ME 529 - Stochastics

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Independent Increment Prcesses

Stationary and Nonstationary Processes

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Independent Increments Stationary Prcesses

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Outline

- Independent Increment Processes
- Cross-Correlation & Cross-Covariance
- Strict-Sense Stationary Process
- Jointly SSS Processes
- Wild-Sense Stationary Process

Uncorrelated Increments Processes

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If increments $X(t_2) - X(t_1)$ and $X(t_4) - X(t_3)$ of process X(t) are uncorrelated (or independent) for any $t_1 < t_2 \le t_3 < t_4$, then X(t) is a process with uncorrelated (or independent) increments.

⇨ ***Poisson, Weiner* are independent increment**

Cross-Correlation Cross-Covariance

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Cross-Correlation

$$R_{XY}(t_1, t_2) = E\{X(t_1)Y(t_2)\} = R_{YX}(t_2, t_1)$$

Cross-Covariance

$$C_{XY}(t_1, t_2) = E\{[X(t_1) - \eta_X(t_1)][Y(t_2) - \eta_Y(t_2)]\}$$
$$= R_{XY}(t_1, t_2) - \eta_X(t_1)\eta_Y(t_2)$$

Uncorrelated & Independent Processes

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Orthogonal Processes $R_{XY}(t_1,t_2)=0$

Uncorrelated Processes $C_{XY}(t_1,t_2)=0$

Independent Processes – group of random variables X(t_1), …, X(t_n) are independent of the group Y(t_1'), …, Y(t_m'):

$$f(x_1,\ldots,x_n,y_1,\ldots,y_m)=f(x_1,\ldots,x_n)f(y_1,\ldots,y_m)$$

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Strict-Sense Stationary (SSS) Process

Statistics not affected by shift in time origin.

$$f(x_1,\ldots,x_n,t_1,\ldots,t_n)=f(x_1,\ldots,x_n,t_1+\tau,\ldots,t_n+\tau)$$

First order density is independent of time

$$f(x;t)=f(x;t+\tau) \Rightarrow f(x;t)=f(x)$$

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Similarly:

Second order density depends on time difference

$$\Rightarrow f(x_1,x_2;t_1,t_2)=f(x_1,x_2;t_1-t_2)$$

Statistics of Stationary processes

$$\Rightarrow E\{X(t)\}=\eta=const \qquad E\{X^2(t)\}=\sigma^2=const$$

$$R(t_1,t_2)=E\{X(t_1)X(t_2)\}=R(t_1-t_2)$$

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Jointly SSS Processes

Joint statistics of X(t) & Y(t) are the same as X(t + τ), Y(t + τ)

$$\Rightarrow f_{XY}(x,y;t_1,t_2)=f_{XY}(x,y,t_1-t_2)$$

$$E\{X(t_1)Y(t_2)\}=R_{XY}(t_1-t_2)$$

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Wide-Sense Stationary (WSS) Process

If mean is independent of time and autocorrelation depends on $\tau = t_1 - t_2$

$$E\{X(t)\} = \eta = const$$

$$E\{X(t+\tau)Y(t)\} = R_{XY}(\tau)$$

Jointly WSS Processes X(t) & Y(t)

If both X(t) & Y(t) are WSS, and cross-correlation depends on time difference

$$E\{X(t+\tau)X(t)\} = R(\tau)$$

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Concluding Remarks

- Cross-Correlation & Cross-Covariance
- Strict-Sense Stationary Process
- Jointly SSS Processes
- Wild-Sense Stationary Process

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Thank you!

Questions?