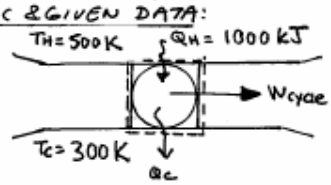
PROBLEM 6.1*

KNOWN: A system undergoes a power cycle for which data are provided.

FIND: Using Eq. 6.2, evaluate σ_{cycle} if η is 60, 40, and 20%. Discuss.

SCHEMATIC & GIVEN DATA:



ASSUMPTION: The system shown in the accompanying figure undergoes a power cycle while receiving Q_H at T_H and discharging Q_C at T_C.

ANALYSIS: For any cycle, $\eta = 1 - Q_C/Q_H \Rightarrow Q_C = (1-\eta)Q_H$. Then, Eq. 6.2 gives

$$\sigma_{cycle} = -\oint \left(\frac{\delta Q}{T}\right)_b = -\left[\frac{Q_H}{T_H} - \frac{Q_C}{T_C}\right] = -Q_H\left[\frac{1}{T_H} - \frac{(1-\eta)}{T_C}\right]$$

(a) $\eta = 60\%$

$$\sigma_{cycle} = -(1000\text{ kJ})\left[\frac{1}{500\text{ K}} - \frac{(1-0.6)}{300\text{ K}}\right] = -0.667\,\frac{\text{kJ}}{\text{K}}$$

① Since σ_{cycle} must be positive or zero in value, this case is impossible.

(b) $\eta = 40\%$

$$\sigma_{cycle} = -(1000\text{ kJ})\left[\frac{1}{500\text{ K}} - \frac{(1-0.4)}{300\text{ K}}\right] = 0$$

Since $\sigma_{cycle} = 0$, this case corresponds to internally reversible operation.

(c) $\eta = 20\%$

$$\sigma_{cycle} = -(1000\text{ kJ})\left[\frac{1}{500\text{ K}} - \frac{(1-0.2)}{300\text{ K}}\right] = +0.667\,\frac{\text{kJ}}{\text{K}}$$

In this case, irreversibilities are present within the system.

---

1. Using Eq. 5.8, the maximum thermal efficiency any cycle can have while operating between reservoirs at T_H and T_C is

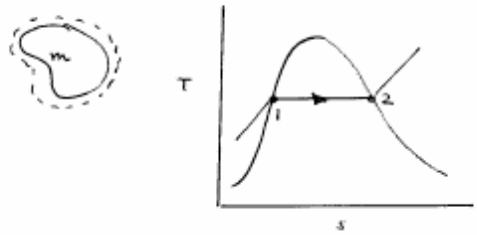
$$\eta_{MAX} = 1 - \frac{T_C}{T_H} = 1 - \frac{300}{500} = 0.4 \ (40\%)$$

PROBLEM 6.11

KNOWN: A fixed mass of water, initially a saturated liquid, undergoes a constant temperature, constant pressure process to a saturated vapor condition.

FIND: (a) Derive expressions for W, Q in terms of m and steam table properties. (b) Show that the process is internally reversible.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system is a fixed mass of water. (2) Kinetic and potential energy effects are absent. (3) In the process, p = constant, T = constant. (4) Volume change is the only work mode.

ANALYSIS: (a) With assumption (4) and since the pressure remains constant,

$$W = m\int_{1}^{2} p\,dv = m\,p\,(v_2 - v_1) = m\,p\,[v_g - v_f] \longleftarrow W$$

With assumption (2) and the above expression for W, an energy balance gives

$$m[u_2 - u_1] = Q - W \Rightarrow Q = m[u_g - u_f] + mp(v_g - v_f)$$

$$= m[(u_g + pv_g) - (u_f + pv_f)]$$

$$= m[h_g - h_f] \longleftarrow Q$$

(b) An entropy balance reduces to

$$m(s_2 - s_1) = \frac{Q}{T} + \sigma$$

Introducing the expression for Q

$$m(s_g - s_f) = \frac{m(h_g - h_f)}{T} + \sigma$$

From the discussion of Sec. 6.3.1, Eq. 6.14, $s_g - s_f = (h_g - h_f)/T$, and so

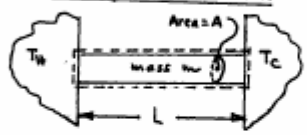
$\sigma = 0 \Rightarrow$ the process is internally reversible.

PROBLEM 6.15

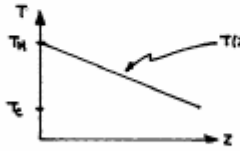
KNOWN: The temperature within a rod initially in contact with hot and cold walls at its ends is linear with position. The rod is insulated overall and eventually comes to a final equilibrium state where the temperature is T_f.

FIND: Evaluate the final temperature and the amount of entropy produced.

SCHEMATIC & GIVEN DATA:



$T_c < T_H$



$$T(z) = T_H - \left(\frac{T_H - T_c}{L}\right) z$$

ASSUMPTIONS: (1) The system is the rod which is insulated on its lateral surface. (2) The rod is modeled as incompressible with constant specific heat c. (3) Initially, the temperature within the rod varies linearly from T_H to T_c.

ANALYSIS: The final temperature can be determined using an energy balance which reduces to give $\Delta U = \cancel{Q}^{0} - \cancel{W}^{0}$ or $\Delta U = 0$. Each element of rod dz changes temperature from T(z) to the final temperature T_f, and thus contributes to the change in internal energy

$$dU = dm\, c\,(T_f - T(z))$$
$$= (\rho A\, dz)\, c\,(T_f - T(z))$$

Accordingly

$$\Delta U = \int_0^L (\rho A dz)\, c\,(T_f - T(z))$$
$$= \rho A c \int_0^L \left[T_f - T_H + \left(\frac{T_H - T_c}{L}\right) z\right] dz$$
$$= \rho A c \left[(T_f - T_H) z + \left(\frac{T_H - T_c}{L}\right)\frac{z^2}{2}\right]_0^L$$
$$= \rho A c L \left[(T_f - T_H) + \left(\frac{T_H - T_c}{2}\right)\right]$$

Since $\Delta U = 0$, $T_f = (T_H + T_c)/2$. $\longleftarrow$ T_f

To find the entropy production, an entropy balance reduces to give $\Delta S = \cancel{\int \frac{dQ}{T}}^{0} + \sigma$ or $\sigma = \Delta S$. With Eq. 6.24, the entropy change of an element of rod dz is

$$dS = dm\, c \ln \frac{T_f}{T(z)}$$
$$= (\rho A dz)\, c \ln \frac{T_f}{T(z)}$$

Accordingly

$$\sigma = \rho A c \int_0^L (\ln T_f - \ln T(z))\, dz = \rho A c \left[(\ln T_f) L - \int_0^L (\ln T(z))\, dz\right]$$

Using the given temperature distribution, the variable of integration can be changed from z to T:

$$dT = -\left(\frac{T_H - T_c}{L}\right) dz \quad \Rightarrow \quad dz = -\left(\frac{L}{T_H - T_c}\right) dT$$

With this, the integral can be expressed as

PROBLEM 6.15 (Contd.)

$$\int_0^L (\ln T(z))\,dz = \int_{T_H}^{T_C} (\ln T)\left(\frac{-L}{T_H - T_C}\right) dT$$

$$= \frac{L}{(T_H - T_C)} \int_{T_C}^{T_H} \ln T \, dT$$

$$= \frac{L}{(T_H - T_C)} \Big[T \ln T - T \Big]_{T_C}^{T_H}$$

$$= \frac{L}{T_H - T_C} \Big[(T_H \ln T_H - T_H) - (T_C \ln T_C - T_C) \Big]$$

$$= L \left[\frac{T_H \ln T_H}{T_H - T_C} - \frac{T_C \ln T_C}{T_H - T_C} - 1 \right]$$

Collecting results

$$\sigma = \rho A c \left[(\ln T_f) L - L \left[\frac{T_H \ln T_H}{T_H - T_C} - \frac{T_C \ln T_C}{T_H - T_C} - 1 \right] \right]$$

$$= mc \left[1 + \ln T_f + \frac{T_C \ln T_C}{T_H - T_C} - \frac{T_H \ln T_H}{T_H - T_C} \right] \longleftarrow \sigma$$

PROBLEM 6.22

FIND: Determine the change in specific entropy, in Btu/lb·°R.

(a) Water. $p_1 = 1000\ lbf/in^2$, $T_1 = 800°F$, $p_2 = 1000\ lbf/in^2$, $T_2 = 100°F$

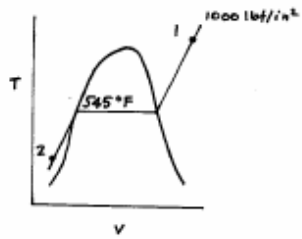


Table A-4E. $s_1 = 1.5665$ Btu/lb·°R

Table A-5E. $s_2 = 0.12901$ Btu/lb·°R

$$s_2 - s_1 = -1.43749 \text{ Btu/lb·°R} \longleftarrow \text{(a)}$$

(b) R-134a. $h_1 = 47.91$ Btu/lb, $T_1 = -40°F$, saturated vapor at $p_2 = 40\ lbf/in^2$

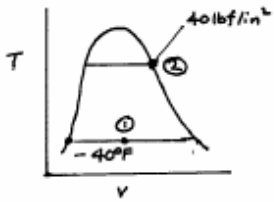


Table A-10E at $-40°F$: $h_f = 0$, $h_{fg} = 95.82$ Btu/lb

$$\therefore\ x_1 = \frac{47.91 - 0}{95.82} = 0.5$$

And

$$s_1 = s_f + x_1(s_g - s_f) = 0 + 0.5(0.2283 - 0) = 0.11415 \text{ Btu/lb·°R}.$$

Table A-11E, $s_2 = 0.2197$ Btu/lb·°R. Then

$$s_2 - s_1 = 0.2197 - 0.11415 = 0.1056 \text{ Btu/lb·°R} \longleftarrow \text{(b)}$$

(c) Air as an ideal gas. $T_1 = 40°F$, $p_1 = 2$ atm, $T_2 = 420°F$, $p_2 = 1$ atm.
With Eq. 6.25a and s° data from Table A-22E at 500°R and 880°R

$$s_2 - s_1 = s^\circ(T_2) - s^\circ(T_1) - \frac{\bar{R}}{M}\ln\frac{p_2}{p_1}$$

$$= (0.71886 - 0.58233)\left(\frac{\text{Btu}}{\text{lb·°R}}\right) - \left(\frac{1.986}{28.97}\ \frac{\text{Btu}}{\text{lb·°R}}\right)\ln\frac{1}{2}$$

$$= 0.18405 \text{ Btu/lb·°R} \longleftarrow \text{(c)}$$

(d) Carbon dioxide as an ideal gas. $T_1 = 820°F$, $p_1 = 1$ atm, $T_2 = 77°F$, $p_2 = 3$ atm.
With Eq. 6.25b and $\bar{s}^\circ$ data from Table A-23E at 1280°R and 537°R

$$\bar{s}_2 - \bar{s}_1 = \bar{s}^\circ(T_2) - \bar{s}^\circ(T_1) - \bar{R}\ln\frac{p_2}{p_1}$$

$$= (51.032 - 60.044)\left(\frac{\text{Btu}}{\text{lbmol·°R}}\right) - \left(1.986\ \frac{\text{Btu}}{\text{lbmol·°R}}\right)\ln\frac{3}{1}$$

$$= -11.1938\ \frac{\text{Btu}}{\text{lbmol·°R}}$$

Then, with the molecular weight of CO_2 from Table A-1E

$$s_2 - s_1 = \frac{\bar{s}_2 - \bar{s}_1}{M}$$

$$= \frac{-11.1938\ \frac{\text{Btu}}{\text{lbmol·°R}}}{44.01\ \text{lb/lbmol}}$$

$$= -0.25435 \text{ Btu/lb·°R} \longleftarrow \text{(d)}$$

PROBLEM 6.25

FIND: Determine the change in specific entropy, in kJ/kg·K, using the ideal gas model and three alternative approaches.

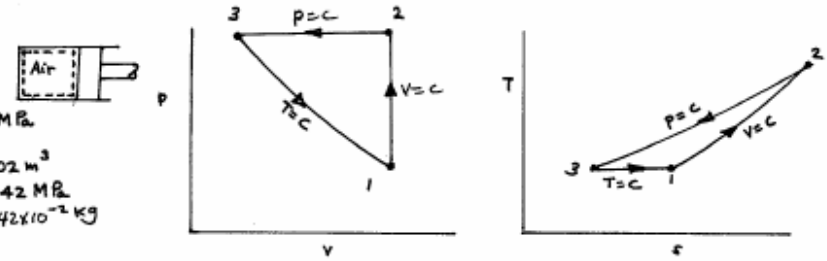
① case	Ideal Gas Table	Constant Specific Heat
	$\Delta s = s^\circ(T_2) - s^\circ(T_1) - R\ln\frac{p_2}{p_1}$ (6.21a)	$\Delta s = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}$ (6.23)
(a) Air $p_1 = 100$ kPa $p_2 = 100$ kPa $T_1 = 293$ K $T_2 = 373$ K	With s° data from Table A-22 $\Delta s = 1.92119 - 1.678298 - 0$ $= 0.24289$ kJ/kg·K	With c_p at 333 K from Table A-20 $\Delta s = 1.007 \ln\frac{373}{293} - 0$ (a) $= 0.2481$ kJ/kg·K
(b) Air $p_1 = 1$ bar $p_2 = 3$ bar $T_1 = 300$ K $T_2 = 650$ K	With s° data from Table A-22 $\Delta s = 2.49364 - 1.70203 - \frac{8.314}{28.97}\ln\frac{3}{1}$ $= 0.47632$ kJ/kg·K	With c_p at 475 K from Table A-20 $\Delta s = 1.0245 \ln\frac{650}{300} - \frac{8.314}{28.97}\ln\frac{3}{1}$ (b) $= 0.47684$ kJ/kg·K
(c) CO_2 $p_1 = 150$ kPa $p_2 = 200$ kPa $T_1 = 303$ K $T_2 = 573$ K	With $\bar{s}^\circ$ data from Table A-23 and M from Table A-1 $\Delta s = \frac{241.033 - 214.284 - 8.314\ln\frac{200}{150}}{44.01}$ $= 0.4769$ kJ/kg·K	With c_p at 438 K from Table A-20 $\Delta s = .9686 \ln\frac{573}{303} - \frac{8.314}{44.01}\ln\frac{200}{150}$ (c) $= 0.4862$ kJ/kg·K
(d) CO $T_1 = 300$ K $T_2 = 500$ K $v_1 = 1.1$ m³/kg $v_2 = 0.75$ m³/kg	With $pv = RT$ $\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)\left(\frac{v_1}{v_2}\right)$ $= \left(\frac{500}{300}\right)\left(\frac{1.10}{0.75}\right) = 2.444$ With $\bar{s}^\circ$ data from Table A-23 and M from Table A-1 $\Delta s = \frac{212.719 - 197.723 - 8.314\ln 2.444}{28.01}$ $= 0.2701$ kJ/kg·K	With c_p at 400 K from Table A-20 $\Delta s = 1.047 \ln\frac{500}{300} - \frac{8.314}{28.01}\ln 2.444$ $= 0.2696$ kJ/kg·K (d)
(e) N_2 $p_1 = 2$ MPa $p_2 = 1$ MPa $T_1 = 800$ K $T_2 = 300$ K	With $\bar{s}^\circ$ data from Table A-23 and M from Table A-1 $\Delta s = \frac{191.682 - 220.907 - 8.314\ln\frac{1}{2}}{28.02}$ $= -0.8373$ kJ/kg·K	With c_p at 550 K from Table A-20 $\Delta s = 1.065 \ln\frac{300}{800} - \frac{8.314}{28.02}\ln\frac{1}{2}$ (e) $= -0.8389$ kJ/kg·K

PROBLEM 6.33

KNOWN: A quantity of air undergoes a thermodynamic cycle consisting of three processes.

FIND: Evaluate the change in entropy for each process and sketch the cycle on p-v coordinates.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) As shown in the accompanying figure, the system consists of the quantity of air. (2) The air behaves as an ideal gas with $c_p = 1\ kJ/kg \cdot K$.

ANALYSIS: The entropy changes can be evaluated using Eqs. 6.22 and 6.23. First, some preliminary results are calculated. Using the ideal gas equation of state

$$T_1 = \frac{p_1 V_1}{m R} = \frac{(0.1 \times 10^6\ N/m^2)(0.02\ m^3)}{(2.42 \times 10^{-2}\ kg)\left(\frac{8314}{28.97}\ \frac{N \cdot m}{kg \cdot K}\right)} = 288\ K$$

Also, since $V_1 = V_2$

$$\left.\begin{aligned} p_1 V &= m R T_1 \\ p_2 V &= m R T_2 \end{aligned}\right\} \Rightarrow \quad T_2 = \frac{p_2}{p_1} T_1 = \left(\frac{0.42}{0.10}\right)(288) = 1210\ K$$

With Eq. 3.44

$$c_v = c_p - R = 1\ \frac{kJ}{kg \cdot K} - \frac{8.314}{28.97}\ \frac{kJ}{kg \cdot K} = 0.713\ kJ/kg \cdot K$$

Process 1-2: With $v_2 = v_1$, Eq. 6.22 reduces so

$$S_2 - S_1 = m c_v \ln \frac{T_2}{T_1} = (0.024\ kg)\left(0.713\ \frac{kJ}{kg \cdot K}\right) \ln \frac{1210}{288}$$

$$= 0.0246\ kJ/K \longleftarrow S_2 - S_1$$

Process 2-3: With $p_1 = p_2$, Eq. 6.23 reduces so

$$S_3 - S_2 = m c_p \ln \frac{T_3}{T_2} = m c_p \ln \frac{T_1}{T_2}$$

$$= (0.024\ kg)\left(1.0\ \frac{kJ}{kg \cdot K}\right) \ln \frac{288}{1210}$$

$$= -0.0344\ kJ/K \longleftarrow S_3 - S_2$$

Process 3-1: Since $T_3 = T_1$, Eq. 6.23 reduces so

$$S_1 - S_3 = -m R \ln \frac{p_1}{p_3} = -m R \ln \frac{p_1}{p_2}$$

$$= -(0.024)\left(\frac{8.314}{28.97}\right) \ln \frac{0.1}{0.42}$$

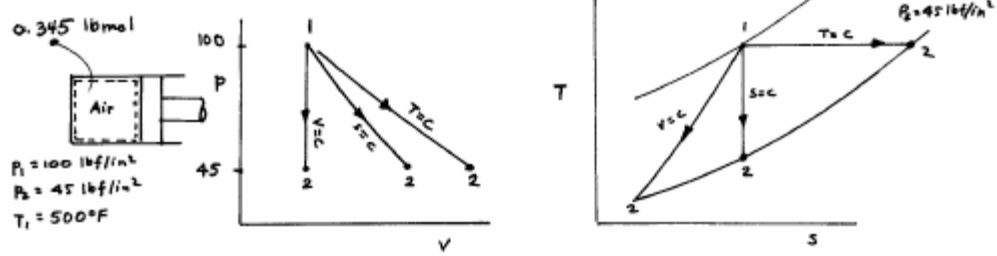
$$= 0.0099\ kJ/K \longleftarrow S_1 - S_3$$

PROBLEM 6.36

KNOWN: Air undergoes an internally reversible process between two specified states.

FIND: For each of three cases, determine the heat transfer and the work, per unit of mass, and show the process on p-v and T-s coordinates.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system consists of air which behaves as an ideal gas. (2) Each process is internally reversible. (3) There is no change in kinetic or potential energy between the end states.

ANALYSIS: (a) Isothermal process. With assumption (2), Eq. 6.25 is applicable. Then, as temperature is constant

$$Q = \int_1^2 T\,dS = m\,T(s_2 - s_1)$$

With Eq. 6.21a and $m = nM$

$$Q = m\,T(s_2 - s_1) = mT\left[-R\ln\frac{p_2}{p_1}\right]$$

$$= \left((0.345)(28.97)\text{ lb}\right)(960°\text{R})\left(-\frac{1.986}{28.97}\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right)\ln\frac{45}{100} = +525.2\text{ Btu} \quad \leftarrow Q$$

Since internal energy depends on temperature alone for an ideal gas, $\Delta U = 0$ because $T_1 = T_2$. Thus, an energy balance reduces with assumption 3 to give $W = Q = +525.2$ Btu. $\leftarrow W$

(b) Adiabatic Process. Since adiabatic, $Q = 0$. Then, an energy balance gives $\leftarrow Q$

$$W = -m(u_2 - u_1)$$

This requires that state 2 be fixed. However, as the process is adiabatic and internally reversible, it is an isentropic process, so $s_2 = s_1$. Eq. 6.21a gives

$$s°(T_2) = s°(T_1) + R\ln\frac{p_2}{p_1}$$

With data from Table A-22E

$$s°(T_2) = 0.74030 + \frac{1.986}{28.97}\ln\frac{45}{100} = 0.68556 \Rightarrow u_2 = 131.36\text{ Btu/lb}$$

At $T_1 = 960°$R, $u_1 = 165.26$ Btu/lb. Finally, with $m = nM = 9.99$ kg

$$W = -(9.99)(131.36 - 165.26) = +338.7\text{ Btu} \quad \leftarrow W$$

(c) Constant volume process. Since volume is constant, $W = 0$. Then, an energy balance gives $Q = m(u_2 - u_1)$. To find u_2, first determine T_2 using the ideal gas equation of state $\leftarrow W$

$$\left.\begin{aligned} p_1 V &= RT_1 \\ p_2 V &= RT_2 \end{aligned}\right\} \Rightarrow T_2 = T_1\left(\frac{p_2}{p_1}\right) = 960\left(\frac{45}{100}\right) = 432°\text{R}$$

Then, from Table A-22E $u_2 = 73.57$ Btu/lb, and

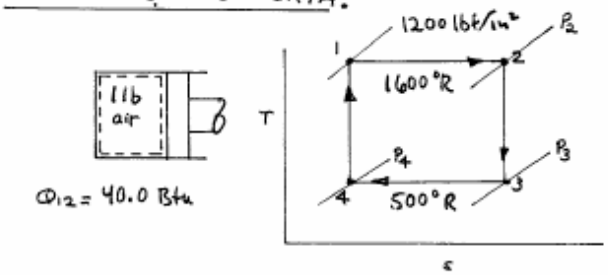
$$Q = (9.99)(73.57 - 165.26) = -916\text{ Btu} \quad \leftarrow Q$$

PROBLEM 6.43*

KNOWN: Data is provided for 1 lb of air undergoing a Carnot power cycle.

FIND: Determine (a) the pressures at the end of the isothermal expansion, adiabatic expansion, and isothermal compression, (b) the net work, and (c) the thermal efficiency.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system is the 1 lb of air. (2) The air is modeled as an ideal gas. (3) The system undergoes a Carnot power cycle.

ANALYSIS: (a) As discussed in Sec. 6.4, for process 1-2 $Q_{12} = m T_H (s_2 - s_1)$. And for an ideal gas at fixed temperature, Eq. 6.25 reduces to read $s_2 - s_1 = -R \ln P_2/P_1$. Combining these $Q_{12} = -mRT_H \ln P_2/P_1$. Solving

$$\ln \frac{P_2}{P_1} = -\frac{Q_{12}}{mRT_H} = \frac{-40.0 \text{ Btu}}{(1.0 \text{ lb})\left(\frac{1.986}{28.97}\frac{\text{Btu}}{\text{lb}^\circ\text{R}}\right)(1600^\circ\text{R})}$$

giving $P_2/P_1 = 0.6944$, $P_2 = 833.3$ lbf/in² ← P_2

For process 2-3 and data from Table A-22E

$$0 = s^\circ(T_3) - s^\circ(T_2) - R \ln \frac{P_3}{P_2} \Rightarrow \ln \frac{P_3}{P_2} = \frac{s^\circ(T_3) - s^\circ(T_2)}{R} = \frac{0.58233 - 0.87130}{(1.986/28.97)}$$

giving $\ln P_3/P_2 = -4.2152$, $P_3/P_2 = 0.01477$, $P_3 = 12.31$ lbf/in² ← P_3

Similarly, for process 4-1

$$\ln \frac{P_1}{P_4} = \frac{s^\circ(T_1) - s^\circ(T_4)}{R} = +4.2152, \quad \frac{P_1}{P_4} = 67.708, \quad P_4 = 17.72 \text{ lbf/in}^2$$ ← P_4

(b) For any cycle, $W_{net} = \eta Q_{in}$. Here, η corresponds to η_{MAX} and $Q_{in} = Q_{12}$.

Thus $$W_{net} = \underbrace{\left[1 - \frac{500}{1600}\right]}_{0.6875 \ (68.75\%)} 40.0 \text{ Btu} = 27.5 \text{ Btu}$$ ← W_{net}, ← η

Alternatively $W_{net} = Q_{net} = Q_{12} + Q_{34}$. From an energy balance, since T is constant, $Q_{34} = W_{34}$, where

$$W_{34} = m\int_3^4 p\,dv = m\int_3^4 \frac{RT}{v}\,dv = mRT \ln \frac{v_4}{v_3} = mRT \ln \frac{P_3}{P_4}$$ ($r = P_3/P_4$)

$$= (1.0)\left(\frac{1.986}{28.97}\right)(500) \ln\left(\frac{12.31}{17.72}\right) = -12.5 \text{ Btu}$$

So $Q_{net} = 40.0 - 12.5 = 27.5$ Btu, and $W_{net} = 27.5$ Btu

PROBLEM 6.61*

KNOWN: Data is provided for a quantity of propane rapidly compressed.

FIND: Determine whether the reported value for the work is feasible.

SCHEMATIC & GIVEN DATA:

$W = -72.5$ kJ

$p_1 = 4$ bar, $T_1 = 30°C$

$p_2 = 14$ bar, $T_2 = 100°C$

ASSUMPTIONS: 1. The propane is the closed system. 2. Heat transfer with the surroundings occurs at T_b, which cannot be less than 293 K (20°C) and is most likely in the interval from 303 K (30°C) to 373 K (100°C). Kinetic and potential energy effects can be ignored.

ANALYSIS: An energy balance reduces to $\Delta U = Q - W$, or with data from Table A-15

$$Q = m(u_2 - u_1) + W = (0.5\text{ kg})[1480.79 - 1377.49]\frac{\text{kJ}}{\text{kg}} + (-72.5\text{ kJ}) = -20.85\text{ kJ}$$

An entropy balance takes the form,

$$\Delta S = \frac{Q}{T_b} + \sigma \Rightarrow \sigma = m[s_2 - s_1] - \frac{Q}{T_b}$$

With Eq. 6.21a and data from Table A-15

$$\sigma = (0.5\text{ kg})\left[(5.4433 - 5.6328)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right] - \frac{(-20.85\text{ kJ})}{T_b}$$

$$\sigma = -0.09475\frac{\text{kJ}}{\text{kg}} + \frac{20.85\text{ kJ}}{T_b}$$

For $T_b \geq 293$ K, $\sigma < 0$. Thus, the value for W cannot be correct.

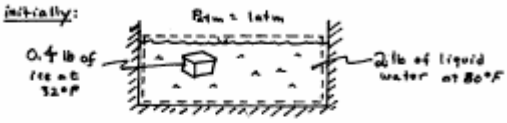
PROBLEM 6.73

KNOWN: A system initially consisting of 0.4 lb of ice at 32°F and 2 lb of liquid water at 80°F attains an equilibrium state adiabatically at a constant pressure of 1 atm.

FIND: Determine the final temperature and the amount of entropy produced.

SCHEMATIC & GIVEN DATA:

initially:



h_{if} = 144 Btu/lb

ASSUMPTIONS: (1) As shown in the accompanying figure, the system consists of the ice and liquid. (2) $Q = 0$ and pressure remains constant. (3) The liquid is incompressible with constant specific heat.

ANALYSIS: (a) The final temperature can be determined using an energy balance. Thus, $\Delta U = \cancel{Q}^{0} - W$, where $W = \int_1^2 p\,dV = p\Delta V$ since pressure is constant. So

$$\Delta U = -p\Delta V \Rightarrow \Delta H = 0 \quad \text{for the constant pressure process}$$

If all the ice does not melt, the final temperature would be 32°F. To investigate this, let T_f be the final temperature when all of the ice is regarded to have melted. Then with c = 1.0 Btu/lb·°R for liquid water from Table A-19E

$$\left\{\Delta H\big]_{ice} + \Delta H\big]_{\substack{melt \\ 32°F \to T_f}}\right\} + \Delta H\big]_{\substack{liquid \\ 80°F \to T_f}} = 0$$

$$\left\{(0.4\,lb)\left(144\,\frac{Btu}{lb}\right) + (0.4\,lb)\left(1\,\frac{Btu}{lb\cdot°R}\right)(T_f - 492)°R\right\} + (2\,lb)\left(1\,\frac{Btu}{lb\cdot°R}\right)(T_f - 540)°R = 0$$

$$\Rightarrow T_f = 508°R \ (48°F) \longleftarrow T_f$$

As $T_f > 32°F$, it can be concluded that all the ice melts.

(b) An entropy balance reduces to give

$$\Delta S = \cancel{\int_1^2 \left(\frac{\delta Q}{T}\right)_b}^{0} + \sigma \Rightarrow \sigma = \Delta S$$

Thus

$$\sigma = \left\{\Delta S\big]_{\substack{ice \\ @32°F}} + \Delta S\big]_{\substack{melt \\ 32°F \to T_f}}\right\} + \Delta S\big]_{\substack{liquid \\ 80°F \to T_f}}$$

The two liquid terms can be evaluated using Eq. 6.24. The entropy change for the melting ice can be found using the TdS equation, Eq 6.12b, which at constant pressure and temperature reduces to give

$$ds = \frac{dh}{T} \Rightarrow s_f - s_i = \frac{h_f - h_i}{T} = \frac{144\,Btu/lb}{492\,°R} = 0.2927\,Btu/lb\cdot°R$$

Thus

$$\sigma = (0.4\,lb)(0.2927\,\tfrac{Btu}{lb\cdot°R}) + (0.4\,lb)\left(1\,\tfrac{Btu}{lb\cdot°R}\right)\ln\frac{508}{492} + (2\,lb)\left(1\,\tfrac{Btu}{lb}\right)\ln\left(\frac{508}{540}\right)$$

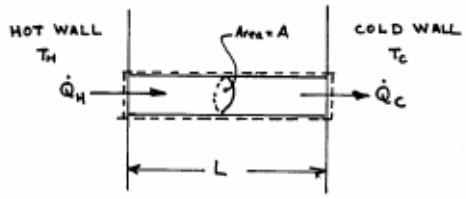
$$= (0.1171) + (0.0128) + (-0.1222) = 0.0077\ Btu/°R \longleftarrow \sigma$$

PROBLEM 6.80

KNOWN: Energy is conducted steadily through a copper rod from a hot wall to a cold wall. The rate of heat transfer, temperatures, and geometrical parameters are specified.

FIND: (a) Determine an expression for the rate of entropy production within the rod in terms of specified quantities, and (b) plot $\dot{Q}_H$ and $\dot{\sigma}$ versus L for a given set of numerical values for these quantities.

SCHEMATIC & GIVEN DATA:



$$\dot{Q}_H = kA(T_H - T_C)/L \qquad (1)$$

$T_H = 600\ K\ (327°C)$
$T_C = 350\ K\ (77°C)$
$k = 0.4\ kW/m \cdot K$
$A = 0.1\ m^2$

ASSUMPTIONS: (1) As shown in the accompanying figure, the system is the copper rod. (2) The system is at steady state. (3) The rod is insulated on its lateral surface. (4) An expression for evaluating $\dot{Q}_H$ is provided.

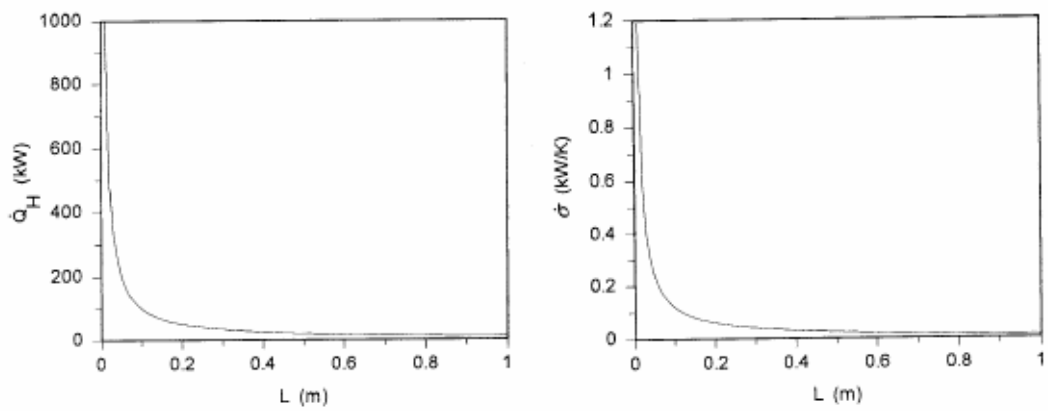
ANALYSIS: (a) At steady state an entropy rate balance reduces to give

$$\cancel{\frac{dS}{dt}}^{\,0} = \frac{\dot{Q}_H}{T_H} - \frac{\dot{Q}_C}{T_C} + \dot{\sigma} \Rightarrow \dot{\sigma} = \frac{\dot{Q}_C}{T_C} - \frac{\dot{Q}_H}{T_H}$$

Noting that the energy transfers are positive in the directions of the arrows, an energy rate balance gives $\dot{Q}_C = \dot{Q}_H$. Collecting results

$$\dot{\sigma} = \dot{Q}_H\left[\frac{1}{T_C} - \frac{1}{T_H}\right] = \frac{kA(T_H - T_C)^2}{L\,T_H T_C} \qquad (2)$$

(b) If $T_H = 600K\ (327°C)$, $T_C = 350K\ (77°C)$, $k = 0.4\ kW/m \cdot K$, $A = 0.1\ m^2$, the variations of $\dot{Q}$ and $\dot{\sigma}$ with L are



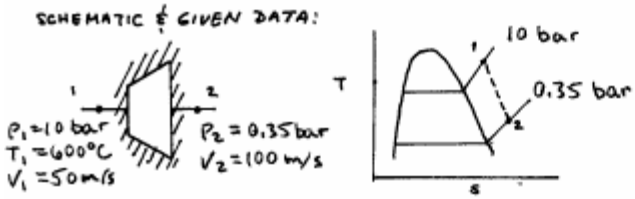
The plots show that both heat transfer and entropy production rate decrease rapidly with increasing L. $\dot{Q}_H$ decreases due to greater resistance to heat transfer as the length of the rod increases. The variation of $\dot{\sigma}$ is associated with the changing temperature gradient, dT/dx, in the rod. Higher temperature gradient ($L \to 0$; $dT/dx \to \infty$) corresponds to high rates of entropy production, and conversely. As a final point, Eq. (2) above indicates that $\dot{\sigma}$ is positive, illustrating the irreversible nature of heat transfer.

PROBLEM 6.86*

KNOWN: An insulated turbine at steady state has steam entering at 10 bar, 600°C, 50 m/s and exiting at 0.35 bar, 100 m/s. The work developed is claimed to be (a) 1000 kJ/kg, (b) 500 kJ/kg.

FIND: Determine if the claim can be correct.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The turbine operates at steady-state and is well-insulated. (2) Potential energy effects can be ignored.

ANALYSIS: At steady-state the entropy rate balance reduces with the mass rate balance: $\dot{m}_1 = \dot{m}_2 = \dot{m}$ to give

$$0 = \cancel{\sum \frac{\dot{Q}_j}{T_j}}^{\,0} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$$

Thus,

$$s_2 - s_1 = \frac{\dot{\sigma}_{cv}}{\dot{m}} \geq 0$$

Accordingly, the exiting specific entropy must be greater than, or equal to, the entering specific entropy.

From Table A-4 at 10 bar, 600°C, $s_1 = 8.0290$ kJ/kg·K. The exit specific entropy that corresponds to the claimed work value can be determined using an energy rate balance to fix state 2. Thus, at steady-state

$$0 = \cancel{\dot{Q}_{cv}}^{\,0} - \dot{W}_{cv} + \dot{m}\left(h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + \cancel{g(z_1 - z_2)}^{\,0}\right)$$

Solving for h_2, and introducing h_1 from Table A-4: $h_1 = 3697.9$ kJ/kg

$$h_2 = -\frac{\dot{W}_{cv}}{\dot{m}} + h_1 + \frac{V_1^2 - V_2^2}{2} = -\frac{\dot{W}_{cv}}{\dot{m}} + 3697.9 + \left[\frac{(50)^2 - (100)^2}{2}\left(\frac{m}{s}\right)^2\right]\left|\frac{1\ N}{1\ kg\cdot m/s^2}\right|\left|\frac{1\ kJ}{10^3\ N\cdot m}\right|$$

$$= -\frac{\dot{W}_{cv}}{\dot{m}} + 3694.2\ \frac{kJ}{kg} \qquad (1)$$

(a) $\dot{W}_{cv}/\dot{m} = 1000$ kJ/kg. Eq. (1) gives $h_2 = 2694.2$ kJ/kg. Interpolating in Table A-4 at 0.35 bar with h_2 gives $s_2 = 7.8868$ kJ/kg·K. Since this value is less than s_1, the claimed value cannot be correct.

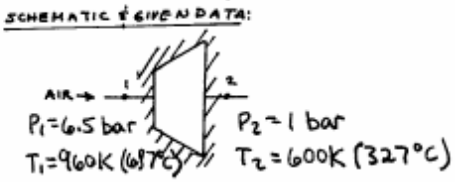
(b) $\dot{W}_{cv}/\dot{m} = 500$ kJ/kg. Eq. (1) gives $h_2 = 3194.2$ kJ/kg. Interpolating in Table A-4 at 0.35 bar with h_2 gives $s_2 = 8.8987$ kJ/kg·K. Since this value is greater than s_1, the claimed value could be correct.

PROBLEM 6.88*

KNOWN: Air enters and exits an insulated turbine at specified pressures and temperatures.

FIND: Determine the work developed per kg of air flowing, and whether the expansion is internally reversible, irreversible, or impossible.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The turbine operates at steady state and is well-insulated. (2) Changes in kinetic and potential energy from inlet to exit can be neglected. (3) Air is modeled as an ideal gas. ①

ANALYSIS: An energy rate balance at steady state for a control volume enclosing the turbine reduces with a mass rate balance: $\dot{m}_1 = \dot{m}_2 = \dot{m}$ to give

$$0 = \cancel{\dot{Q}_{cv}}^{\,0} - \dot{W}_{cv} + \dot{m}\left(h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}}^{\,0} + \cancel{g(z_1 - z_2)}^{\,0}\right)$$

or

$$\frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2$$

Then, with data from Table A-22

$$\frac{\dot{W}_{cv}}{\dot{m}} = 1000.55 - 607.02 = 393.53 \ \frac{kJ}{kg}$$

An entropy rate balance at steady state gives

$$0 = \cancel{\sum \frac{\dot{Q}_j}{T_j}}^{\,0} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$$

or

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = s_2 - s_1$$

With Eq 6.25a and data from Table A-22

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = s^\circ(T_2) - s^\circ(T_1) - R \ln P_2/P_1$$

$$= 2.40902 - 2.92128 - \frac{8.314}{28.97} \ln \frac{1}{6.5} = 0.0249 \ kJ/kg \cdot K$$

Since $\dot{\sigma}_{cv}/\dot{m}$ is positive, the expansion is irreversible. However, the small value suggests that the expansion approaches ideality.

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1. From Table A-1, the critical pressure of air is $P_c = 37.7$ bar and the critical temperature is $T_c = 133$ K. Thus, at the inlet and exit, respectively

$$P_{R1} = \frac{6.5}{37.7} = 0.17 \qquad P_{R2} = \frac{1}{37.7} = 0.027$$

$$T_{R1} = \frac{960}{133} = 7.22 \qquad T_{R2} = \frac{600}{133} = 4.51$$

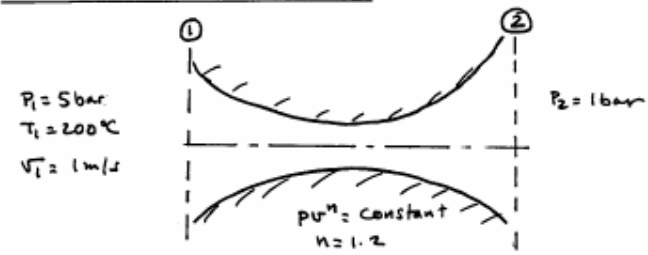
Referring to Fig A-1, these states fall into the region where the ideal gas model is appropriate.

PROBLEM 6.175

KNOWN: Data are provided for CO expanding through a nozzle operating at steady state.

FIND: Determine the velocity at the nozzle exit and the rate of heat transfer per kg of CO flowing.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The nozzle is at steady state. (2) The expansion is a polytropic process with $n = 1.2$. (3) CO is modeled as an ideal gas. (4) Potential energy effects are ignored.

ANALYSIS: (a) As a polytropic process is internally reversible, Eq. 6.53a is applicable. Then, with $\dot{W}_{cv} = 0$, the Bernoulli equation, Eq. 6.54 results:

$$\int_1^2 v\,dp + \frac{V_2^2 - V_1^2}{2} + \cancel{g(z_2 - z_1)}^{\,0} = 0 \qquad (1)$$

Using the relationship pv^n = constant, the integral can be performed (Sec. 6.9), giving

$$\int_1^2 v\,dp = \frac{n}{n-1}(p_2 v_2 - p_1 v_1)$$

$$= \frac{nR}{n-1}(T_2 - T_1) \qquad (2)$$

where the ideal gas equation of state has been used to obtain the last expression. For a polytropic process of an ideal gas (Eq. 3.56b)

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(n-1)/n} \qquad (3)$$

Collecting Eqs. (1)-(3)

$$V_2 = \sqrt{V_1^2 + 2\frac{nRT_1}{n-1}\left[1 - (p_2/p_1)^{(n-1)/n}\right]} \qquad (4)$$

$$= \sqrt{\left(1\frac{m}{s}\right)^2 + 2\left(\frac{1.2}{1.2-1}\right)\left(\frac{8314}{28}\frac{N\cdot m}{kg\cdot K}\right)(473K)\left|\frac{1\,kg\cdot m/s^2}{1\,N}\right|\left[1 - \left(\frac{1}{5}\right)^{0.2/1.2}\right]} = 629.7\,\frac{m}{s} \longleftarrow V_2$$

(b) An energy balance reduces to give

$$\frac{\dot{Q}_{cv}}{\dot{m}} = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2}$$

With Eq. (3), $T_2 = T_1\left(p_2/p_1\right)^{\frac{n-1}{n}} = (473)\left(\frac{1}{5}\right)^{\frac{0.2}{1.2}} = 362K$

Then with data from Table A-23

$$\frac{\dot{Q}_{cv}}{\dot{m}} = \frac{(10531 - 13797)}{28}\frac{kJ}{kg} + \left[\frac{(629.7)^2 - (1)^2}{2}\left(\frac{m^2}{s^2}\right)\right]\left[\frac{1\,N}{1\,kg\cdot m/s^2}\right]\left[\frac{1\,kJ}{10^3\,N\cdot m}\right] = 81.7\,\frac{kJ}{kg} \longleftarrow \dot{Q}_{cv}/\dot{m}$$