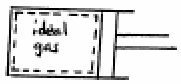
PROBLEM 5.15

KNOWN: The gas temperature scale introduced in Sec. 1.6.2 and the Kelvin scale defined in Sec. 5.4.

FIND: Demonstrate that the two scales are identical.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system shown in the figure consists of a gas obeying $pv = RT'$, where T' denotes temperature on the gas scale. (2) The system undergoes a reversible cycle while communicating thermally with reservoirs at T_H' and T_C'. (3) The cycle consists of four processes in series: 1-2 isothermal at T_H', 2-3 adiabatic, 3-4 isothermal at T_C', 4-1 adiabatic. (4) Kinetic and potential energy effects are absent.

(1)

ANALYSIS: With assumption (4), an energy balance in differential form reads

$$du = \delta(Q/m) - \delta(W/m)$$

Then, since the processes are internally reversible, $\delta(W/m) = p\,dv$. Also, for the ideal gas, $du = c_v\,dT'$. Collecting results

$$c_v\,dT' = \delta(Q/m) - p\,dv \Rightarrow c_v\,dT' = \delta(Q/m) - \frac{RT'}{v}dv \Rightarrow \underline{c_v\,dT' = \delta(Q/m) - RT'\,d\ln v}$$

Process 1-2: The gas temperature is constant at T_H', so

$$\cancel{c_v\,dT'}^{\,0} = \delta(Q/m)_{1-2} - RT_H'\,d\ln v \Rightarrow (Q/m)_{1-2} = RT_H'\ln(v_2/v_1) \qquad (1)$$

Process 2-3: There is no heat transfer, and the gas temperature varies from T_H' to T_C', so

$$c_v\,dT' = \cancel{\delta(Q/m)}^{\,0} - RT'\,d\ln v \Rightarrow c_v\frac{dT'}{T'} = -R\,d\ln v \Rightarrow \int_{T_H'}^{T_C'} \frac{c_v\,dT'}{T'} = -R\ln\left(\frac{v_3}{v_2}\right) \qquad (2)$$

Process 3-4: The gas temperature is constant at T_C', and the heat rejected in the process is

$$(Q/m)_{3-4} = RT_C'\ln(v_4/v_3) \Rightarrow |(Q/m)_{3-4}| = RT_C'\ln\left(\frac{v_3}{v_4}\right) \qquad (3)$$

Process 4-1: There is no heat transfer, and the gas temperature varies from T_C' to T_H', so

$$\int_{T_C'}^{T_H'} \frac{c_v\,dT'}{T'} = -R\ln\left(\frac{v_1}{v_4}\right) \Rightarrow \int_{T_H'}^{T_C'} \frac{c_v\,dT'}{T'} = -R\ln\left(\frac{v_4}{v_1}\right) \qquad (4)$$

From Equations (2) and (4), $v_3/v_2 = v_4/v_1$ or $v_3/v_4 = v_2/v_1$. Using this result together with Equations (1) and (3) gives

$$\frac{|(Q/m)_{3-4}|}{(Q/m)_{1-2}} = \frac{T_C'}{T_H'}$$

which corresponds to Equation 5.6 underlying the Kelvin scale. This implies $T' \equiv T$, where T denotes temperature on the Kelvin scale.

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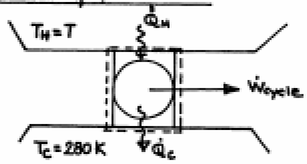
1. The reversible cycle considered here corresponds to the Carnot cycle discussed in Sec. 5.6.

PROBLEM 5.26

KNOWN: Steady state operating data are provided for a system undergoing a power cycle while receiving and discharging energy by heat transfer with a hot and cold reservoir at temperatures T and 280 K, respectively

FIND: Determine the minimum theoretical value for T, in K.

SCHEMATIC & GIVEN DATA:



$\dot{W}_{cycle} = 40 kW$
$\dot{Q}_C = 1000 kJ/min$

ASSUMPTIONS: (1) The system is shown on the accompanying figure. (2) The system undergoes a power cycle. (3) The data provided is for operation at steady state.

ANALYSIS: An energy rate balance gives

① $$\dot{W}_{cycle} = \dot{Q}_H - \dot{Q}_C$$

or

$$\dot{Q}_H = \dot{W}_{cycle} + \dot{Q}_C$$
$$= 40 kW\left(\frac{1 kJ/s}{1 kW}\right) + 1000 \frac{kJ}{min}\left(\frac{min}{60 s}\right)$$
$$= 56.67 kJ/s$$

We know that $\eta \leq \eta_{max}$; that is

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_H} \leq 1 - \frac{T_C}{T}$$

Inserting values

$$\frac{40 kJ/s}{56.67 kJ/s} \leq 1 - \frac{280K}{T}$$

$$\Rightarrow \quad \frac{280}{T} \leq 1 - \frac{40}{56.67} = 0.294$$

$$\Rightarrow \quad \underbrace{952K}_{T_{min}} \leq T \quad \longleftarrow T_{min}$$

---

1. At steady state, the cycle energy balance and thermal efficiency can be expressed in terms of rates:

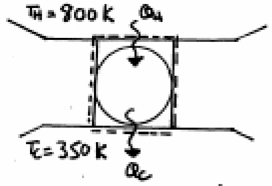
$$\dot{W}_{cycle} = \dot{Q}_H - \dot{Q}_C$$
$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_H}$$

PROBLEM 5.29*

KNOWN: A system undergoes a cycle while receiving and discharging energy by heat transfer with two reservoirs of known temperatures.

FIND: For each of two specified values of thermal efficiency, determine if the claimed operation is feasible.

SCHEMATIC & GIVEN DATA:



(a) $\eta = 56\%$

(b) $\eta = 40\%$

ASSUMPTION: The system shown in the schematic undergoes a power cycle.

ANALYSIS: The maximum thermal efficiency for any power cycle under the stated conditions is given by Eq. 5.8:

$$\eta_{max} = 1 - \frac{T_C}{T_H} = 1 - \frac{350}{800} = 0.5625 \ (56.25\%)$$

(a) $\eta = 56\%$. Since this claimed value is nearly the maximum theoretical value, the cycle must be nearly ideal in operation. Although not ruled out by the second law, it is unlikely for an actual power cycle to perform at such a level.

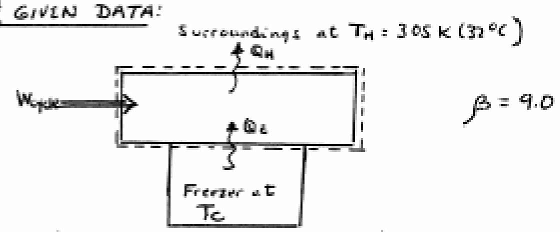
(b) $\eta = 40\%$. Since the claimed value is less than η_{max}, the cycle is feasible.

PROBLEM 5.41

KNOWN: A tray of ice cubes is placed in a freezer having a coefficient of performance of 9.0 operating in a room at 32°C.

FIND: Determine if the cubes would remain frozen.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system shown in the accompanying figure undergoes a refrigeration cycle. (2) The freezer compartment and the surroundings play the roles of the cold and hot reservoirs, respectively.

ANALYSIS: As discussed in Sec. 5.52, the actual coefficient of performance must be less than, or equal to, the coefficient of performance of a reversible refrigeration cycle operating between reservoirs at the specified temperatures. Then, with Equation 5.9 and the given value of 9.0 for the coefficient of performance

$$9.0 \leq \frac{T_C}{T_H - T_C} = \frac{1}{(305/T_C) - 1}$$

or

$$\frac{305}{T_C} \leq 1 + \frac{1}{9}$$

giving

$$T_C \gtrapprox 275 \text{ K}$$

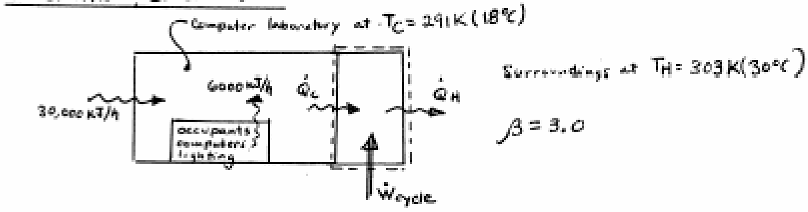
However, to maintain the frozen cubes the freezer must be at 273 K, or less. Accordingly, the cubes would not remain frozen.

PROBLEM 5.47

KNOWN: A refrigeration cycle maintains a computer laboratory at 18°C on a day when the outside temperature is 30°C. Steady state operating data are provided.

FIND: Determine the power required, in kW, and compare with the minimum theoretical power required.

SCHEMATIC & GIVEN DATA



ASSUMPTIONS: (1) The system shown in the accompanying figure undergoes a refrigeration cycle. (2) All data provided are for operation at steady state. (3) The computer laboratory and surroundings play the roles of cold and hot reservoirs, respectively.

ANALYSIS: At steady state, the refrigeration cycle must remove energy from the computer laboratory at the same rate as energy enters from all sources: $\dot{Q}_C = (30,000 + 6000)\ kJ/h = 36,000\ kJ/h$. The coefficient of performance is given as 3.0. Thus

$$\beta = \frac{\dot{Q}_C}{\dot{W}_{cycle}} \Rightarrow 3.0 = \frac{36,000\ kJ/h}{\dot{W}_{cycle}} \Rightarrow \dot{W}_{cycle} = 12,000\ \frac{kJ}{h}$$

or

$$\dot{W}_{cycle} = \left(12000\ \frac{kJ}{h}\right)\left(\frac{h}{3600\ s}\right)\left(\frac{1\ kW}{1\ kJ/s}\right) = \mathbf{3.33}\ kW \longleftarrow \dot{W}_{cycle}$$

From Sec 5.5.2, we know that the actual coefficient of performance must be less than, or equal to, the coefficient of performance of a reversible refrigeration cycle operating between reservoirs at the specified temperatures. Then, with Equation 5.9

$$\frac{\dot{Q}_C}{\dot{W}_{cycle}} \leq \frac{T_C}{T_H - T_C} = \frac{291}{303 - 291} = 24.25$$

Accordingly

$$\frac{36,000\ kJ/h}{24.25} \leq \dot{W}_{cycle}$$

$$1484.5\ \frac{kJ}{h} \leq \dot{W}_{cycle}$$

or

$$\dot{W}_{cycle} \geq \left(1484.5\ \frac{kJ}{h}\right)\left|\frac{1\ h}{3600\ s}\right|\left|\frac{1\ kW}{1\ kJ/s}\right| = 0.41\ kW \longleftarrow (\dot{W}_{cycle})_{min}$$

Forming the ratio of the actual power required to the minimum theoretical value

$$\frac{3.33\ kW}{0.41\ kW} = 8.12$$

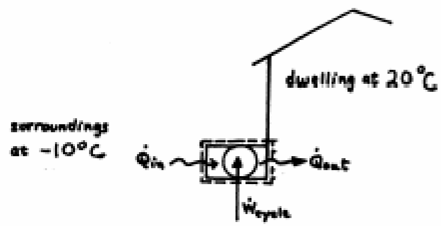
it can be concluded that the actual requirement is over 8 times greater than the minimum theoretical requirement.

PROBLEM 5.55*

KNOWN: A heat pump maintains a dwelling at a specified temperature.

FIND: Determine the minimum theoretical cost.

SCHEMATIC & GIVEN DATA:



$\dot{Q}_{out} = 25{,}000\ kJ/h$

cost of electricity = 0.08 \$/kW·h

ASSUMPTIONS: (1) The system shown on the accompanying figure undergoes a heat pump cycle. (2) The data are for operation at steady state. (3) The dwelling and the surroundings play the roles of hot and cold reservoirs, respectively.

ANALYSIS: The minimum theoretical cost for any heat pump cycle under the stated conditions is the cost for a reversible cycle operating between reservoirs at $T_H = 293\,K$ (20°C) and $T_C = 263\,K$ (−10°C). The power required by such a cycle can be obtained from

$$(\dot{W}_{cycle})_{min} = \frac{\dot{Q}_{out}}{\gamma_{max}}$$

where γ_{max} is

$$\gamma_{MAX} = \frac{293}{293-263} = 9.77$$

Accordingly

$$(\dot{W}_{cycle})_{Min} = \frac{(25{,}000\ \frac{kJ}{h})}{(9.77)}\left|\frac{1\ kW}{1\ kJ/s}\right|\left|\frac{1\ h}{3600\ s}\right| = 0.71\ kW$$

Then, the minimum cost for one day of operation is

$$\text{Minimum theoretical cost: } \dot{\$} = (0.71\,kW)\left|\frac{24\,h}{day}\right|(\$\,0.08/day)$$

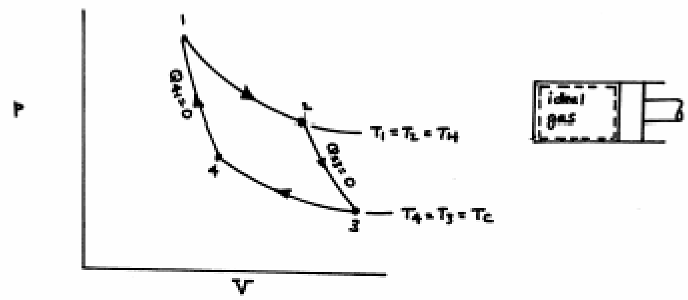
$$= \$\,1.36/day \longleftarrow \frac{MIN}{COST}$$

PROBLEM 5.64

KNOWN: A carnot cycle is executed by an ideal gas with constant specific heat ratio k.

FIND: Show that (a) $V_4 V_2 = V_1 V_3$, (b) $T_2/T_3 = (p_2/p_3)^{(k-1)/k}$, (c) $T_2/T_3 = (V_3/V_2)^{k-1}$

SCHEMATIC & GIVEN DATA.



ASSUMPTIONS: (1) The system shown in the figure consists of an ideal gas. (2) The specific heat ratio k is constant (required in part (b) only). (3) The system undergoes a Carnot cycle.

ANALYSIS: (a) The thermal efficiency is

$$\eta = \frac{W_{cycle}}{Q_{in}} = \frac{W_{12} + W_{23}}{Q_{12}}$$

Since internal energy of an ideal gas depends on temperature only, an energy balance for process 1-2 reduces to $U_2 - U_1 = Q_{12} - W_{12}$, where $U_2 = U_1$. Thus, $Q_{12} = W_{12}$. Furthermore

$$W_{12} = \int_1^2 p\,dV = \int_1^2 \frac{mRT_H}{V}\,dV = mRT_H \ln\frac{V_2}{V_1}$$

Similarly, $W_{34} = mRT_C \ln V_4/V_3$. Collecting results

$$\eta = 1 - \frac{mRT_C \ln V_3/V_4}{mRT_H \ln V_2/V_1} = 1 - \left(\frac{\ln V_3/V_4}{\ln V_2/V_1}\right)\frac{T_C}{T_H}$$

However, for the Carnot cycle $\eta = 1 - T_C/T_H$, so it is necessary that

$$\frac{\ln(V_3/V_4)}{\ln(V_2/V_1)} = 1 \Rightarrow \ln\left(\frac{V_3}{V_4}\right) = \ln\left(\frac{V_2}{V_1}\right) \Rightarrow V_4V_2 = V_3V_1 \longleftarrow \quad (a)$$

(b) As process 2-3 is adiabatic, a energy balance in differential form reads

$$dU = \cancel{\delta Q}^{\,0} - \delta W$$

where $\delta W = p\,dV$ and with assumption 1 $dU = mc_v\,dT$. Collecting results and using $pV = mRT$ and $c_v = R/(k-1)$ (Eq. 3.47b)

$$\frac{1}{k-1}\,d\ln T = -\,d\ln V$$

Integration for constant k (assumption 2) gives

$$\ln\frac{T_3}{T_2} = -\ln\left(\frac{V_3}{V_2}\right)^{k-1} \Rightarrow \frac{T_3}{T_2} = \left(\frac{V_2}{V_3}\right)^{k-1} \longleftarrow \quad (c)$$

Finally, using $V = mRT/p$

$$\frac{T_3}{T_2} = \left[\frac{T_2}{T_3}\,\frac{p_3}{p_2}\right]^{k-1} \Rightarrow \frac{T_3}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{k-1}{k}} \longleftarrow \quad (b)$$