ME 529 - Stochastics

Transformation of Several Random Variables

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Transformation of Random Variables

Outline

- A Function of Several Random Variables
- Two Functions of Two Random Variables
- Uncorrelated Random Variables
- Orthogonality
- Jointly Normal Random Variables

A Function of Several Random Variables

$$Z(\xi) = g[X(\xi), Y(\xi)]$$

By definition

$$F_Z(z) = P\{Z(\xi) \le z\} = P\{g(x,y) \le z\}$$

D_Z: region of xy-plane such that g(x,y) ≤ z

$$F_Z(z) = P\{(X,Y) \in D_Z\} = \iint_{D_Z} f_{XY}(x,y)\,dxdy$$

Two Functions of Two Random Variables

$$Z = g(X,Y) \qquad W = h(X,Y)$$

By definition

$$F_{ZW}(z,w) = P\{Z \le z \cap W \le w\} = P\{g(X,Y) \le z \cap h(X,Y) \le w\}$$

D_{ZW}: region of xy-plane such that g(x,y) ≤ z and h(x,y) ≤ w

$$F_{ZW} = P\{(X,Y) \in D_{ZW}\} = \iint_{D_{ZW}} f_{XY}(x,y)\,dxdy$$

Uncorrelated Random Variables

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Uncorrelated Random Variables X_i

$E\{X_i X_j\} = E\{X_i\}E\{X_j\}$ **or** $E\{(X_i-\eta_i)(X_j-\eta_j)\} = 0$ $\quad i \neq j$

$$\sigma^2_{X_1+\ldots+X_n} = \sigma^2_{X_1} + \sigma^2_{X_2} + \ldots + \sigma^2_{X_n}$$

Orthogonal Random Variables

$E\{X_i X_j\} = 0 \quad i \neq j$

If X_i are independent, they are uncorrelated

Jointly Normal Random Variables

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Jointly Normal Random Variables X, Y

$$f_{XY}(x,y) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-r^2}} \exp\left\{-\frac{1}{2(1-r^2)}\left[\frac{(x-\eta_1)^2}{\sigma_1^2} - \frac{2r(x-\eta_1)(x-\eta_2)}{\sigma_1\sigma_2} + \frac{(y-\eta_2)^2}{\sigma_2^2}\right]\right\}$$

$\eta_1 = E\{X\}$ $\quad \eta_2 = E\{Y\}$ $\quad \sigma_1^2 = E\{(X-\eta_1)^2\}$

$\sigma_2^2 = E\{(Y-\eta_2)^2\}$ $\quad r = \dfrac{E\{(X-\eta_1)(Y-\eta_2)\}}{\sigma_1\sigma_2}$

X and Y are uncorrelated if r = 0

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Concluding Remarks

- A Function of Several Random Variables
- Two Functions of Two Random Variables
- Uncorrelated Random Variables
- Orthogonality
- Jointly Normal Random Variables

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Thank you!

Questions?