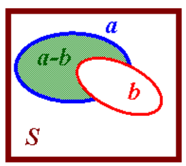




Mutually exclusive sets.



Complements



Difference of Sets

Probability Space

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Random Experiment $\Im$

By an experiment $\Im$ we mean a set (space) *S* of outcomes ξ. Elements of *S* are outcomes or *elementary events*. *S* is a probability (sample) space. Subsets of *S* are called *events*. Space *S* is the *sure (certain) event*. Empty set *O* is the *impossible event*.

Mutually Exclusive Events $a \cap b = 0$

Probability Space

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Axioms of Probability

To each event *a*, a measure *P(a)* is assigned subject to the following axioms

$i)\ P(a) \ge 0$

$ii)\ P(S) = 1$

$iii)\ P(a \cup b) = P(a) + P(b)$ **If** $a \cap b = 0$

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Corollaries

$P(0) = 0$

$P(a) = 1 - P(\bar{a}) \le 1$

If $a \cap b \ne 0$ ⇨ $P(a \cup b) = P(a) + P(b) - P(a \cap b)$

If $b \subset a$ ⇨ $P(a) = P(b) + P(a \cap \bar{b}) \ge P(b)$

Field

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Field F is a nonempty class of sets such that

If $a \in F$ ⇨ $\bar{a} \in F$

If $a \in F$ **&** $b \in F$ ⇨ $a \cup b \in F$

Corollaries

If $a \in F$ **&** $b \in F$ ⇨ $a \cap b \in F$, $a - b \in F$

Also $0 \in F$ **&** $S \in F$

Borel Field

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If a field has the property that if the sets $a_1, a_2, \ldots, a_n, \ldots$ belong to it then so does the set $a_1 \cup a_2 \cup a_3 \cup \ldots \cup a_n \cup \ldots$, then the field is called a Borel field. Note that the class of all subsets of *S* is a Borel field.

Probability Experiment F: (S, F, P)

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Probability Experiment ℑ: (S, F, P)

1. Set S of outcomes ξ; this set is called space or sure (certain) event
2. Borel field F consisting of certain subsets of S called events
3. Measure $P(a)$ assigned to every event a; this measure is called probability of event a, satisfies axioms 1 to 3

Probability Space- Example

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Example. Probability Experiment of Tossing a Coin, ℑ: (S, F, P)

$$S = \{h, t\}$$

$$F : 0, \{h\}, \{t\}, \{h, t\}$$

$$P(h) = p \quad P\{t\} = q \quad p + q = 1$$

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Concluding Remarks

- Definitions
- Set Operations
- Probability Space
- Borel Field
- Probability Experiment

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Thank you!

Questions?