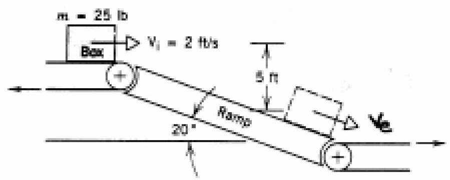
PROBLEM 2.17

KNOWN: A box slides down a ramp. The mass of the box and its velocity at the top of the ramp are known. The ramp geometry is also specified.

FIND: (a) In the absence of friction, determine the velocity of the box at the base of the ramp and the changes in kinetic and potential energy for the box. (b) Determine the changes in kinetic and potential energy of the box when friction is acting and the velocity at the base of the ramp is known. Compare with the results of part (a).

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The box is the closed system. 2. In part (a), friction is negligible. 3. The acceleration of gravity is 32.0 ft/s².

ANALYSIS: (a) In the absence of any resultant force acting on the system, including friction, Eq. 2.9 reduces to

$$\tfrac{1}{2} m (V_e^2 - V_i^2) + mg(z_e - z_i) = 0 \qquad (*)$$

Solving

$$V_e = \sqrt{V_i^2 + 2g(z_i - z_e)}$$

$$= \sqrt{\left(2\tfrac{ft}{s}\right)^2 + 2\left(32\tfrac{ft}{s^2}\right)(5 ft)} = 18\ ft/s \longleftarrow V_e$$

As shown by Eq (*), the kinetic and potential energy changes have the same magnitude but are opposite in sign. Thus

$$\Delta PE = mg(z_e - z_i) = (25\,lb)\left(32.0\tfrac{ft}{s^2}\right)(-5 ft)\left|\frac{1\,lbf}{32.2\,lb\cdot ft/s^2}\right|$$
$$= -124.2\ ft\cdot lbf \longleftarrow \Delta PE$$

and $\Delta KE = +124.2\ ft\cdot lbf$. $\longleftarrow \Delta KE$

(b) Whether there is friction or not, the potential energy change in this case is the same as determined in part (a): −124.2 ft·lbf. If V_e = 9 ft/s, the change in kinetic energy is smaller in magnitude:

$$\Delta KE = \tfrac{1}{2} m\left[V_e^2 - V_i^2\right] = \tfrac{1}{2}(25\,lb)\left[(9\,ft/s)^2 - (2\,ft/s)^2\right]\left|\frac{1\,lbf}{32.2\,lb\cdot ft/s^2}\right| = 29.9\ ft\cdot lbf \longleftarrow \Delta KE$$

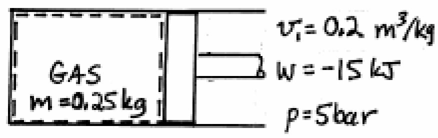
Referring again to Eq. 2.9, the decrease in potential energy in this case can be accounted for in terms of the increase in kinetic energy of the box and work to overcome friction.

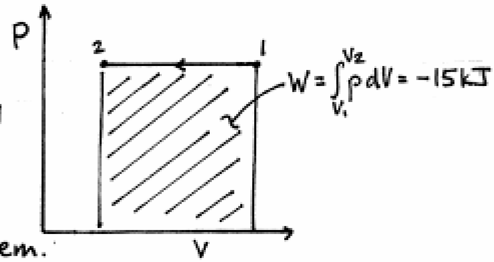
PROBLEM 2.25*

KNOWN: A known amount of gas undergoes a constant-pressure process in a piston-cylinder assembly beginning at a specified specific volume. The work is known.

FIND: Determine the final volume.

SCHEMATIC & GIVEN DATA:

① 



ASSUMPTIONS: (1) The gas is a closed system.
(2) Pressure is constant during the process.

ANALYSIS: Using Eq. 2.17

$$W = \int_{V_1}^{V_2} p\,dV = p(V_2 - V_1)$$

$$= p(V_2 - m v_1)$$

Solving for V_2 and inserting values

$$V_2 = \frac{W}{p} + m v_1$$

$$= \frac{(-15\ kJ)}{(5\ bar)} \left| \frac{1\ bar}{10^5\ N/m^2} \right| \left| \frac{10^3\ N \cdot m}{1\ kJ} \right| + (0.25\ kg)(0.2\ m^3/kg)$$

$$= 0.02\ m^3 \longleftarrow V_2$$

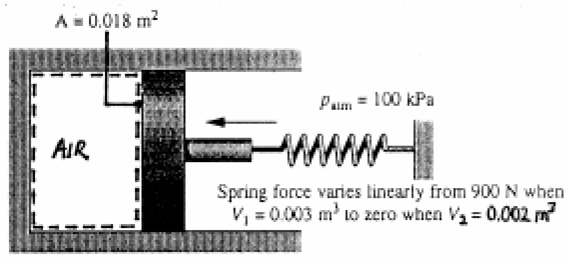
PROBLEM 2.31

KNOWN: Warm air cools slowly in a piston-cylinder assembly from a known initial volume to a known final volume. During the process, a spring exerts a force on the piston that varies linearly from a known initial value to a final value of zero.

FIND: Determine the initial and final pressures of the air, and the work.

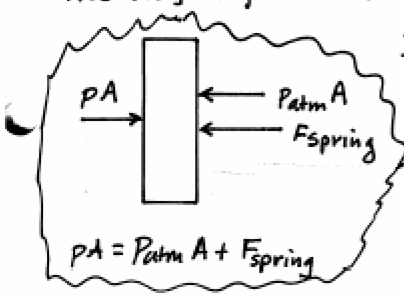
SCHEMATIC & GIVEN DATA:

$V_1 = 0.003\ m^3$
$V_2 = 0.002\ m^3$



ASSUMPTIONS: (1) The air is a closed system. (2) The process occurs slowly, so there is no acceleration of the piston. (3) There is no friction between the piston and the cylinder wall. (4) The spring force varies linearly with volume.

ANALYSIS: The initial and final pressures of the air are determined from a free-body diagram of the piston, as follows. That is, $\Sigma F = 0$, so



Initially: $F_{spring} = 900\ N$

$$p_1 = p_{atm} + \frac{F_{spring,1}}{A}$$

$$= 100\ kPa + \frac{(900\ N)}{(0.018\ m^2)}\left|\frac{1\ kPa}{10^3\ N/m^2}\right| = 150\ kPa \longleftarrow p_1$$

Finally: $F_{spring} = 0 \Rightarrow p_2 = 100\ kPa \longleftarrow p_2$

Now, the work is determined using Eq. 2.17, $W = \int_{V_1}^{V_2} p\,dV$, but from above

$$p = p_{atm} + \frac{F_{spring}}{A}$$

Since the spring force varies linearly from 900 N to zero as volume goes from $V_1 = 0.003\ m^3$ to $V_2 = 0.002\ m^3$

$$F_{spring} = \left(\frac{900\ N}{0.001}\right)(V - 0.002)$$

and

$$W = \int_{V_1}^{V_2}\left(p_{atm} + \frac{F_{spring}}{A}\right)dV = \int_{V_1}^{V_2}\left[100 + \left(\frac{900}{0.001}\right)\frac{(V-0.002)}{(0.018)|10^3|}\right]dV$$

$$= \int_{V_1}^{V_2}\left[\cancel{100} + 50000V - \cancel{100}\right]dV = \left(\frac{50000}{2}\right)V^2\Bigg|_{V_1 = 0.003\ m^3}^{V_2 = 0.002\ m^3}$$

① $$= -0.125\ kPa\cdot m^3\left|\frac{10^3\ N/m^2}{1\ kPa}\right|\left|\frac{1\ kJ}{10^3\ N\cdot m}\right| = -0.125\ kJ \longleftarrow W$$

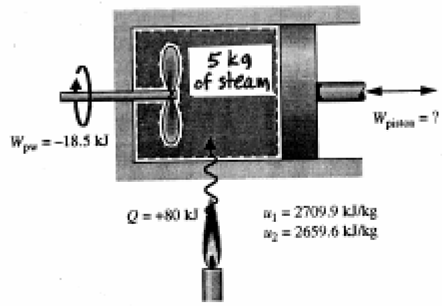
1. The negative sign denotes that the piston does work on the air as the air cools. Also, the atmosphere and the spring do work on the piston.

PROBLEM 2.57

KNOWN: Five kg of steam undergo an expansion in a piston-cylinder assembly from state 1 to state 2. During the process there is a known heat transfer to the steam and a known work transfer of energy to the steam by a paddle wheel. The change in specific internal energy of the steam is also known.

FIND: Determine the amount of energy transfer by work from the steam to the piston during the process.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The steam is the closed system. 2. There is no change in the kinetic or potential energy from state 1 to state 2.

ANALYSIS: The net work can be determined from an energy balance. That is, with assumption 2

$$\cancelto{0}{\Delta KE} + \cancelto{0}{\Delta PE} + \Delta U = Q - W$$

or

$$W = Q - \Delta U$$

The net work is the sum of the work associated with the paddlewheel W_{pw} and the work done on the piston W_{piston}:

$$W = W_{pw} + W_{piston}$$

From the given information $W_{pw} = -18.5$ kJ, where the minus sign is required because the paddle wheel transfers energy to the system. Collecting results

$$W_{pw} + W_{piston} = Q - \Delta U$$

or

$$\begin{aligned} W_{piston} &= Q - \Delta U - W_{pw} \\ &= Q - m(u_2 - u_1) - W_{pw} \\ &= 80\text{ kJ} - 5\text{ kg}\,(2659.6 - 2709.9)\tfrac{\text{kJ}}{\text{kg}} - (-18.5\text{ kJ}) \\ &= +350\text{ kJ} \longleftarrow W_{piston} \end{aligned}$$

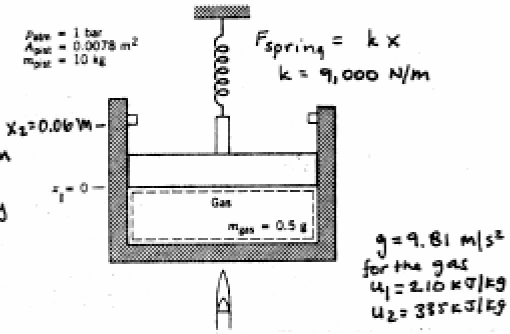
where the positive sign indicates that energy is transferred from the steam to the piston as the steam expands during the process.

PROBLEM 2.70

<u>KNOWN:</u> A gas undergoes a process in a piston-cylinder assembly. The piston is constrained by a spring with a linear force-displacement relation.

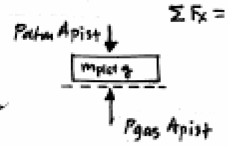
<u>FIND:</u> Determine (a) the initial pressure of the gas, (b) the work done by the gas on the piston, and (c) the heat transfer.

<u>SCHEMATIC & GIVEN DATA:</u>



<u>ASSUMPTIONS:</u> (1) The gas is a closed system. (2) There is no friction between the piston and cylinder wall. (3) The process occurs slowly with no acceleration of the piston. (4) The acceleration of gravity is constant. (5) Kinetic and potential energy effects are negligible.

<u>ANALYSIS:</u> (a) Initially, the spring exerts no force on the piston, which is at rest. Thus, with assumption 2 the force exerted by the gas on the bottom of the piston equals the piston weight plus the force of the atmosphere acting on the top of the piston. That is

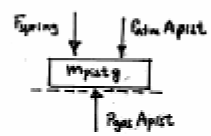


$$\Sigma F_x = 0 \Rightarrow \quad P_{gas} A_{pist} = (m_{pist}\,g) + P_{atm} A_{pist}$$

$$P_{gas} = P_{atm} + \frac{m_{pist}\,g}{A_{pist}} = 1\text{ bar}\left|\frac{100\text{ kPa}}{1\text{ bar}}\right| + \frac{(10\text{ kg})(9.81\text{ m/s}^2)}{(7.8\times10^{-3}\text{ m}^2)}\left|\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right|\left|\frac{1\text{ kPa}}{10^3\text{ N/m}^2}\right|$$

$$= 112.6\text{ kPa} \longleftarrow P_{gas}$$

(b) As the piston moves from x = 0 to x = 0.06 m, the spring force acts. Then, with assumptions 2 and 3, $\Sigma F_x = 0$ reads



$$P_{gas} A_{pist} = (m_{pist}\,g) + P_{atm} A_{pist} + F_{spring}$$
$$= (m_{pist}\,g) + P_{atm} A_{pist} + kx$$

The work done by the gas on the piston is given by

$$W = \int_{x_1}^{x_2} (P_{gas} A_{pist})\,dx = \int_{x_1}^{x_2} [P_{atm} A_{pist} + m_{pist}\,g + kx]\,dx$$

$$= \left[[P_{atm} A_{pist} + m_{pist}\,g]\,x + \frac{kx^2}{2}\right]_0^{0.06}$$

$$= \left[\left(10^5\frac{\text{N}}{\text{m}^2}\right)(7.8\times10^{-3}\text{m}^2) + (10\text{ kg})\left(9.81\frac{\text{m}}{\text{s}^2}\right)\left|\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right|\right](0.06\text{m})$$

$$+ \left(9\times10^3\frac{\text{N}}{\text{m}}\right)\left(\frac{(0.06\text{m})^2}{2}\right)$$

$$= [780\text{ N} + 98.1\text{ N}](0.06\text{ m}) + 16.2\text{ N}\cdot\text{m} = 68.89\text{ N}\cdot\text{m}$$

$$= 68.89\text{ N}\cdot\text{m}\left|\frac{1\text{ J}}{1\text{ N}\cdot\text{m}}\right| = 68.89\text{ J} \longleftarrow W$$

(c) The energy balance for the system consisting of the gas reduces to $\Delta U = Q - W$. Then, with $\Delta U = m_{gas}(u_2 - u_1)$

$$Q = m_{gas}(u_2 - u_1) + W \Rightarrow \quad Q = (0.5\text{ g})(335 - 210)\frac{\text{J}}{\text{g}} + 68.89\text{ J} = 131.39\text{ J} \longleftarrow Q$$

PROBLEM 2.75

KNOWN: A closed system undergoes a cycle consisting of three processes.

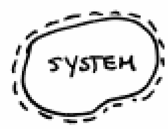
FIND: Sketch the cycle on a p-V diagram and calculate the net work for the cycle and the heat transfer for process 2-3.

SCHEMATIC & GIVEN DATA: The following data are given for each process:

Process 1-2: adiabatic compression with $pV^{1.4}$ = const. from $p_1 = 50$ lbf/in², $V_1 = 3$ ft³ to $V_2 = 1$ ft³

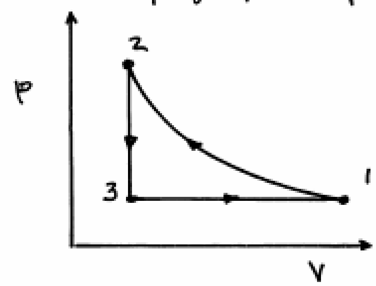
Process 2-3: constant volume

Process 3-1: Constant pressure, $U_1 - U_3 = 46.7$ Btu



ASSUMPTIONS: (1) The system is closed. (2) Kinetic and potential energy effects are negligible. (3) Process 1-2 is polytropic.

ANALYSIS: (a) Since process 1-2 is a polytropic compression, the p-V diagram for the cycle is



(b) Use Eq. 2.17 to evaluate the work for process 1-2

$$W_{12} = \int_{V_1}^{V_2} p\,dV = \text{const.} \int_{V_1}^{V_2} \frac{dV}{V^{1.4}} = p_1 V_1^{1.4} \frac{(V_2^{-.4} - V_1^{-.4})}{(-.4)}$$

$$= \left(50 \frac{\text{lbf}}{\text{in}^2}\right)(3\ \text{ft}^3)^{1.4} \left[\frac{(1\ \text{ft}^3)^{-.4} - (3\ \text{ft}^3)^{-.4}}{(-.4)}\right] \left|\frac{144\ \text{in}^2}{1\ \text{ft}^2}\right| \left|\frac{1\ \text{Btu}}{778\ \text{ft·lbf}}\right|$$

$$= -38.3\ \text{Btu}$$

For process 2-3: $W_{23} = 0$

Finally, for process 3-1 use Eq. 2.17: $W_{31} = \int_{V_3}^{V_1} p\,dV = p_1(V_1 - V_3)$

$$W_{31} = \left(50 \frac{\text{lbf}}{\text{in}^2}\right)(3-1)\ \text{ft}^3 \left|\frac{144\ \text{in}^2}{1\ \text{ft}^2}\right| \left|\frac{1\ \text{Btu}}{778\ \text{ft-lb}}\right| = +18.51\ \text{Btu}$$

Thus $W_{cycle} = W_{12} + W_{23} + W_{31} = -19.79$ Btu ← W_{cycle}

(c) For the overall cycle $Q_{cycle} = W_{cycle}$

$$\cancel{Q_{12}}^{\,0} + Q_{23} + Q_{31} = W_{cycle}$$

$$Q_{23} = W_{cycle} - Q_{31}$$

For process 3-1: $\Delta KE + \Delta PE + (U_1 - U_3) = Q_{31} - W_{31} \Rightarrow Q_{31} = U_1 - U_3 + W_{31}$

$$Q_{31} = 46.7 + 18.51 = +65.21\ \text{Btu}$$

Finally $Q_{23} = W_{cycle} - Q_{31} = -19.79 - 65.21 = -85$ Btu ← Q_{23}