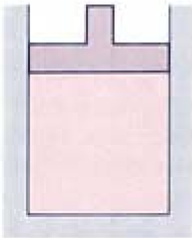
Example 1

Problem Statement: A piston/cylinder device contains one kilogram of a substance at 0.8 MPa with a specific volume of 0.2608 m^3/kg. The substance undergoes an isobaric process until its specific volume becomes 0.001115 m^3/kg. Find the total work done in the process. Is it work done on or by the system? Show the process on a P-v diagram and indicate the area that represents the work.

Solution:

Diagram: 

Given:

$m = 1$ kg
$P_1 = P_2 = 0.8$ MPa = constant
$v_1 = 0.2608\ m^3/kg$
$v_2 = 0.001115\ m^3/kg$

Find:

- Draw P-v diagram and indicate area of work
- W (specify in or out)

Assumptions: Sealed piston-cylinder (m = constant)

Governing Relations:

$$w_{OUT,12} = \int_1^2 P dv = P \int_1^2 dv = P(v_2 - v_1)$$

Property Data: None

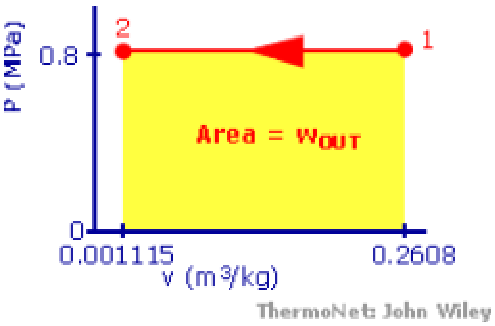
Quantitative Solution:

$$w_{12,out} = (0.8\,\text{MPa})(0.001115 - 0.2608)\frac{m^3}{kg}\frac{10^6\ N}{m^3\ MPa} = -207.7\frac{kJ}{kg}$$

$$\text{For 1 kg } W_{12,out} = (1\,\text{kg})\left(-207.7\frac{kJ}{kg}\right) = -207.7\,\text{kJ}$$

Work on System

The P-v diagram is



Note that specific volume (v) and not total volume (V) is shown on the diagram, so that the area under the curve is specific work (kJ/kg). Multiplying the specific work by the mass of the system (1 kg) yields the total work.

Discussion of Results: The process line on the P-v diagram goes from right to left, which has a negative area beneath it. A negative area on a P-v diagram indicates that work is done on the system (W_{IN}), which is consistent with our analytic answer. Comparing the sign of the area under a P-v diagram with your analytic result is a quick way to catch errors.

Example 2

Problem Statement: Air with mass of 1 kg and initially at P = 101.3 kPa is contained within a cylinder and has a specific volume of 0.850 m^3/kg. The piston within the cylinder has a diameter of 0.2 m. A cup on top of the piston is then filled at a constant rate until it contains 50 kg of water, thus compressing the air. The compression proceeds slowly and the air undergoes a process where Pv = constant.

(a) Draw a P-v diagram of the process. Label the initial state 1 and the final state 2.
(b) Calculate the work done on the gas during the process.

Solution:

Diagram:

Given:

m = 1 kg
P = 101.3 kPa
A = 0.2 m
v_1 = 0.850 m^3/kg
Pv = constant
$m_{w,1}$ = 0 kg
$m_{w,2}$ = 50 kg

Find:

(a) Draw a P-v diagram of the process. Label the initial state 1 and the final state 2.
(b) Calculate the work done on the gas during the process.

Assumptions: The piston is frictionless

Governing Relations:

$$(1) \quad W_{OUT,12} = \int P dV$$

Property Data: -

Quantitative Solution:

a) To draw the P-v diagram, we need P and v information at states 1 and 2. We are given P_1 and v_1, and we need to find P_2 and v_2. We'll start be finding P_2. A force balance on the piston yields:

$$\sum F = 0 = P_{air}A = P_{atm}A + m_{pist}g + m_{water}g$$

At the initial state, the mass of water mwater = 0 and the pressure of the air in the cylinder is due to the combined effects of the atmospheric pressure acting on the piston area and the weight of the piston. Since these remain constant throughout the process, they can be combined into one pressure Po. A general expression for the pressure of the air in the cylinder is then:

$$P_{air}A = P_oA + m_{water}g$$
$$\Rightarrow P_{air} = P_o + \frac{m_{water}g}{A}$$

The pressure at State 2 can then be calculated from:

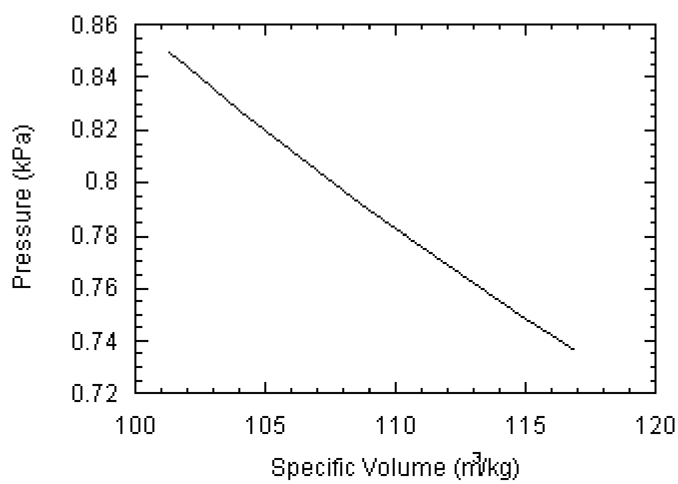
$$P_{2,air} = 101.3\,\text{kPa} + \frac{50\,\text{kg} \cdot 9.81\,\text{m}/\text{s}^2}{\frac{\pi}{4}(0.2)^2\,\text{m}^2} \cdot \frac{1\,\text{Pa}}{1\,\text{kg}/\text{m} \cdot \text{s}^2} \cdot \frac{1\,\text{kPa}}{1000\,\text{Pa}}$$
$$P_{2,air} = \underline{116.9\,\text{kPa}}$$

We can use the relationship Pv = Constant to find v_2. Since we know P_1 and v_1,

$$\text{Constant} = P_1 v_1 = (101.3\,\text{kPa})\left(0.850\,\frac{\text{m}^3}{\text{kg}}\right) = 86.105\,\frac{\text{kPa} \cdot \text{m}^3}{\text{kg}}$$

$$P_2 v_2 = \text{Constant} \Rightarrow v_2 = \frac{\text{Constant}}{P_2} = \frac{86.105\,\frac{\cancel{\text{kPa}} \cdot \text{m}^3}{\text{kg}}}{116.9\,\cancel{\text{kPa}}} = 0.736\,\frac{\text{m}^3}{\text{kg}}$$

The P-v diagram is then



Note that while this appears to be a straight line, it is in fact a curve.

b) We can calculate the work by rewriting equation (1),

$$W_{OUT,12} = \int_1^2 PdV = \int_1^2 Pd(mv) = m\int_1^2 Pdv \qquad (2)$$

To integrate (2), we need to express P as a function of v, (i.e., we need to find P = P(v)). Since we know Pv =

Constant from A to B, and we calculated the value of the Constant in a) above,

$$Pv = \text{Constant} \quad \Rightarrow \quad P = \frac{\text{Constant}}{v} \quad (3)$$

$$(3) \Rightarrow (2) \quad W_{OUT,12} = m\int_1^2 \frac{\text{Constant}}{v}dv = m \cdot \text{Constant} \cdot \ln\left(\frac{v_2}{v_1}\right)$$

$$W_{OUT,12} = (1\ \cancel{kg})\left(86.105\frac{\cancel{kPa}\cdot\cancel{m^3}}{\cancel{kg}}\right)\left(\frac{\cancel{kN}}{\cancel{kPa}\cdot\cancel{m^2}}\right)\left(\frac{kJ}{\cancel{kN}\cdot\cancel{m}}\right)\ln\left(\frac{0.736\frac{\cancel{m^3}}{\cancel{kg}}}{0.850\frac{\cancel{m^3}}{\cancel{kg}}}\right)$$

$$W_{OUT,12} = -12.4kJ$$

$$W_{IN,12} = -W_{OUT,12} = 12.4kJ$$

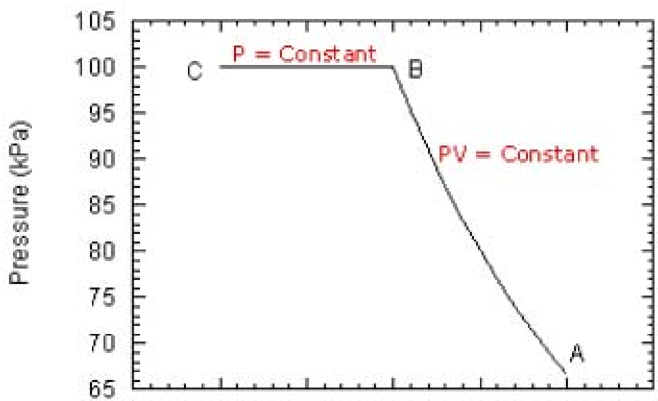
$$\underline{\underline{W_{IN,12} = -W_{OUT,12} = 12.4kJ}}$$

Discussion of Results: Work is done on the system (W_{IN}), which is consistent with the P-v diagram, where the area under the curve that indicates W_{OUT} is negative because the curve is swept out from right to left.

Example 3

Problem Statement: Find the work done, in kilojoules, by a gas going from state A to state C. At state A the volume is 3 m^3, at state B the pressure and volume are 100 kPa and 2 m^3, and at state C the volume is 1 m^3. For the process from A to B, PV = constant, and the process from B to C is **isobaric.**

Solution:



Find: The work in kJ

Assumptions: -

Governing Relations:

$$(1) \quad W_{OUT} = \int P dV$$

Property Data: -

Quantitative Solution:

To find the total work for the process, we will find the work from A to B and B to C, and then sum the two together.

Process A to B:

To integrate (1) from A to B, we need to express P as a function of V from states A to B since P is not constant, (i.e., we need to find P = P(V)). Since we know PV = constant from A to B, and we know P_B and V_B,

$$PV = \text{Constant} = P_B V_B \quad \Rightarrow \quad (2) \quad P = \frac{P_B V_B}{V}$$

$$(2) \Rightarrow (1) \quad W_{OUT,AB} = \int_A^B \frac{P_B V_B}{V} dV = P_B V_B \int_A^B \frac{1}{V} dV = P_B V_B \ln\left(\frac{V_B}{V_A}\right)$$

$$W_{OUT,AB} = (100 \cancel{kPa})(2 \cancel{m^3})\left(\frac{\cancel{kN}}{\cancel{m^2} \cdot \cancel{kPa}}\right)\left(\frac{kJ}{\cancel{kN} \cdot \cancel{m}}\right)\ln\left(\frac{2 \cancel{m^3}}{3 \cancel{m^3}}\right)$$

$$W_{OUT,AB} = -81.09 kJ$$

Process B to C:

Integrating (1) from B to C is easy since P = constant from A to B, and we know P_B (and P_C),

$$W_{OUT,BC} = \int_B^C P_B dV = P_B \int_B^C dV = P_B (V_C - V_B)$$

$$W_{OUT,BC} = (100 \cancel{kPa})(1 - 2)\cancel{m^3}\left(\frac{\cancel{kN}}{\cancel{m^2} \cdot \cancel{kPa}}\right)\left(\frac{kJ}{\cancel{kN} \cdot \cancel{m}}\right)$$

$$W_{OUT,BC} = -100.0 kJ$$

Process A to C:

The total work is then,

$$W_{OUT,AC} = W_{OUT,AB} + W_{OUT,BC} = -81.09 kJ - 100.0 kJ = -181.09 kJ$$

Since $W_{OUT,AC} = -W_{IN,AC}$,

$$\underline{\underline{W_{IN,AC} = 181.09kJ}}$$

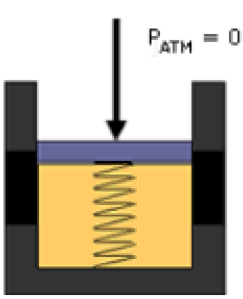
Discussion of Results: The process line on the P-v diagram goes from right to left, which has a negative area beneath it. A negative area on a P-v diagram indicates that work is done on the system (W_{IN}), which is consistent with our analytic answer. Comparing the sign of the area under a P-v diagram with your analytic result is a quick way to catch sign errors.

Example 4

Problem Statement: A weightless and frictionless piston moves 10 cm. against a gas contained in a cylinder that has an inside diameter of 15 cm. A spring is also within the cylinder, positioned as shown. The spring exerts no force on the piston in the initial position, but is touching the piston. The spring has a spring constant of 2×10^4 N/m. During the piston motion, the gas in the cylinder is maintained at 1 atm.

(a) How much work was done by the piston?
(b) What fraction of the work is done in compressing the gas?
(c) What fraction of the total work done by the piston is done in the first 5 cm of the piston travel?

Solution:

Diagram: 

Given:

x = 10 cm
d= 15 cm
k= 2 $x10_4$ N/m
P_{GAS} = 1 atm = constant

a) With pressure constant, F = kx and let dS = dX, the work done by the piston can be expressed as:

$$W_{pist} = P\int dV + k\int x\,dx$$
$$W_{pist} = P(V_2 - V_1) + \frac{1}{2}k\left(x_2^2 - x_1^2\right)$$

The first term can be rearranged using the relationship V = Ax. Since we are considering work done by the piston, x will be considered positive in the downward direction. Taking the initial position of the piston to be $x_1 = 0$ gives:

$$W_{pist} = PA(x_2 - x_1) + \frac{1}{2}k\left(x_2^2 - x_1^2\right)$$
$$W_{pist} = 101.3\frac{kN}{m^2}\cdot\pi\left(\frac{0.15}{2}\right)^2 m^2\cdot(0.1)m + \frac{1}{2}\left(2x10^4\frac{N}{m}\cdot\frac{1kN}{1000N}\right)\cdot\left(0.1^2\,m^2\right)$$
$$W_{pist} = 0.179\,kJ + 0.100\,kJ = 179\,J + 100\,J = \underline{\underline{279\,J}}$$

b) The fraction of the work that was done compressing the gas is:

$$\frac{W_{air}}{W_{air} + W_{spring}} = \frac{179J}{279J} = 0.64 = \underline{\underline{64\%}}$$

c) The fraction of the total work done in the first 5 cm of compression is:

$$W_{5cm} = \left[PA(x_2 - x_1) + \frac{1}{2}k\left(x_2^2 - x_1^2\right)\right]_0^{0.05}$$
$$W_{5cm} = 101.3\frac{kN}{m^2}\cdot\pi\left(\frac{0.15}{2}\right)^2 m^2\cdot(0.05)m + \frac{1}{2}\left(2x10^4\frac{N}{m}\cdot\frac{1kN}{1000N}\right)\cdot\left(0.05^2\,m^2\right)$$
$$W_{5cm} = 89.5J + 25J = \underline{114.5J}$$
$$\frac{W_{5cm}}{W_{tot}} = \frac{114.5J}{279J} = 0.41 = \underline{\underline{41\%}}$$

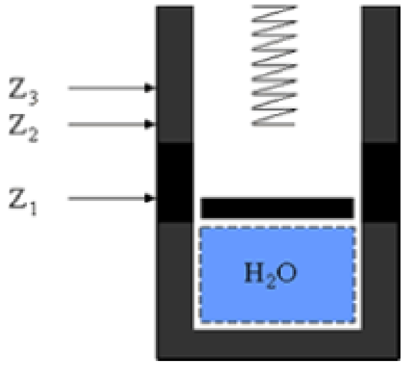
Example 5

Problem Statement: A piston-cylinder device contains 50 kg of liquid water with a specific volume of 0.0010 m^3/kg. The cross-sectional area of the piston is 0.1 m^2 and it has a mass of 500 kg. Heat is now transferred to the water, causing part of it to evaporate and expand. When the volume reaches 0.2 m^3, the piston reaches a linear spring whose spring constant is 100 kN/m. More heat is transferred to the water until the piston rises 20 cm more. Find,

- The final pressure (P_3)
- The work done during this process (W_{13})
- Also show the process on a P-V diagram.

Solution:

Diagram:



Given:

m_W = 50 kg
v_1 = 0.0010 m^3/kg
V_2 = 0.2 m^3
A_P = 0.1 m^2
m_P = 500 kg
k = 100 kN/m, spring constant from 2 to 3
$z_3 - z_2$ = 20 cm

Find:

a) P_3
b) W_{13}
c) Draw P-V diagram

Assumptions:

Frictionless Piston
Atmospheric Pressure, P_0 = 101.3 kPa
g = 9.81 m/s^2

Governing Relations:

W_{OUT} is area under a P-V curve.

Property Data: -

Quantitative Solution:

a) Between states 1 and 2 the piston is not in contact with the spring. Therefore, applying a force balance at the lower face of the piston yields (click equation for a force balance diagram)

$$PA_p = P_oA_p + m_pg$$

Solving for pressure of the system,

$$P = P_o + \frac{m_pg}{A_p}$$

The process from state 1 to state 2 is a constant pressure process, so

$$P_2 = P_1 = P_o + \frac{m_pg}{A_p}$$

$$= 101.3\,kPa + \frac{(500\,\cancel{kg})\left(9.81\,\frac{\cancel{m}}{\cancel{s^2}}\right)\left(\frac{N\cdot\cancel{s^2}}{\cancel{m}\cdot\cancel{kg}}\right)}{\left(0.1\,\cancel{m^2}\right)}\left(\frac{kPa}{1000\,\cancel{N}/\cancel{m^2}}\right) = \underline{\underline{150\,kPa}}$$

The piston comes into contact with the spring at state 2. The force balance now becomes (click equation for a force balance diagram):

$$PA_p = P_o A_p + m_p g + k(z - z_2)$$

or

$$P = \left(P_o A_p + \frac{m_p g}{A_p}\right) + \frac{k}{A_p}(z - z_2)$$

$$P_3 = P_2 + \frac{k}{A_p}(z_3 - z_2) = 150\,\text{kPa} + \frac{\left(100\,\frac{\cancel{kN}}{\cancel{m}}\right)}{0.10\,\cancel{m^2}}\left(\frac{\text{kPa}\cdot \cancel{m^2}}{\cancel{kN}}\right)(0.2\,\cancel{m})$$

$$\underline{\underline{P_3 = 350\,\text{kPa}}}$$

b) The pressure remains constant during process 1-2 and changes linearly (a straight line) during process 2-3. Then the boundary work during this process is simply the total area under the process curve (click for P-v diagrams),

$$W_{13,OUT} = \underbrace{P_1(V_2 - V_1)}_{W_{OUT,12}} + \underbrace{\left(\frac{P_2 + P_3}{2}\right)(V_3 - V_2)}_{W_{OUT,23}}$$

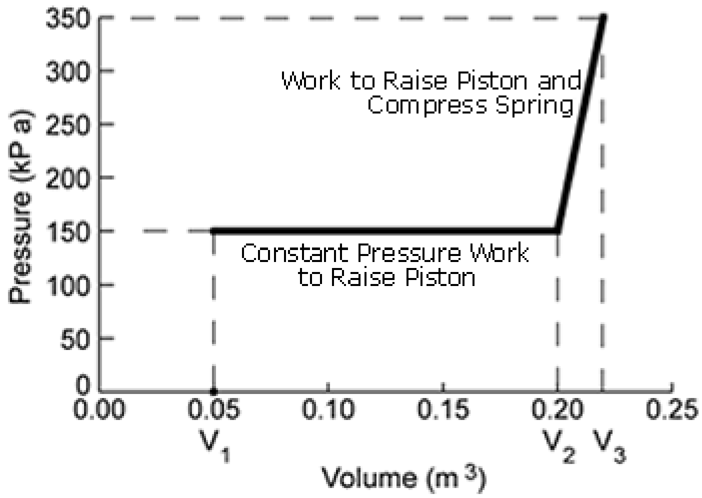
The volume at state 3 is

$$V_3 = V_2 + A_p(z_3 - z_2) = 0.2\,m^3 + 0.1\,m^2(0.2\,m) = 0.22\,m^3$$

Noting that kPa = kN/m^2, the work is

$$W_{13,OUT} = \left[150\,\frac{kN}{m^2}(0.2 - 0.05)\,m^3 + \left(\frac{150 + 350}{2}\right)\frac{kN}{m^2}(0.22 - 0.2)\,m^2\right]\left(\frac{kJ}{kN\cdot m}\right)$$

$$\underline{\underline{W_{13,OUT} = 27.5\,kJ}}$$



c)

Discussion of Results: The above is based on the principle of using the simple concept that work = area under the process path on a P-V diagram. The work can also be calculated from the expression for an expansion or compression process:

$$(1) \quad W_{OUT} = \int P dV$$

From state 1 to state 2 P is constant, therefore

$$W_{OUT,12} = P_1 \int_1^2 dV = P_1 (V_2 - V_1) \quad \Leftarrow \quad P_1 = \left(P_0 + \frac{mg}{A_P}\right)$$

From state 2 to state 3 the compression of the spring adds force and the work integral becomes

$$(2) \quad W_{OUT,23} = \int_2^3 \left(P_2 + \frac{F_S}{A_P}\right) dV$$

and

$$(3) \quad F_S = k(z - z_2) = k\left(\frac{V - V_2}{A_P}\right)$$

Now,

$$(3) \Rightarrow (2) \quad W_{OUT,23} = \int_2^3 \left[P_2 + \frac{k}{A_P^2}(V - V_2)\right] dV$$

$$= P_2 (V_3 - V_2) + \frac{k}{A_P^2}\left(\frac{V_3^2 + V_2^2}{2} - V_2 V_3\right)$$

The total boundary work is thus

$W_{OUT,TOT} = W_{OUT,12} + W_{OUT,23}$

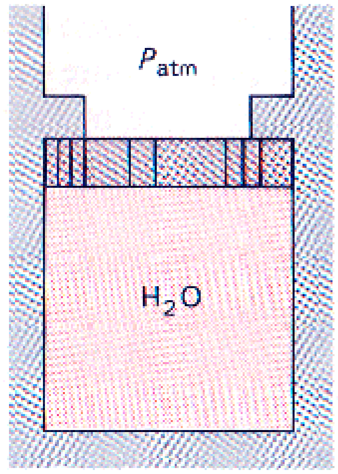
$$W_{OUT,TOT} = P_1 (V_2 - V_1) + P_2 (V_3 - V_2) + \frac{k}{A_P^2}\left(\frac{V_3^2 + V_2^2}{2} - V_2 V_3\right) \quad \Leftarrow \quad P_2 = P_1$$

$$W_{OUT,TOT} = P_1 (V_3 - V_1) + \frac{k}{A_P^2}\left(\frac{V_3^2 + V_2^2}{2} - V_2 V_3\right)$$

$$W_{OUT,TOT} = 27.5 kJ$$

Example 6

Problem Statement: A piston of mass equal to 900 kg and area equal to 9.8 x 10-3 m^2 is supported against stops (see figure) by water at 20 MPa with a specific volume of 0.005874 m^3/kg (state 1). The water is cooled until v = 0.002240 m^3/kg. Atmospheric pressure is 101 kPa. Determine the work per unit mass for the process (w12,out). Draw the process on the P-v diagram.



Solution:

Diagram: See above

Given:

m_P = 900 kg
A_P = 9.8 $x10^{-3}$ m^2
P_1 = 20 MPa
v_1 = 0.005874 m^3/kg
v_2 = 0.002240 m^3/kg
P_{ATM} = 101 kPa

Find:

a) $w_{OUT,12}$
b) Draw P-v diagram

Assumptions: Frictionless piston

Governing Relations:

$$(1) \quad w_{OUT} = \int P dv$$

Property Data: -

Quantitative Solution:

The water undergoes two distinct process from 1 to 2, and we'll introduce a new state 1' to indicate a state between 1 and 2. From 1 to 1', the pressure of the water decreases while the piston remains stationary, which is thus an isometric process. At state 1', the pressure of the water is equal to the pressure due to weight of the piston and atmospheric pressure. As the water is further cooled, the piston moves down in an isobaric process from states 1' to 2. We can calculate the pressure at state 1' as

$$(2) \quad P_{1'} = P_{ATM} + \frac{F_P}{A_P}$$

$$(3) \quad F_P = \frac{m_P g}{g_C}$$

$$(3) \Rightarrow (2) \quad P_{1'} = P_{ATM} + \frac{m_P g}{A_P g_C}$$

$$P_{1'} = 101\,\text{kPa} + \frac{(900\,\cancel{kg})\left(9.8\,\frac{\cancel{m}}{\cancel{s^2}}\right)}{\left(9.8\text{x}10^{-3}\,\cancel{m^2}\right)\left(1\,\frac{\cancel{kg}\cdot\cancel{m}}{\cancel{N}\cdot\cancel{s^2}}\right)} \frac{10^{-3}\,\text{kPa}}{\left(1\,\frac{\cancel{N}}{\cancel{m^2}}\right)} = 1{,}001\,\text{kPa}$$

Looking at the work for the process from states 1 to 1' during which v is constant,

$$W_{OUT,1-1'} = \int_{1}^{1'} P\,\underbrace{\cancel{dv}}_{0} = 0$$

For the isobaric process from states 1' to 2

$$W_{OUT,1'-2} = \int_{1'}^{2} P dv = P_{1'} \int_{1'}^{2} dv = P_{1'}\left(v_2 - v_{1'}\right)$$

$$= (1001\,\cancel{kPa})\,[0.002240 - 0.005874]\left(\frac{\cancel{m^3}}{kg}\right)\left(\frac{\cancel{kN}}{\cancel{m^2}\cdot\cancel{kPa}}\right)\left(\frac{kJ}{\cancel{kN}\cdot\cancel{m}}\right)$$

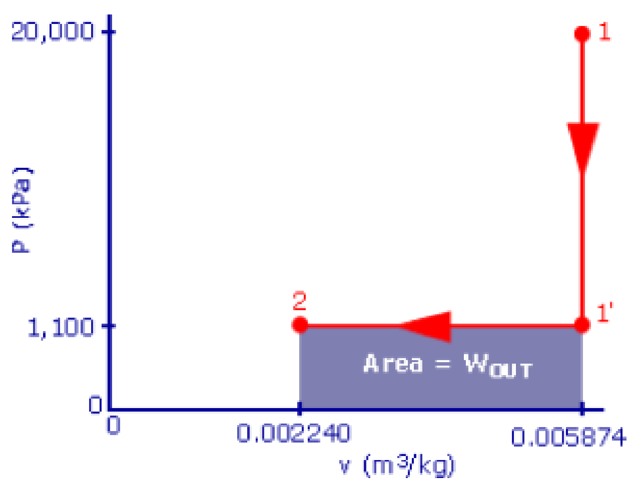
$$= -3.64\,\frac{kJ}{kg}$$

The total work from 1 to 2 is the sum of the works from 1 to 1' and 1' to 2

$$W_{OUT,12} = W_{OUT,1-1'} + W_{OUT,1'-2} = 0 - 3.64\,kJ = -3.64\,kJ$$

$$\underline{\underline{W_{IN,12} = -W_{OUT,12} = 3.64\,kJ}}$$

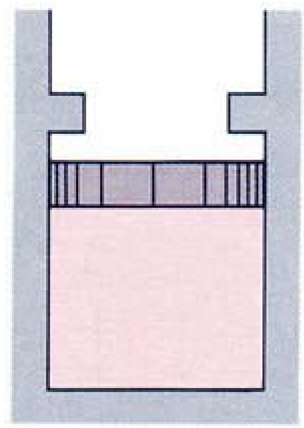
The P-v diagram is



Discussion of Results: Any process that is isochoric or isometric does not have any boundary work. However, it may have other kinds of work, such as shaft or electrical.

Example 7

Problem Statement: A piston encloses 1 kg of a gas with a specific volume of 0.1751 m^3/kg and pressure of 1.0 MPa. The substance is heated in the cylinder shown in the figure until the piston hits the stops, at which point the gas has a specific volume of 0.1900 m^3/kg. Heat transfer continues until the pressure of the gas reaches 1026 kPa. Determine the work done by the gas.



Solution:

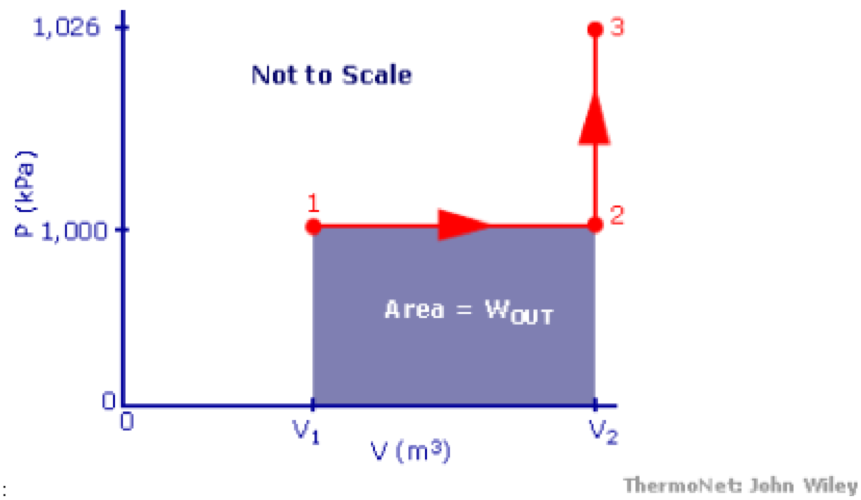


Diagram:

Given:

m = 1 kg
P_1 = 1.0 MPa = 1000 kPa
v_1 = 0.1751 m^3/kg
v_2 = 0.1900 m^3/kg
P_3 = 1026 kPa

Find:

$W_{OUT,13}$

Assumptions: Frictionless piston

Governing Relations:

$$(1) \quad W_{OUT} = \int PdV = \int Pd(mv) = m\int Pdv$$

Property Data: -

Quantitative Solution:

The work from 1 to 3 is the sum of the work from 1 to 2 and 2 to 3:

States 1 to 2: The process from 1 to 2 is isobaric, and therefore the work using eqn (1) is

$$W_{OUT,12} = m\int_1^2 P dv = mP_1\int_1^2 dv$$
$$= \left(1\,\cancel{kg}\right)\left(1{,}000\,\cancel{kPa}\right)(0.1900 - 0.1751)\left(\frac{\cancel{m^3}}{\cancel{kg}}\right)\left(\frac{\cancel{kN}}{\cancel{m^2}\cdot\cancel{kPa}}\right)\left(\frac{kJ}{\cancel{kN}\cdot\cancel{m}}\right)$$
$$= 15kJ$$

States 2 to 3: The process from 2 to 3 is isochoric, and therefore the work using eqn (1) is

$$W_{OUT,23} = m\int_1^2 P\,\underbrace{\cancel{dv}}_{0} = 0kJ$$

Therefore, the work from 1 to 3 is

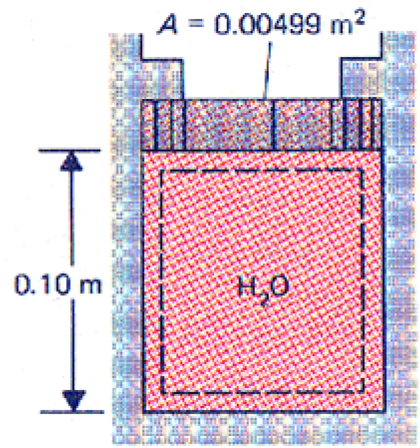
$$W_{OUT,13} = W_{OUT,12} + W_{OUT,13} = 15kJ + 0kJ = 15kJ$$

$$\underline{\underline{W_{OUT,13} = 15kJ}}$$

Discussion of Results: We could have calculated the work done by the gas by calculating the area under the P-V curve. Note that the area under a vertical line (e.g., 2 to 3) is zero, which is consistent with our numeric solution.

Example 8

Problem Statement: A piston-cylinder device with one set of stops has dimensions as shown. The piston is free to move. The H_2O inside is initially steam with a pressure of 6.0 MPa and specific volume of 0.03564 m^3/kg . The steam is cooled until it has a specific volume of 0.01847 m^3/kg. The piston requires a pressure of 5.5 MPa to support it. Determine:



a) the initial and final mass of H2O, and
b) the total work done (kJ).

Solution:

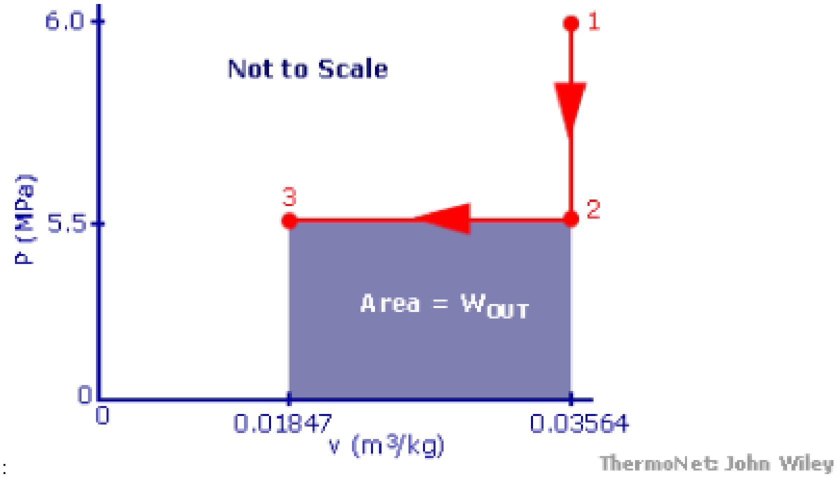


Diagram:

Given:

P_1 = 6.0 MPa = 6000 kPa
v_1 = v_2 = 0.03564 m^3/kg
P_2 = P_3 = 5.5 MPa = 5500 kPa
v_3 = 0.01847 m^3/kg
A_P = 0.00499 m^2
z_1 = 0.10 m^2

Find:

a) m_1 and m_3
b) W_{13} (kJ).

Assumptions:

- Frictionless piston
- Sealed piston-cylinder

Governing Relations:

$$(1) \quad W_{OUT} = \int P dV = \int P d(mv) = m \int P dv$$

Property Data: -

Quantitative Solution:

a) At state 1, the mass is

$$v_1 = \frac{V_1}{m_1} \Rightarrow m_1 = \frac{V_1}{v_1} \quad (2)$$

$$(3) \quad V_1 = A_P z_1$$

$$(3) \Rightarrow (2) \quad m_1 = \frac{A_P z_1}{v_1} = \frac{\left(0.00499\ \cancel{m^2}\right)\left(0.1\ \cancel{m}\right)}{\left(0.03564\ \frac{\cancel{m^3}}{kg}\right)} = 0.014kg$$

Based on the assumption of a sealed piston-cylinder, there is no mass entering or leaving the system between states 1 and 3. Therefore,

$\underline{m_1 = m_3 = 0.014\ kg}$

b) The work from 1 to 3 is the sum of the work from 1 to 2 and 2 to 3:

States 1 to 2: The process from 1 to 2 is isochoric, and therefore the work using eqn (1) is

$$W_{OUT,12} = m\int_1^2 P\,\underbrace{\cancel{dv}}_{0} = 0kJ$$

States 2 to 3: The process from 2 to 3 is isobaric, and therefore the work using eqn (1) is

$$W_{OUT,12} = m\int_1^2 Pdv = mP_1\int_1^2 dv$$

$$= \left(0.014\ \cancel{kg}\right)\left(5,500\ \cancel{kPa}\right)(0.01847 - 0.03564)\left(\frac{\cancel{m^3}}{\cancel{kg}}\right)\left(\frac{\cancel{kN}}{\cancel{m^2}\cdot\cancel{kPa}}\right)\left(\frac{kJ}{\cancel{kN}\cdot\cancel{m}}\right)$$

$$= -1.322kJ$$

Therefore, the work from 1 to 3 is

$$W_{OUT,13} = W_{OUT,12} + W_{OUT,13} = 0kJ - 1.322kJ = -1.322kJ$$

$$\underline{\underline{W_{IN,13} = -W_{OUT,13} = 1.322kJ}}$$

Example 9

Problem Statement: The numerical values of the properties x, y, and z are given for parts a, b and c. Linearly interpolate and obtain the specified property.

a) Find y for x = 112.3

x	y
110	1.2074
115	1.0351

b) Find z for x = 27 and y = 150

c) Find z for x = 32 and y = 181

x = 10		x = 50	
y	z	y	z
150	19.513	150	3.8894
200	21.820	200	4.3561

Solution:

Diagram: Not Applicable

Given:

Part a)

x	y
110	1.2074
115	1.0351

Parts b) and c)

x = 10		x = 50	
y	z	y	z
150	19.513	150	3.8894
200	21.820	200	4.3561

Find:

a) y for x = 112.3
b) z for x = 27 and y = 150
c) z for x = 32 and y = 181

Assumptions: None

Governing Relations: None

Property Data: None

Quantitative Solution:

a) We want to find y for x = 112.3. Let x_1 = 110, x_2 = 115, y_1 = 1.2074 and y_2 = 1.0351. Using linear interpolation we can setup the following ratio:

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$$

Solving for y yields,

$$y = y_1 + (y_2 - y_1)\frac{x - x_1}{x_2 - x_1} = 1.2074 + (1.0351 - 1.2074)\frac{(112.3 - 110)}{(115 - 110)}$$

$$\underline{\underline{y = 1.1281}}$$

b) We want to find z for x = 27. Let x_1 = 10, x_2 = 50, z_1 = 19.513 and z_2 = 3.8894 from the table above. Setting up our interpolation ratio of x and z and solving for z,

$$\frac{x - x_1}{x_2 - x_1} = \frac{z - z_1}{z_2 - z_1} \Rightarrow z = z_1 + (z_2 - z_1)\left(\frac{x - x_1}{x_2 - x_1}\right)$$

Solving for z yields,

$$z = 19.513 + (3.8894 - 19.513)\left(\frac{27-10}{50-10}\right)$$

$$\underline{\underline{z = 12.8730}}$$

c) Since both our desired x and y data points are between x and y data points in the table, we must **double interpolate**. For double interpolations, we must effectively create a new line of data at y = 181 for x = 10 and x = 50, and then interpolate between these values of x. This requires three interpolations:

1. We'll first find z at y = 181 and x = 10. Let y_1 = 150, y_2 = 200, z_1 = 19.513 and z_2 = 21.820. Therefore,

$$\frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1} \Rightarrow z = z_1 + (z_2 - z_1)\left(\frac{y - y_1}{y_2 - y_1}\right)$$

Solving for x yields,

$$z = 19.513 + (21.820 - 19.513)\left(\frac{181-150}{200-150}\right) = 20.943$$

2. Next we'll find z at y = 181 and x = 50. Let y_1 = 150, y_2 = 200, z_1 = 3.8894 and z_2 = 4.3561. Starting from the same ratio as above,

$$z = 3.8894 + (4.3561 - 3.8894)\left(\frac{181-150}{200-150}\right) = 4.1788$$

3. We have now created a line of data at y = 181 in the table as shown below.

x = 10		x = 50	
y	z	y	z
150	19.513	150	3.8894
181	20.943	181	4.1788
200	21.820	200	4.3561

We'll interpolate for z at x = 32 and y = 181 by letting letting x_1 = 10, x_2 = 50, z_1 = 20.943 and z_2 = 4.1788.

$$\frac{x - x_1}{x_2 - x_1} = \frac{z - z_1}{z_2 - z_1} \Rightarrow z = z_1 + (z_2 - z_1)\left(\frac{x - x_1}{x_2 - x_1}\right)$$

$$z = 20.943 + (4.1788 - 20.943)\left(\frac{32-10}{50-10}\right)$$

$$\underline{\underline{z = 11.723}}$$

Discussion of Results: These are the types of interpolation that are required to obtain properties from many tables.