ME 529 - Stochastics

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Correlation & Power Spectrum of Stationary Processes

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Correlation and Spectrum

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Outline

- Autocorrelation
- Autocovariance
- Cross-Correlation
- Power Spectrum
- Cross Spectrum

Autocorrelation

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Autocorrelation of a stationary process X(t)

$$R(\tau) = E\{X(t+\tau)X(t)\}$$

For real processes

$$R(\tau) = R(-\tau)$$

Autocovariance of WSS process X(t)

$$C(\tau) = E\{(X(t)-\eta)(X(t+\tau)-\eta)\} = R(\tau) - \eta^2$$

$$\eta = E\{X(t)\}$$

Cross-Correlation

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Cross-Correlation of jointly WSS processes

$$R_{XY}(\tau) = E\{X(t+\tau)Y(t)\} = R_{YX}(-\tau)$$

Cross-Covariance

$$C_{XY}(\tau) = R_{XY}(\tau) - \eta_X \eta_Y = C_{YX}(-\tau)$$

If Z(t) = aX(t) +bY(t) and X,Y are jointly WSS

$$R_{ZZ}(\tau) = a^2 R_{XX}(\tau) + ab(R_{XY}(\tau) + R_{YX}(\tau)) + b^2 R_{YY}(\tau)$$

Properties of Correlations

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Properties of Correlations

$$R(0) \ge 0 \qquad R(\tau) \le R(0)$$

R(τ) is positive definite

$$\sum_i \sum_j a_i a_j^* R(\tau_i - \tau_j) \ge 0$$

Properties of Cross-Correlation

$$R_{XY}^2(\tau) \le R_{XX}(0) R_{YY}(0) \qquad 2R_{XY}(\tau) \le R_{XX}(0) + R_{YY}(0)$$

Power Spectrum

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Power Spectrum WSS Process Is Defined As

$$S(\omega) = \int_{-\infty}^{+\infty} e^{-i\omega t} R(\tau)\, d\tau$$

Fourier Inverse

$$R(\tau) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-i\omega\tau} S(\omega)\, d\omega$$

Power Spectrum

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Symmetry

$$S(\omega) = \int_{-\infty}^{+\infty} R(\tau)\cos\omega\tau\, d\tau = 2\int_0^{\infty} R(\tau)\cos\omega\tau\, d\tau$$

$$R(\tau) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} S(\omega)\cos\omega\tau\, d\omega = \frac{1}{\pi}\int_0^{\infty} S(\omega)\cos\omega\tau\, d\omega$$

Variance of X

$$\sigma_X^2 = R(0) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} S(\omega)\, d\omega = \frac{1}{\pi}\int_0^{\infty} S(\omega)\, d\omega$$

Cross-Spectrum

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Cross Spectrum of WSS Processes Is Defined As

$$S_{XY}(\omega) = \int_{-\infty}^{+\infty} R_{XY}(\tau) e^{-i\omega\tau}\, d\tau$$

Inversion Formula

$$R_{XY}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} S_{XY}(\omega) e^{i\omega\tau}\, d\tau$$

τ = 0

$$R_{XY}(0) = E\{X(t)Y(t)\} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} S_{XY}(\omega)\, d\omega$$

If X, Y are orthogonal

$$R_{XY}(\tau) = 0 \qquad S_{XY}(\omega) = 0$$