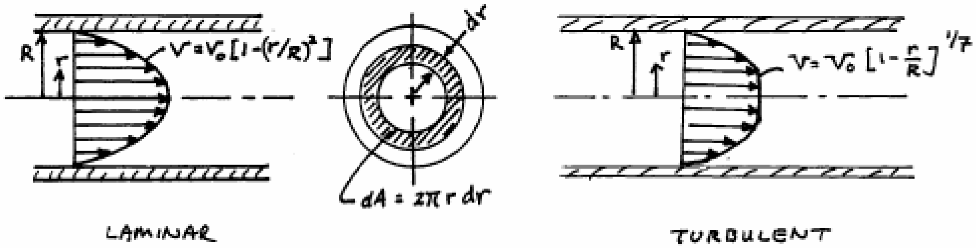
PROBLEM 4.8

KNOWN: Velocity distributions are given for laminar and turbulent flow of an incompressible liquid in a circular pipe.

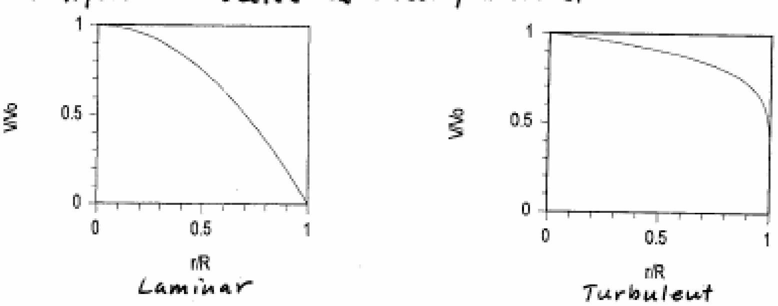
FIND: For each distribution, plot V/V_0 vs. r/R, derive expressions for the mass flow rate, average flow velocity, and specific kinetic energy. Determine the percent error if the specific kinetic energy is evaluated in terms of average velocity. Discuss.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: The liquid is modeled as incompressible.

ANALYSIS: (a)



(b) Using Eq. 4.9, the mass flow rate for laminar flow is

$$\dot{m} = \int_A \rho V\,dA$$

$$= \int_0^R \rho V_0\left[1-\left(\frac{r}{R}\right)^2\right]2\pi r\,dr$$

$$= \rho V_0 2\pi \int_0^R \left[r - \frac{r^3}{R^2}\right]dr$$

$$= \rho V_0 2\pi \left[\frac{r^2}{2} - \frac{r^4}{4R^2}\right]\Bigg|_{r=0}^{r=R} = \rho V_0 2\pi\left[\frac{R^2}{2} - \frac{R^2}{4}\right]$$

$$= \rho V_0 \pi \frac{R^2}{2}$$

Using Eq. 4.11a, the average velocity is

$$V_{ave} = \frac{\dot{m}}{\rho A} = \frac{\rho V_0 \pi R^2/2}{\rho(\pi R^2)}$$

$$= \frac{V_0}{2}$$

PROBLEM 4.8 (Contd.)

Using Eq. 4.9, the mass flow rate for turbulent flow is

$$\dot{m} = \int_A \rho V \, dA$$

$$= \int_0^R \rho V_0 \left[1 - \frac{r}{R}\right]^{\frac{1}{7}} 2\pi r \, dr$$

$$= \rho V_0 2\pi \int_0^R r \left[1 - \frac{r}{R}\right]^{\frac{1}{7}} dr$$

To evaluate the integral, let $u = 1 - r/R$ and $dr = -R\,du$. Then

$$\dot{m} = \rho V_0 2\pi \int_1^0 R(1-u)\, u^{1/7} (-R\,du) = \rho V_0 2\pi \left[-R^2 \int_1^0 (u^{1/7} - u^{8/7})\, du\right]$$

$$= \rho V_0 2\pi \left[-R^2 \left(\frac{7}{8} u^{8/7} - \frac{7}{15} u^{15/7}\right)_{u=1}^{u=0}\right]$$

$$= \rho V_0 2\pi \left[R^2 \frac{49}{120}\right] = \rho V_0 \pi \left(\frac{49}{60}\right) R^2$$

Using Eq. 4.11a, the average velocity is

$$V_{ave} = \frac{\dot{m}}{\rho A} = \frac{\rho V_0 \pi \left(\frac{49}{60}\right) R^2}{\rho (\pi R^2)} = \frac{49}{60} V_0$$

(c) The specific kinetic energy (kinetic energy per unit of mass) carried through an area normal to the flow is

$$ke = \frac{\int_A \frac{V^2}{2} \rho V \, dA}{\dot{m}} = \frac{\rho \int_A \frac{V^3}{2} dA}{\rho \int_A V \, dA} = \frac{\int_A \frac{1}{2} V^3 dA}{\int_A V \, dA} = \frac{\int_A \frac{1}{2} V^3 dA}{V_{ave} A} \qquad (*)$$

Forming the ratio of Eq. (*) to the specific kinetic energy calculated as $V_{ave}^2/2$, we have the kinetic energy coefficient (or kinetic energy correction factor):

$$\alpha = \frac{\int_A \frac{1}{2} V^3 dA}{\frac{1}{2} V_{ave}^3 \cdot A} = \frac{\int_A V^3 dA}{V_{ave}^3 A}$$

For Laminar Flow: $\alpha = 2$ ➡ % ERROR = 50

① For Turbulent Flow: $\alpha = 1.058$ ➡ % ERROR = 5.5

The flatter turbulent velocity profile adheres most closely to the idealization of one-dimensional flow.

---

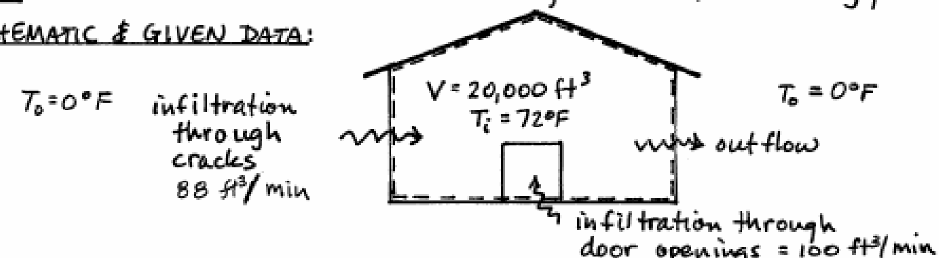
1. For further discussion, see R. W. Fox and A. T. McDonald, Introduction to Fluid Mechanics, 5th ed., J. Wiley & Sons, New York, pp 354-356.

PROBLEM 4.11

KNOWN: Air enters a building through cracks around doors and windows and due to door openings. The internal volume of the building is known.

FIND: Estimate the number of air changes within the building per hour.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume is at steady state. (2) The air behaves as an ideal gas. (3) The inside and outside air pressures are nearly equal.

ANALYSIS: At steady state, the mass balance reduces to

$$\dot{m}_{outflow} = \dot{m}_{cracks} + \dot{m}_{doors}$$

$$\rho_i (AV)_{outflow} = \rho_o \left[(AV)_{cracks} + (AV)_{doors} \right]$$

where ρ_i and ρ_o are the inside and outside densities, respectively. Assuming ideal gas behavior

$$\frac{\rho_o}{\rho_i} = \frac{\cancel{p_o}/RT_o}{\cancel{p_i}/RT_i} = \frac{T_i}{T_o}$$

Thus

$$(AV)_{outflow} = \left(\frac{T_i}{T_o}\right) \left[(AV)_{cracks} + (AV)_{doors} \right]$$

Inserting values

$$(AV)_{outflow} = \left(\frac{532\ °R}{460\ °R}\right) \left[88\ ft^3/min + 100\ ft^3/min \right] = 217\ ft^3/min$$

$$= 217 \frac{ft^3}{min} \left| \frac{60\ min}{1\ h} \right| = 13020 \frac{ft^3}{h}$$ ◄ $(AV)_{outflow}$

$$\text{Air changes per hour} = \frac{13{,}020\ ft^3/h}{20{,}000\ ft^3/\text{air change}}$$

$$\approx 0.65 \text{ air changes}/h$$ ◄ air changes

PROBLEM 4.13*

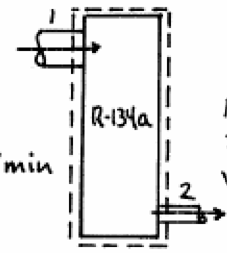
KNOWN: Refrigerant 134a flows through a refrigeration condenser. Data are known at the inlet and exit. The mass flow at the inlet is given.

FIND: Determine (a) the inlet velocity, (b) the diameter of the exit pipe.

SCHEMATIC & GIVEN DATA:

ASSUMPTIONS: (1) The control volume is at steady state. (2) The flow is one-dimensional at the inlet and exit.

$P_1 = 9$ bar
$T_1 = 50\ °C$
$d_1 = 2.5$ cm
$\dot{m}_1 = 6$ kg/min



$P_2 = 9$ bar
$T_2 = 30\ °C$
$V_2 = 2.5$ m/s

ANALYSIS: (a) Solving Eq. 4.11b for V_1

$$V_1 = \frac{\dot{m}_1 v_1}{A_1} = \frac{\dot{m}_1 v_1}{\left(\frac{\pi d_1^2}{4}\right)} = \frac{4\dot{m}_1 v_1}{\pi d_1^2}$$

From Table A-12, $v_1 = 0.02472$ m³/kg. Thus

$$V_1 = \frac{(4)(6\ \text{kg/min})(0.02472\ \text{m}^3/\text{kg})}{\pi\ (2.5)^2\text{cm}^2}\left|\frac{1\ \text{min}}{60\ \text{s}}\right|\left|\frac{10^4\ \text{cm}^2}{1\ \text{m}^2}\right|$$

$$= 5.04\ \text{m/s} \leftarrow$$ V_1

(b) To find the diameter of the exit pipe, begin with mass rate balance

$$\cancel{\frac{dm_{cv}}{dt}}^{\,0} = \dot{m}_1 - \dot{m}_2 \Rightarrow \dot{m}_2 = \dot{m}_1$$

Thus, with $\dot{m}_2 = A_2 V_2 / v_2$

$$A_2 = \frac{\dot{m}_2 v_2}{V_2}$$

From Table A-11, $T_2 < T_{sat\,@\,9\,bar}$. Thus, the refrigerant is a sub-cooled liquid. From Table A-10,

$$v_2 \approx v_{f\,@\,30\,°C} = 0.8417 \times 10^{-3}\ \text{m}^3/\text{kg}$$

Inserting values

$$A_2 = \frac{(6\ \text{kg/min})(0.8417\times10^{-3}\ \text{m}^3/\text{kg})}{(2.5\ \text{m/s})}\left|\frac{1\ \text{min}}{60\ \text{s}}\right| = 3.367\times10^{-5}\ \text{m}^2$$

Finally, with $A = \pi d^2/4$

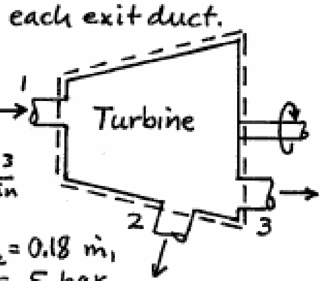
$$d_2 = \left(\frac{4A_2}{\pi}\right)^{1/2} = 0.0065\ \text{m} = 0.65\ \text{cm} \leftarrow$$ d_2

PROBLEM 4.14*

KNOWN: Data are given for steam flowing through a turbine with one inlet and two exits.

FIND: Determine the diameter of each exit duct.

SCHEMATIC & GIVEN DATA:

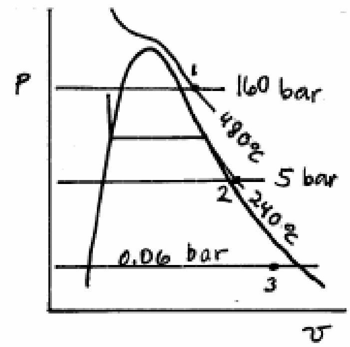


$P_1 = 160$ bar
$T_1 = 480$ °C
$(AV)_1 = 800\ \frac{m^3}{min}$

$\dot{m}_2 = 0.18\ \dot{m}_1$
$P_2 = 5$ bar
$T_2 = 240$ °C
$V_2 = 25$ m/s

$P_3 = 0.06$ bar
$x_3 = 0.94$
$V_3 = 400$ m/s

ASSUMPTIONS: (1) The control volume is at steady state. (2) The flow at the inlet and each exit is one-dimensional.



ANALYSIS: The mass flow rate at the inlet is

$$\dot{m}_1 = \frac{(AV)_1}{v_1}$$

From Table A-4, $v_1 = 0.01842\ m^3/kg$. Thus

$$\dot{m}_1 = \frac{(800\ m^3/min)}{(0.01842\ m^3/kg)}\left|\frac{1\ min}{60\ s}\right| = 723.9\ kg/s$$

and $\dot{m}_2 = 0.18\ \dot{m}_1 = 130.3\ kg/s$

Applying the mass rate balance

$$\cancel{\frac{dm_{cv}}{dt}}^{0} = \dot{m}_1 - \dot{m}_2 - \dot{m}_3 \Rightarrow \dot{m}_3 = \dot{m}_1 - \dot{m}_2 = 593.6\ kg/s$$

Now, with $\dot{m} = AV/v$, and from Table A-4; $v_2 = 0.4646\ m^3/kg$

$$A_2 = \frac{\dot{m}_2 v_2}{V_2} = \frac{(130.3\ kg/s)(0.4646\ m^3/kg)}{(25\ m/s)} = 2.421\ m^2$$

Noting that $A = \pi d^2/4$

$$d_2 = \sqrt{\frac{4A_2}{\pi}} = 1.756\ m \longleftarrow d_2$$

From Table A-3, at $P_3 = 0.06$ bar and $x_3 = 0.94$

$$v_3 = v_{f3} + x_3(v_{g3} - v_{f3})$$
$$= 1.0064\times10^{-3} + (0.94)(23.739 - 1.0064\times10^{-3}) = 22.315\ m^3/kg$$

Thus

$$A_3 = \frac{\dot{m}_3 v_3}{V_3} = \frac{(593.6)(22.315)}{(400)} = 33.115\ m^2$$

and

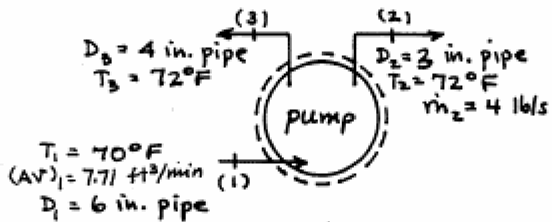
$$d_3 = \sqrt{\frac{4A_3}{\pi}} = 6.49\ m \longleftarrow d_3$$

PROBLEM 4.19

KNOWN: A pump operating at steady state provides water through two exit pipes. Data are known at the inlet and each exit.

FIND: Determine the water velocity in each exit pipe.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume is at steady state.
(2) The water is incompressible and $v \approx v_f(T)$.

ANALYSIS: For the velocity in the 3 in. pipe, use Eq. 4.11b

$$V_2 = \frac{\dot{m}_2 v_2}{A_2} = \frac{4\dot{m}_2 v_2}{\pi D_2^2}$$

From Table A-2E at 72°F; $v_2 \approx v_f$ = 0.01606 ft³/lb. Thus

$$V_2 = \frac{4\,(4\ \text{lb/s})(0.01606\ \text{ft}^3/\text{lb})}{\pi\,(3/12)^2\ \text{ft}^2} = 1.309\ \text{ft/s} \longleftarrow V_2$$

To find the velocity in the 4 in. pipe, use the mass rate balance to determine the mass flow rate. That is

$$\cancel{\frac{dm_{cv}}{dt}}^{\,0} = \dot{m}_1 - \dot{m}_2 - \dot{m}_3 \Rightarrow \dot{m}_3 = \dot{m}_1 - \dot{m}_2$$

At the inlet

$$\dot{m}_1 = \frac{A_1 V_1}{v_1} = \frac{(7.71\ \text{ft}^3/\text{min})}{(0.01605\ \text{ft}^3/\text{lb})}\left|\frac{1\ \text{min}}{60\ \text{s}}\right| = 8.01\ \text{lb/s}$$

where v_1 is obtained from Table A-2E. Finally,

$$\dot{m}_3 = \dot{m}_1 - \dot{m}_2 = 8.01 - 4 = 4.01\ \text{lb/s}$$

and

$$V_3 = \frac{4\dot{m}_3 v_3}{\pi D_3^2} = \frac{4\,(4.01\ \text{lb/s})(0.01606\ \text{ft}^3/\text{lb})}{\pi\,(4/12)^2\ \text{ft}^2}$$

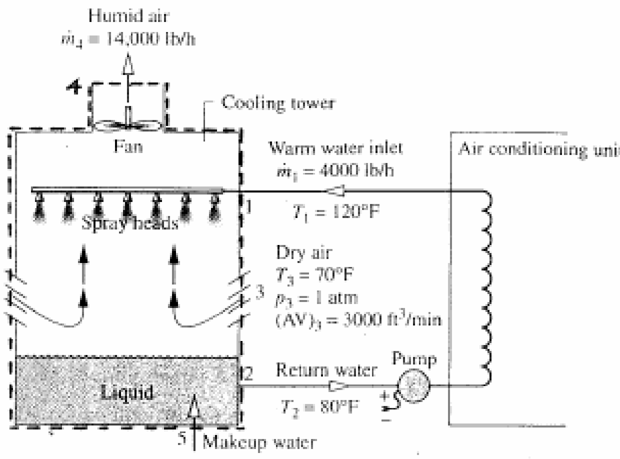
$$= 0.738\ \text{ft/s} \longleftarrow V_3$$

PROBLEM 4.21

KNOWN: Data are know for inlet and exit streams of an air conditioner cooling tower operating at steady state.

FIND: Determine the mass flow rate of the makeup water.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume is at steady state. (2) The dry air stream at location 3 behaves as an ideal gas.

ANALYSIS: The mass rate balance for the control reduces as follows:

$$\cancel{\frac{dm_{cv}}{dt}}^{0} = \dot{m}_1 + \dot{m}_3 + \dot{m}_5 - \dot{m}_2 - \dot{m}_4$$

$$\therefore \quad \dot{m}_5 = \dot{m}_2 - \dot{m}_1 + \dot{m}_4 - \dot{m}_3$$

From the schematic, we see that circuit through the air conditioning unit is closed. Therefore, at steady state $\dot{m}_1 = \dot{m}_2$. Thus

$$\dot{m}_5 = \dot{m}_4 - \dot{m}_3 \qquad (*)$$

To get $\dot{m}_3$, use Eq. 4.11 b and the ideal gas equation

$$\dot{m}_3 = \frac{(AV)_3}{v_3} = \frac{p_3 (AV)_3}{R T_3}$$

$$= \frac{(1 \text{ atm})(3000 \text{ ft}^3/\text{min})}{\left(\frac{1545}{28.97} \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(530°\text{R})} \left|\frac{14.696 \text{ lbf/in}^2}{1 \text{ atm}}\right| \left|\frac{144 \text{ in}^2}{1 \text{ ft}^2}\right| \left|\frac{60 \text{ min}}{1 \text{ h}}\right|$$

$$= 13480 \text{ lb/h}$$

Finally, from (*)

$$\dot{m}_5 = 14{,}000 - 13{,}480 = 520 \text{ lb/h} \longleftarrow \dot{m}_5$$

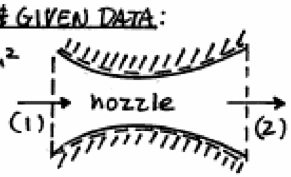
PROBLEM 4.25*

KNOWN: Steam flows through a well-insulated nozzle with known conditions at the inlet and exit.

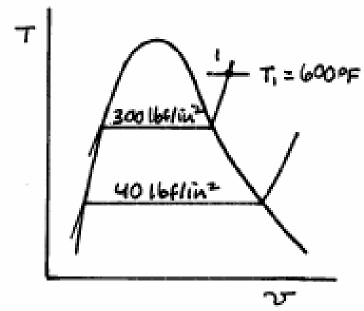
FIND: Determine the exit temperature.

SCHEMATIC & GIVEN DATA:

$p_1 = 300$ lbf/in²
$T_1 = 600$°F
$V_1 = 100$ ft/s



$p_2 = 40$ lbf/in²
$V_2 = 1800$ ft/s



ASSUMPTIONS: (1) The control volume is at steady state. (2) There is no heat transfer, and $\dot{W}_{cv} = 0$. (3) Potential energy effects are negligible.

ANALYSIS: The pressure is known at the exit. The state is fixed by determining h_2 using the steady-state energy balance

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g\cancelto{0}{(z_1 - z_2)}\right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Solving for h_2

$$h_2 = h_1 + \left(\frac{V_1^2 - V_2^2}{2}\right)$$

From Table A-4E, $h_1 = 1314.5$ Btu/lb. Thus

$$h_2 = (1314.5 \text{ Btu/lb}) + \left(\frac{100^2 - 1800^2}{2}\right)\frac{\text{ft}^2}{\text{s}^2}\left|\frac{1 \text{ lbf}}{32.2 \text{ lb}\cdot\text{ft/s}^2}\right|\left|\frac{1 \text{ Btu}}{778 \text{ ft}\cdot\text{lbf}}\right|$$

$$= 1250.03 \text{ Btu/lb}$$

Interpolating in Table A-4E at $p_2 = 40$ lbf/in², $h_2 = 1250.03$ Btu/lb gives

$$T_2 = 686.6 \text{ °F} \longleftarrow \underline{T_2}$$

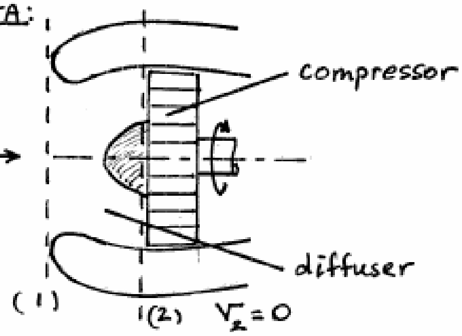
PROBLEM 4.31

KNOWN: Air enters the diffuser at the inlet of a jet engine and decelerates to zero velocity before entering the jet engine's compressor.

FIND: Determine the temperature of the air entering the compressor.

SCHEMATIC & GIVEN DATA:

$V_1 = 1000$ km/h
$P_1 = 0.6$ bar
$T_1 = 8°C$



ASSUMPTIONS: (1) The control volume is at steady state. (2) Heat transfer is negligible and $\dot{W}_{cv} = 0$. (3) Potential energy effects are negligible. (4) The air behaves as an ideal gas. ①

ANALYSIS: Since $h = h(T)$ for an ideal gas, the exit temperature can be found by evaluating h_2. Beginning with the steady-state energy balance

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}\left[(h_1 - h_2) + \cancelto{0}{\frac{(V_1^2 - V_2^2)}{2}} + g(z_1 - z_2)\right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$, and using assumption (3)

$$0 = (h_1 - h_2) + \tfrac{1}{2}V_1^2$$

or

$$h_2 = h_1 + \tfrac{1}{2}V_1^2$$

From Table A-22; $h_1 = 281.3$ kJ/kg, and

$$h_2 = 281.3\,\frac{\text{kJ}}{\text{kg}} + \frac{1}{2}(1\times10^6)^2\,\frac{\text{m}^2}{\text{h}^2}\left|\frac{1\ \text{h}^2}{3600^2\ \text{s}^2}\right|\left|\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right|$$

$$= 319.9\ \text{kJ/kg}$$

Interpolating in Table A-22; $T_2 \approx 319.6$ K ← T_2

1. The applicability of the ideal gas model can be checked by reference to the compressibility chart.

PROBLEM 4.34

KNOWN: Air expands through a turbine with known conditions at the inlet and exit. The power developed is known.

FIND: Determine the mass flow rate and the exit area.

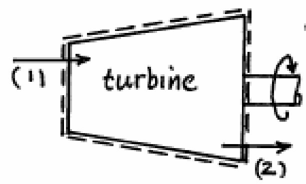
SCHEMATIC & GIVEN DATA:

air

$p_1 = 10$ bar

$T_1 = 900$ K

$V_1 << V_2$



$\dot{W}_{cv} = 3200$ kW

$p_2 = 1$ bar

$T_2 = 500$ K

$V_2 = 100$ m/s

ASSUMPTIONS: (1) The control volume is at steady state. (2) Heat transfer is negligible. (3) Potential energy effects and kinetic energy at the inlet can ① be neglected. (4) The air behaves as an ideal gas.

ANALYSIS: Begin with a steady-state energy balance

$$0 = \cancel{\dot{Q}_{cv}}^{\,0} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancel{\left(\frac{V_1^2 - V_2^2}{2}\right)}^{\,0} + g\cancel{(z_1 - z_2)}^{\,0}\right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Solving for $\dot{m}$

$$\dot{m} = \frac{\dot{W}_{cv}}{(h_1 - h_2) - \frac{V_2^2}{2}}$$

From Table A-22; $h_1 = 932.93$ kJ/kg and $h_2 = 503.02$ kJ/kg. Thus

$$\dot{m} = \frac{(3200 \text{ kW})\,|1 \text{ kJ/s}/1 \text{ kW}|}{(932.93 - 503.02)\text{ kJ/kg} - \left(\frac{100^2 \text{ m}^2/\text{s}^2}{2}\right)\left|\frac{1 \text{ N}}{\text{kg}\cdot\text{m/s}^2}\right|\left|\frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}}\right|}$$

$$= 7.53 \text{ kg/s} \leftarrow \qquad \dot{m}$$

The exit area is

$$A_2 = \frac{v_2 \dot{m}}{V_2} = \frac{R T_2 \dot{m}}{p_2 V_2}$$

$$= \frac{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(500\text{K})(7.53 \text{ kg/s})}{(1 \text{ bar})(100 \text{ m/s})}\left|\frac{1 \text{ bar}}{10^5 \text{ N/m}^2}\right|\left|\frac{10^3 \text{N}\cdot\text{m}}{1 \text{ kJ}}\right|$$

$$= 0.108 \text{ m}^2 \leftarrow \qquad A_2$$

---

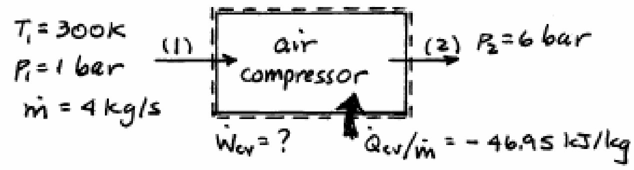
1. The applicability of the ideal gas model can be checked by reference to the compressibility chart.

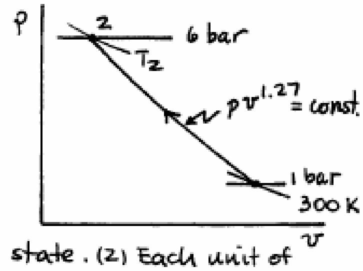
PROBLEM 4.43

KNOWN: Air is compressed at steady state from a given initial state to a given final pressure. The mass flow is known, and each unit of mass undergoes a specified process in going from inlet to exit.

FIND: Determine the compressor power.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume is at steady state. (2) Each unit of

1 mass undergoes a process described by $pv^{1.27} = constant$. (3) The air can be modeled as an ideal gas. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: To determine the power, begin with mass and energy rate balances at steady state

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancel{\left(\frac{V_1^2 - V_2^2}{2}\right)}^{0} + g\cancel{(z_1 - z_2)}^{0}\right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$ and the indicated terms are deleted by assumption 4. Rearranging and solving for $\dot{W}_{cv}$

$$\dot{W}_{cv} = \dot{m}\left[(\dot{Q}_{cv}/\dot{m}) + (h_1 - h_2)\right] \qquad (*)$$

Now, specific enthalpy h_1 is read from Table A-22 at 300K: $h_1 = 300.19\ \frac{kJ}{kg}$.
To get T_2, we use Eq. 3.56 with $n = 1.27$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} \Rightarrow T_2 = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} T_1 = \left(\frac{6}{1}\right)^{\frac{1.27-1}{1.27}} (300\,K) = 439.1\ K$$

Interpolating in Table A-22; $h_2 = 440.7$ kJ/kg. Inserting values in (*)

$$\dot{W}_{cv} = \left(4\ \frac{kg}{s}\right)\left[\left(-46.95\ \frac{kJ}{kg}\right) + (300.19 - 440.7)\ \frac{kJ}{kg}\right]\left|\frac{1\ kW}{1\ kJ/s}\right|$$

2 $$= -750\ kW \qquad \dot{W}_{cv}$$

---

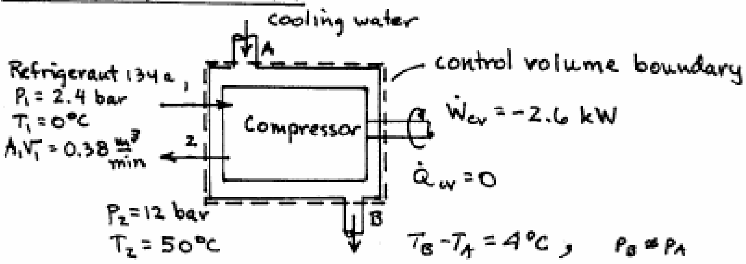
1. The applicability of the ideal gas model can be checked by reference to the generalized compressibility chart.
2. The negative sign for power denotes energy transfer by work into the control volume, as expected.

PROBLEM 4.51

KNOWN: Refrigerant 134a is compressed in a water-jacketed compressor between two known states. The inlet volumetric flow rate, input power, and cooling water temperature rise are given.

FIND: Determine the mass flow rate of the cooling water.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown is at steady-state. (2) There is no heat transfer with the surroundings. (3) Kinetic and potential energy effects can be neglected. (4) The cooling water is incompressible with $c = 4.179$ kJ/kg·K from Table A-19.

ANALYSIS: The energy rate balance applied to the overall compressor and water jacket at steady state reduces to

$$0 = \cancel{\dot{Q}_{cv}}^{\,0} - \dot{W}_{cv} + \dot{m}_1\left(h_1 + \underline{\frac{V_1^2}{2} + gz_1}\right) - \dot{m}_2\left(h_2 + \underline{\frac{V_2^2}{2} + gz_2}\right)$$
$$+ \dot{m}_A\left(h_A + \underline{\frac{V_A^2}{2} + gz_A}\right) - \dot{m}_B\left(h_B + \underline{\frac{V_B^2}{2} + gz_B}\right)$$

where $\dot{Q}_{cv} = 0$ by assumption(2), and the underlined terms drop out by assumption(3). Since the water and refrigerant streams are separate

$$\dot{m}_1 = \dot{m}_2 \equiv \dot{m}_R$$
$$\dot{m}_A = \dot{m}_B \equiv \dot{m}_w$$

Thus

$$0 = -\dot{W}_{cv} + \dot{m}_R(h_1 - h_2) + \dot{m}_w(h_A - h_B)$$

Applying Eq. 3.20b for the water stream

$$h_A - h_B = c(T_A - T_B) + v\,\cancel{(P_A - P_B)}^{\,0}$$

Inserting this result and solving for $\dot{m}_w$

$$\dot{m}_w = \frac{-\dot{W}_{cv} + \dot{m}_R(h_1 - h_2)}{c(T_B - T_A)} \qquad (*)$$

From Table A-12; $h_1 = 248.89$ kJ/kg, $v_1 = 0.08574$ m³/kg and $h_2 = 275.52$ kJ/kg.
Evaluating $\dot{m}_R$

$$\dot{m}_R = \frac{(AV)_1}{v_1} = \frac{(0.38\ m^3/min)}{(0.08574\ m^3/kg)}\left|\frac{1\ min}{60\ s}\right| = 0.0739\ kg/s$$

Finally, inserting values in (*)

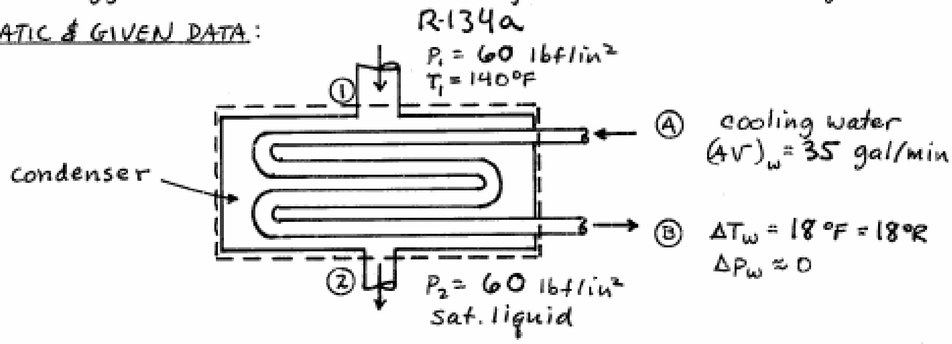
$$\dot{m}_w = \frac{-(-2.6\ kW)\left|\frac{1\ kJ/s}{1\ kW}\right| + (0.0739\ kg/s)(248.89 - 275.52)\frac{kJ}{kg}}{(4.179\ kJ/kg\cdot K)(4\ °C)} = 0.0378\ \frac{kg}{s} \leftarrow \dot{m}_w$$

PROBLEM 4.58*

KNOWN: Refrigerant 134a and cooling water pass in separate streams through a condenser (heat exchanger). The volumetric flow rate of cooling water and other data are given at the inlets and exits.

FIND: Determine (a) the mass flow rate of R-134a, and (b) the rate of energy transfer from the condensing R-134a to the cooling water.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume is at steady state. (2) Heat transfer from the outside of the condenser is negligible. (3) Kinetic and potential energy changes from inlet to exit are negligible. (4) The cooling water is modeled as an incompressible liquid with constant specific heat.

ANALYSIS: (a) Since the R-134a and cooling water are separate streams

$$\underline{\text{R-134a}}: \dot{m}_1 = \dot{m}_2 \equiv \dot{m}_R$$
$$\underline{\text{Water}}: \dot{m}_A = \dot{m}_B \equiv \dot{m}_w$$

The mass flow rate of R-134a is found from the steady-state energy balance

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + \dot{m}\left[(h_1 - h_2) + \cancel{\left(\frac{V_1^2 - V_2^2}{2}\right)}^{0} + g\cancel{(z_1 - z_2)}^{0}\right] + \dot{m}_w\left[(h_A - h_B) + \cancel{\left(\frac{V_A^2 - V_B^2}{2}\right)}^{0} + g\cancel{(z_A - z_B)}^{0}\right]$$

Or

$$\dot{m}_R = \frac{\dot{m}_w (h_B - h_A)}{(h_1 - h_2)}$$

For the water, using Eq. 3.20b

$$h_B - h_A = c\,(T_B - T_A) + v\,\cancel{(p_B - p_A)}^{0}$$

and

$$\dot{m}_w = \frac{(AV)_w}{v_w}$$

From Table A-19E; $c \approx 1$ Btu/lb·°R, and $v_w = 1/\rho = 0.0161$ ft³/lb. For the R-134a, $h_1 = 129.53$ Btu/lb from Table A-12E and $h_2 = 27.24$ Btu/lb from Table A-11E. Inserting values

$$\dot{m}_R = \left(\frac{(35\,\text{gal/min})\left|\frac{0.13368\ \text{ft}^3}{1\ \text{gal}}\right|}{(0.0161\ \text{ft}^3/\text{lb})}\right)\frac{(1\ \text{Btu/lb}\cdot°\text{R})(18\ °\text{R})}{(129.53 - 27.24)\ \text{Btu/lb}}\left|\frac{60\ \text{min}}{1\ \text{h}}\right|$$

$$= 3068.3\ \text{lb/h} \longleftarrow \dot{m}_R$$

PROBLEM 4.58 (Cont'd)

(b) For a control volume enclosing only the R-134 a

$$0 = \dot{Q}_R - \cancel{\dot{W}_R}^{0} + \dot{m}_R\left[(h_1 - h_2) + \cancel{\left(\frac{V_1^2 - V_2^2}{2}\right)}^{0} + g\cancel{(z_1 - z_2)}^{0}\right]$$

where $\dot{Q}_R$ denotes the heat transfer rate for the R-134 a only. Thus

$$\dot{Q}_R = \dot{m}_R (h_2 - h_1)$$

$$= (3068.3 \text{ lb/h})(27.24 - 129.53)$$

① $$= -3.140 \times 10^5 \text{ Btu/h} \longleftarrow \dot{Q}_R$$

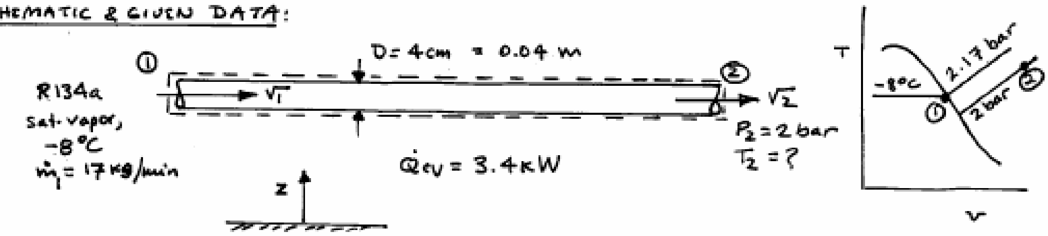
1. The negative value for $\dot{Q}_R$ denotes energy transfer heat from the R-134 a to the cooling water, as expected.

PROBLEM 4.64

KNOWN: Refrigerant 134a flows through a horizontal pipe at steady state, for which operating data are provided.

FIND: Determine the exit temperature and velocity, and the inlet velocity.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The control volume shown in the schematic is at steady state. 2. For the control volume, $\dot{W}_{cv} = 0$ and there is no change in potential energy from inlet to exit.

ANALYSIS: At steady state, the mass rate balance reduces to $\dot{m}_1 = \dot{m}_2 = \dot{m}$. The inlet velocity is found as follows:

$$\dot{m} = \frac{A_1 V_1}{v_1} \Rightarrow V_1 = \frac{\dot{m} v_1}{A_1} = \frac{4\dot{m} v_1}{\pi d^2}$$

with v_1 from Table A-10

$$V_1 = \frac{4(17 \text{ kg/min})\left|\frac{1 \text{ min}}{60 \text{ s}}\right|(0.0919 \text{ m}^3/\text{kg})}{\pi (0.04 \text{ m})^2} = 20.72 \text{ m/s} \longleftarrow V_1$$

Similarly, V_2 is found from $\dot{m}_1 = \dot{m}_2$ as follows:

$$\dot{m}_1 = \dot{m}_2 \Rightarrow \frac{\cancel{A_1} V_1}{v_1} = \frac{\cancel{A_2} V_2}{v_2} \Rightarrow V_2 = \frac{v_2}{v_1} V_1 \qquad (*)$$

In this expression, v_1 and V_1 are known. However, $v_2 = v(T_2, P_2)$ is unknown.

Another relation is obtained using the energy rate balance

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}}^{\,0} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(\cancel{z_1 - z_2}^{\,0})\right]$$

$$= \dot{Q}_{cv} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right)\right]$$

Inserting known values, including h_1 from Table A-10

$$0 = (3.4 \text{ kW})\left|\frac{1 \text{ kJ/s}}{1 \text{ kW}}\right| + \left(17 \frac{\text{kg}}{\text{min}}\right)\left|\frac{1 \text{ min}}{60 \text{ s}}\right|\left[(242.54 - h_2)\frac{\text{kJ}}{\text{kg}} + \left(\frac{(20.72 \text{ m/s})^2 - V_2^2}{2}\right)\left|\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right|\left|\frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}}\right|\right] \qquad (**)$$

where $h_2 = h(T_2, P_2)$.

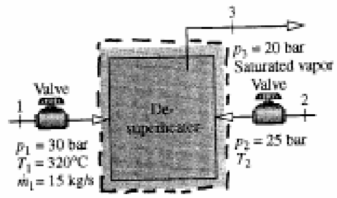
Equations (*) and (**) can be solved simultaneously by referring to Table A-12 for v_2 and h_2 as functions of T_2 and P_2. The process is iterative. To avoid iteration, IT can be used effectively, as follows:

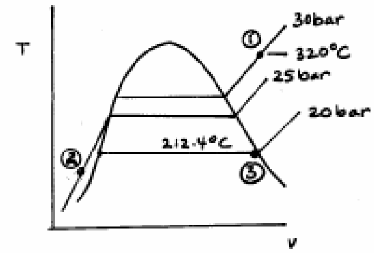
PROBLEM 4.66

KNOWN: Data are provided for a desuperheater operating at steady state.

FIND: (a) For a specified temperature for the entering liquid, determine the liquid mass flow rate. (b) Plot the liquid mass flow rate versus the liquid temperature.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: 1. A control volume enclosing the desuperheater with inlets at 1 and 2 and an exit at 3 is at steady state. 2. For the control volume, $\dot{W}_{cv} = 0$ and heat transfer with the surroundings can be ignored. Kinetic and potential energy effects can be ignored.

ANALYSIS: The mass rate balance at steady state reads $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$.

The energy rate balance at steady state reduces as follows:

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_1\left[h_1 + \cancel{\frac{V_1^2}{2}} + \cancel{gz_1}\right] + \dot{m}_2\left[h_2 + \cancel{\frac{V_2^2}{2}} + \cancel{gz_2}\right] - \dot{m}_3\left[h_3 + \cancel{\frac{V_3^2}{2}} + \cancel{gz_3}\right]$$

$$0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - (\dot{m}_1 + \dot{m}_2) h_3$$

Solving for $\dot{m}_2$

$$\dot{m}_2 = \dot{m}_1 \left[\frac{h_3 - h_1}{h_2 - h_3}\right] \qquad (1)$$

(a) From Table A-4; h_1 = 3043.4 kJ/kg. From Table A-3; h_3 = 2799.5 kJ/kg. Further, at T_2 = 200°C, p_2 = 25 bar; Table A-5 gives h_2 = 852.8 kJ/kg. Thus

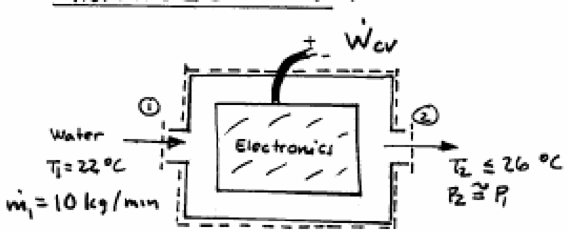
$$\dot{m}_2 = \left(15\ \frac{kg}{s}\right)\left[\frac{2799.5 - 3043.4}{852.8 - 2799.5}\right] = 1.88\ kg/s \qquad \underset{\text{(part a)}}{\dot{m}_2}$$

PROBLEM 4.71*

KNOWN: Data are provided for a water-jacketed housing filled with electronic components, which is at steady state.

FIND: Determine the maximum power input to satisfy a limit on the temperature of the water exiting the enclosure.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The control volume shown in the schematic is at steady state. 2. For the control volume, heat transfer with the surroundings can be ignored, as can kinetic/potential energy effects. 3. For the water

① entering and exiting the housing; $h \approx h_f(T)$.

ANALYSIS: At steady state, $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. An energy rate balance reads

$$0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancel{\left(\frac{V_1^2 - V_2^2}{2}\right)} + g\cancel{(z_1 - z_2)}\right]$$

giving

$$\dot{W}_{cv} = \dot{m}(h_1 - h_2)$$
$$= \dot{m}(h_f(T_1) - h_f(T_2))$$

Since $T_2 \leq 26°C$

$$\dot{W}_{cv} \geq \dot{m}(h_f(22°C) - h_f(26°C))$$
$$\geq \left(10 \frac{kg}{min}\right)(92.33 - 109.07)\frac{kJ}{kg}\left|\frac{1\ min}{60\ s}\right|\left|\frac{1\ kW}{1\ kJ/s}\right|$$

② $$\boxed{\dot{W}_{cv} \geq -2.79\ kW}$$

1. Alternatively, Eq. 3.20b with c from Table A-19 can be used.

2. By the usual sign convention, $\dot{W}_{cv}$ represents power output. Hence, the constraint on power input can be expressed alternatively in terms of the magnitude of $\dot{W}_{cv}$ as

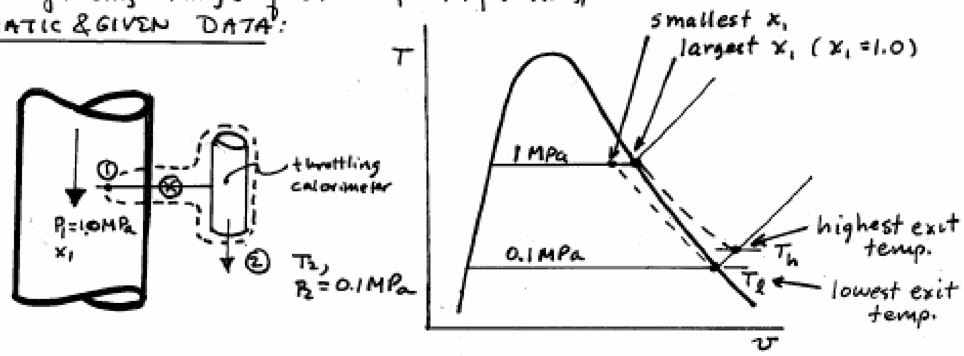
$$0 \leq |\dot{W}_{cv}| \leq 2.79\ kW$$

PROBLEM 4.76

KNOWN: Data are provided for a throttling calorimeter attached to a large pipe carrying a two-phase liquid–vapor mixture. Operation is at steady state. The substance is H_2O.

FIND: Determine the range of throttling calorimeter exit temperatures for which the device can determine the quality of steam in the pipe, and the corresponding range of steam quality values.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The control volume shown in the figure is at steady state. 2. The expansion through the calorimeter adheres to the throttling process model: $h_2 \approx h_1$, where $h_1 = h_{f1} + x_1(h_{g1} - h_{f1})$.

ANALYSIS: With assumption 2

$$h(T_2, p_2) = h_{f1} + x_1(h_{g1} - h_{f1}) \qquad (1)$$

where the state at the exit is fixed by T_2, p_2, and thus must be superheated vapor or, in the limit, saturated vapor. Also, the quality x_1 is required to be less than or, in the limit, equal to 1.0.

Referring to the T-v diagram, the highest exit temperature, T_h, corresponds to a steam quality of 1.0: That is, with data from Table A-3, Eq. (1) gives

$$h(T_h, p_2) = h_g(p_1)$$
$$= 2778.1 \text{ kJ/kg}$$

Interpolating in Table A-4 at 0.1 MPa gives $T_h = 151°C$.

The lowest exit temperature, T_l, corresponds to saturated vapor exiting the calorimeter, for which $h_2 = 2675.5$ kJ/kg and $T_l = 99.6°C$. Eq. (1) gives

$$x_1 = \frac{h_2 - h_{f1}}{h_{g1} - h_{f1}} = \frac{2675.5 - 762.81}{2015.3} = 0.949$$

In summary

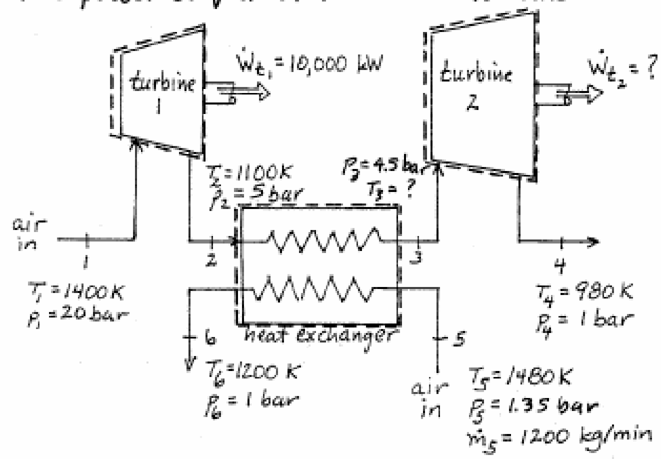
$$99.6 \le T_2 \le 151 \ °C$$
$$0.949 \le x_1 \le 1.0$$

PROBLEM 4.82

KNOWN: Air flows through two turbine stages and an inter-connecting heat exchanger. A separate hot air stream passes in counter flow through the heat exchanger. Data are known at various locations.

FIND: Determine the temperature of the main air stream exiting the heat exchanger and the power output of the second turbine.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volumes are at steady state. (2) Heat transfer to the surroundings can be neglected. (3) Kinetic and potential energy effects are negligible. (4) The air behaves as an ideal gas. (5) For the heat exchanger, $\dot{W}_{cv} = 0$.

ANALYSIS: First, find the air flow rate at 1. Begin with steady-state energy and mass balances for turbine 1

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \dot{W}_{t_1} + \dot{m}_1\left[(h_1 - h_2) + \cancel{\left(\frac{V_1^2 - V_2^2}{2}\right)}^{0} + g\,\cancel{(z_1 - z_2)}^{0}\right]$$

and $\quad 0 = \dot{m}_1 - \dot{m}_2 \Rightarrow \dot{m}_1 = \dot{m}_2$

Solving for $\dot{m}_1$

$$\dot{m}_1 = \frac{\dot{W}_{t_1}}{(h_1 - h_2)}$$

From Table A-22; $h_1 = 1515.42$ kJ/kg and $h_2 = 1161.07$ kJ/kg. Thus

$$\dot{m}_1 = \frac{(10{,}000 \text{ kW})}{(1515.42 - 1161.07)\text{ kJ/kg}}\left|\frac{1 \text{ kJ/s}}{1 \text{ kW}}\right| = 28.22 \text{ kg/s}$$

Turning next to the heat exchanger

$$\dot{m}_1 = \dot{m}_2 = \dot{m}_3 = \dot{m}_4$$
$$\dot{m}_5 = \dot{m}_6$$

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + \dot{m}_2\left[(h_2 - h_3) + \cancel{\left(\frac{V_2^2 - V_3^2}{2}\right)}^{0} + g\,\cancel{(z_2 - z_3)}^{0}\right] + \dot{m}_5\left[(h_5 - h_6) + \cancel{\left(\frac{V_5^2 - V_6^2}{2}\right)}^{0} + g\,\cancel{(z_5 - z_6)}^{0}\right]$$

$$0 = \dot{m}_2(h_2 - h_3) + \dot{m}_5(h_5 - h_6)$$

or

$$h_3 = h_2 + \frac{\dot{m}_5}{\dot{m}_2}(h_5 - h_6)$$

Again, from Table A-22; $h_5 = 1611.79$ kJ/kg and $h_6 = 1277.79$ kJ/kg. Thus

PROBLEM 4.82 (Cont'd)

$$h_3 = 1161.07 + \left[\frac{(1200\ \text{kg/min})}{(28.22\ \text{kg/s})|60\ \text{s}/1\text{min}|}\right](1611.79 - 1277.79)$$

$$= 1397.8\ \text{kJ/kg}$$

Interpolating in Table A-22

$$T_3 = 1301.5\text{K} \longleftarrow \underline{T_3}$$

Now, writing the steady-state energy balance for turbine 2

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \dot{W}_{t_2} + \dot{m}_3\left[(h_3 - h_4) + \cancel{\left(\frac{V_3^2 - V_4^2}{2}\right)}^{0} + g\cancel{(z_3 - z_4)}^{0}\right]$$

or

$$\dot{W}_{t_2} = \dot{m}_3 (h_3 - h_4)$$

From Table A-22; $h_4 = 1023.25$ kJ/kg, and

$$\dot{W}_{t_2} = (28.22\ \text{kg/s})(1397.8 - 1023.25)\ \text{kJ/kg}\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right|$$

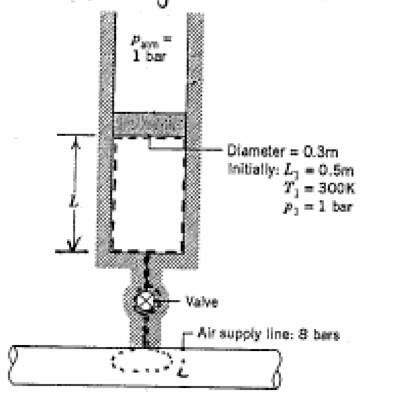
$$= 10{,}570\ \text{kW} \longleftarrow \underline{\dot{W}_{t_2}}$$

PROBLEM 4.99

KNOWN: Air is admitted slowly into an insulated piston-cylinder assembly from a supply line until the volume inside the cylinder has doubled.

FIND: Plot the final temperature and mass inside the cylinder for a given range of supply temperatures.

SCHEMATIC & GIVEN DATA:



$300 \le T_{supply} \le 500K$

ASSUMPTIONS: (1) For the control volume shown, $\dot{Q}_{cv} = 0$. (2) Kinetic and potential energy effects can be neglected. (3) The weight of the piston and friction between the piston and cylinder wall can be neglected. (4) The air behaves as an ideal gas.

ANALYSIS: The mass rate balance takes the form $dm_{cv}/dt = \dot{m}_i$. With the assumptions listed, the energy balance is

$$\frac{dU_{cv}}{dt} = \cancel{\dot{Q}_{cv}}^{\,0} - \dot{W}_{cv} + \dot{m}_i h_i$$

The line conditions are constant, so h_i is constant. Combining the mass and energy rate balances and integrating

$$m_2 u_2 - m_1 u_1 = -W_{cv} + h_i (m_2 - m_1) \qquad (1)$$

To evaluate the work, note that the pressure in the cylinder is always atmospheric since the process is slow and since assumption (3) applies. Thus

$$W_{cv} = \int_{V_1}^{V_2} p\,dV = p(V_2 - V_1)$$

From the given data

$$V_1 = \left(\frac{\pi d_{pist}^2}{4}\right) L_1 = \left(\frac{\pi (.3^2) m^2}{4}\right)(0.5\,m) = 0.03534\ m^3$$

and $\quad V_2 = 0.07068\ m^2$

$$W_{cv} = (1\,bar)(0.07068 - 0.03534)m^3 \left|\frac{10^5 N/m^2}{1\,bar}\right| \left|\frac{1\,kJ}{10^3\,N\cdot m}\right|$$

$$= 3.534\ kJ$$

From Table A-22, at $T_1 = 300K$; $u_1 = 214.07$ kJ/kg. Also, $h_i = h_i(T_{supply})$. For $T_{supply} = 300K$, $h_i = 300.19$ K.

Using the ideal gas model

$$m_1 = \frac{P_1 V_1}{RT_1} = \frac{(1\,bar)(0.03534\,m^3)}{\left(\frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\right)(300K)} \left|\frac{10^5 N/m^2}{1\,bar}\right| \left|\frac{1\,kJ}{10^3 N\cdot m}\right| = 0.04105\ kg$$

and

$$m_2 = \frac{P_2 V_2}{RT_2} = \frac{(1)(0.07068)|100|}{\left(\frac{8.314}{28.97}\right)(T_2)} = \frac{24.628}{T_2} \qquad (2)$$

Incorporating these results into (1) and rearranging

PROBLEM 4.99 (Cont'd)

$$m_2 u_2 - h_i (m_2 - m_1) = m_1 u_1 - W_{cv}$$

$$\frac{24.628\, u_2}{T_2} - h_i \left(\frac{24.628}{T_2} - 0.04105\right) = 5.254 \qquad (3)$$

Equations (2) and (3) can be solved for given values of h_i (T_{supply}) by an iterative procedure and data from Table A-22. For $T_{supply} = 300K$,

$$T_2 = 300K$$

$$m_2 = 0.0821 \text{ kg}$$

Plotting for the range of T_{supply} values