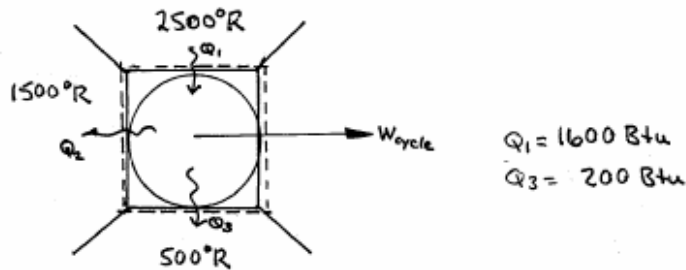


## PROBLEM 6.2\*

**KNOWN:** A system undergoes a reversible power cycle while communicating thermally with three reservoirs at specified temperatures. The values for the energy transfer heat between two of the reservoirs and the system are also specified.

**FIND:** Determine the thermal efficiency.

**SCHEMATIC & GIVEN DATA:**



**ASSUMPTIONS:** 1. The system shown in the accompanying figure undergoes a power cycle with no internal irreversibilities. 2. All heat transfers take place at the indicated temperatures.

**ANALYSIS:** The thermal efficiency is

$$\eta = \frac{W_{\text{cycle}}}{Q_{\text{in}}} = \frac{W_{\text{cycle}}}{Q_1}$$

An energy balance gives

$$\begin{aligned} W_{\text{cycle}} &= Q_1 - Q_2 - Q_3 = 1600 - Q_2 - 200 \\ &= 1400 - Q_2 \end{aligned}$$

Since the cycle involves no irreversibilities, and heat transfers are at  $T_1, T_2, T_3$ , Eq. 6.2 reads

$$\begin{aligned} \frac{Q_1}{T_1} - \frac{Q_2}{T_2} - \frac{Q_3}{T_3} &= -\cancel{W_{\text{cycle}}}^0 \\ \Rightarrow 0 &= \frac{1600}{2500} - \frac{Q_2}{1500} - \frac{200}{500} \Rightarrow Q_2 = 360 \text{ Btu} \end{aligned}$$

Thus

$$W_{\text{cycle}} = 1400 - 360 = 1040 \text{ Btu}$$

and

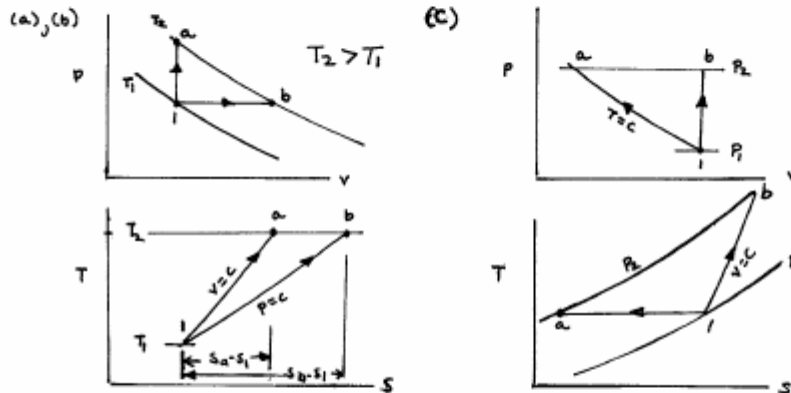
$$\eta = \frac{1040}{1600} = 0.65 \text{ (65\%)}$$

### PROBLEM 6.9

**KNOWN:** A system consisting of an ideal gas with constant specific heat ratio  $k$  undergoes constant volume, constant pressure, and constant temperature processes from the same initial state.

**FIND:** (a) Show that the entropy change is greater for the constant pressure process than for the constant volume process. Sketch the processes on  $p-v$ ,  $T-s$  coordinates. (b) Using the  $T-s$  diagram, show that a line of constant  $v$  has a greater slope than a line of constant  $p$ . (c) Show that the ratio of the entropy change for a constant  $T$  process to the entropy change for a constant  $v$  process is  $(1-k)$ . Sketch the processes on  $p-v$  and  $T-s$  coordinates.

**SCHEMATIC & GIVEN DATA:**



**ASSUMPTION:** The system consists of an ideal gas with constant  $k$ .

**ANALYSIS:** (a) For process 1-a, specific volume is constant. For process 1-b, pressure is constant. Thus, Eqs. 6.22 and 6.23 reduce, respectively, to give

$$\begin{aligned} s_a - s_1 &= c_v \ln \frac{T_2}{T_1} \\ s_b - s_1 &= c_p \ln \frac{T_2}{T_1} \end{aligned} \quad \Rightarrow \quad \frac{s_b - s_1}{s_a - s_1} = \frac{c_p \ln T_2/T_1}{c_v \ln T_2/T_1} = \frac{c_p}{c_v} = k$$

As  $k > 1$ ,  $(s_b - s_1) > (s_a - s_1)$ .

(b) Here, we compare at state 1  $(\partial T/\partial s)_v$  to  $(\partial T/\partial s)_p$ . Since  $(\partial T/\partial s)$  at fixed  $v$  (or fixed  $p$ ) =  $\lim_{\Delta s \rightarrow 0} \frac{\Delta T}{\Delta s}$ , it is evident from the  $T-s$  diagram that a constant specific volume line passing through state 1 has a greater slope than a constant pressure line.

For process 1-a, temperature is constant. For process 1-b, volume is constant. Thus, Eqs. 6.19 and 6.18 reduce, respectively, to give

$$\begin{aligned} s_a - s_1 &= -R \ln \frac{P_2}{P_1} \quad (\text{entropy decreases}) \\ s_b - s_1 &= c_v \ln \frac{T_2}{T_1} \quad (\text{entropy increases}) \end{aligned}$$

Using Eq. 3.44, the first of these can be written as

$$s_a - s_1 = -(c_p - c_v) \ln \frac{P_2}{P_1}$$

Using the ideal gas equation of state with  $v_b = v_1$ , the second can be written as

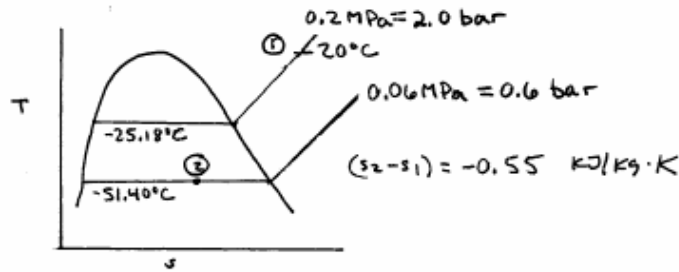
$$s_b - s_1 = c_v \ln \frac{P_2}{P_1}$$

Forming the ratio

$$\frac{s_a - s_1}{s_b - s_1} = \frac{-(c_p - c_v) \ln P_2/P_1}{c_v \ln P_2/P_1} = 1 - k$$

**PROBLEM 6.23\***

**KNOWN:** Refrigerant 22 undergoes a process between two specified states.  
**FIND:** Determine the temperature at the final state, in °C, and the final specific enthalpy, in kJ/kg.  
**SCHEMATIC & GIVEN DATA:**



**ASSUMPTION:** The system is 1 kg of R-22

**ANALYSIS:** From Table A-9,  $s_1 = 1.0786 \text{ kJ/kg}\cdot\text{K}$ . Then

$$s_2 = s_1 + (s_2 - s_1) = 1.0786 + (-0.55) = 0.5286 \text{ kJ/kg}\cdot\text{K}$$

Since  $s_f < s_2 < s_g$  at 0.6 bar, Table A-9, state 2 is in the two-phase liquid-vapor region. Then,  $T_2 = -51.40^\circ\text{C}$  and

$$x_2 = \frac{0.5286 - (-0.0542)}{1.0294 - (-0.0542)} = 0.5378$$

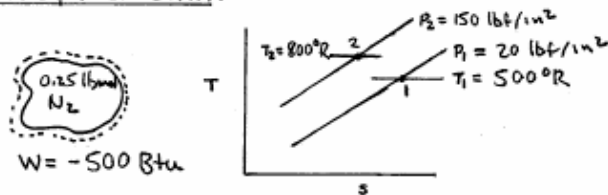
$$\Rightarrow h_2 = h_f + x_2(h_{fg}) = -12.35 + (0.5378)(240.28) = 116.88 \text{ kJ/kg}$$

**PROBLEM 6.81\***

**KNOWN:** 0.25 lbmol of  $\text{N}_2$  undergoes a process between known states.

**FIND:** Determine (a) the heat transfer and (b) the change in entropy.

**SCHEMATIC & GIVEN DATA:**



**ASSUMPTIONS:** (1) The system consists of the  $\text{N}_2$  which behaves as an ideal gas.  
 (2) There are no changes in kinetic and potential energy between the two end states.

**ANALYSIS:** (a) With assumption 2, an energy balance gives

$$Q = W + \Delta U = W + n[\bar{u}_2 - \bar{u}_1]$$

With  $\bar{u}$  data from Table A-23 E

$$Q = -500 \text{ Btu} + (0.25 \text{ lbmol})[(3975.7 - 2479.3) \text{ Btu/lbmol}] = -125.9 \text{ Btu}$$

(b) with Eq. 6.216 and  $\bar{s}^\circ$  data from Table A-23 E

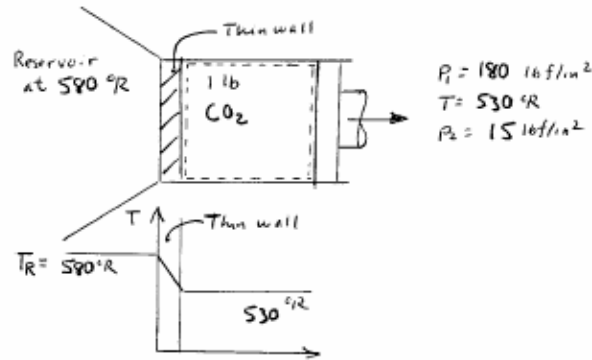
$$\Delta S = n[\bar{s}^\circ(T_2) - \bar{s}^\circ(T_1) - R \ln(P_2/P_1)] = 0.25[48.522 - 45.246 - 1.986 \ln(150/20)] = -0.1814 \text{ Btu/}^\circ\text{R}$$

**PROBLEM 6.5D\***

**KNOWN:** One lb of  $\text{CO}_2$  expands isothermally between specified states while receiving energy through a thin intervening wall from a reservoir.

**FIND:** (a) For the  $\text{CO}_2$  as the system, evaluate  $W, Q, \sigma$ . (b) For the  $\text{CO}_2$  plus the thin wall as the system, evaluate  $\sigma$  and compare with the result of part (a).

**SCHEMATIC & GIVEN DATA:**



**ASSUMPTIONS:** (1) In part (a) the system is the  $\text{CO}_2$  only. In part (b) the system is the  $\text{CO}_2$  plus the thin wall. (2)  $\text{CO}_2$  is modeled as an ideal gas. (3) There is no change in kinetic or potential energy. (4) The state of the thin wall does not change.

**ANALYSIS:** With assumptions (2), (3) an energy balance reduces to  $\Delta U = Q - W$ , where  $\Delta U = 0$  because  $U(T)$  for an ideal gas. Thus  $Q = W$ . To find  $W$ , use the ideal gas equation of state to write

$$W = m \int_1^2 p dV = m \int_1^2 \frac{RT}{V} dV = mRT \ln \frac{V_2}{V_1} = mRT \ln \frac{P_1}{P_2}$$

$$= (1 \text{ lb}) \left( \frac{1.986}{44.01} \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} \right) (530 \text{ }^\circ\text{R}) \ln \left( \frac{180}{15} \right) = 59.43 \text{ Btu}$$

←  $W, Q$

An entropy balance reads

$$\Delta S = \frac{Q}{T} + \sigma \Rightarrow \sigma = \Delta S - \frac{Q}{T}$$

From Eq. 6.2.1b,  $\Delta S = -mR \ln P_2/P_1$ . Also, from above  $Q = mRT \ln P_1/P_2$ . Thus

$$\sigma = -mR \ln \frac{P_2}{P_1} - \frac{mRT \ln \frac{P_1}{P_2}}{T} \equiv 0 \quad (\text{the process is internally reversible}) \leftarrow \sigma$$

(b) For the enlarged system of  $\text{CO}_2$  plus thin wall an entropy balance reads

$$\Delta S = \frac{Q}{T_R} + \sigma \Rightarrow \sigma = \Delta S - \frac{Q}{T_R}$$

where by assumption (4)  $\Delta S$  is the same as in part (a):  $-mR \ln \frac{P_2}{P_1}$ . Introducing

$Q = mRT \ln \frac{P_1}{P_2}$  and simplifying

$$\sigma = -mR \ln \frac{P_2}{P_1} - \frac{mRT \ln \frac{P_1}{P_2}}{T_R} = mR \ln \frac{P_1}{P_2} \left[ 1 - \frac{T}{T_R} \right]$$

$$= (1 \text{ lb}) \left( \frac{1.986}{44.01} \frac{\text{Btu}}{\text{lb} \cdot ^\circ\text{R}} \right) \ln \left( \frac{180}{15} \right) \left[ 1 - \frac{530}{580} \right] = 0.009667 \text{ Btu}/^\circ\text{R}$$

←  $\sigma$

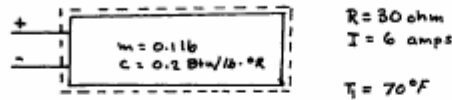
**Discussion:** The enlarged system includes an irreversibility not present in the  $\text{CO}_2$  alone: heat transfer through a finite temperature difference, and so a nonzero value of  $\sigma$  is determined for the enlarged system.

### PROBLEM 6.67

**KNOWN:** Data we provided for a thermally insulated resistor.

**FIND:** Plot the temperature and amount of entropy produced versus  $t$  for  $0 \leq t \leq 3$ s.

**SCHEMATIC & GIVEN DATA:**



**ASSUMPTIONS:** (1) The system shown in the accompanying figure is modeled as an incompressible substance. (2)  $Q = 0$ .

**ANALYSIS:** An energy balance reduces to give

$$\Delta U = \cancel{Q} - W \Rightarrow m(u_2 - u_1) = -W$$

With assumption 1

$$mc(T_2 - T_1) = -W \Rightarrow T_2 = T_1 - W/mc$$

Electrical work is done on the system. Using given data

$$W = -I^2 R \Delta t = -(6 \text{ amps})^2 (30 \text{ ohm})(\Delta t) \left( \frac{W}{(\text{amps})^2 \cdot \text{ohm}} \right)$$

$$= -1080 \Delta t \text{ (W}\cdot\text{s)}$$

Accordingly

$$T_2 = 530^\circ\text{R} - \frac{(-1080 \Delta t \text{ (W}\cdot\text{s)})}{(0.1 \text{ lb})(0.2 \text{ Btu/lb}\cdot^\circ\text{R})} \left( \frac{3.413 \text{ Btu/h}}{1 \text{ W}} \right) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right)$$

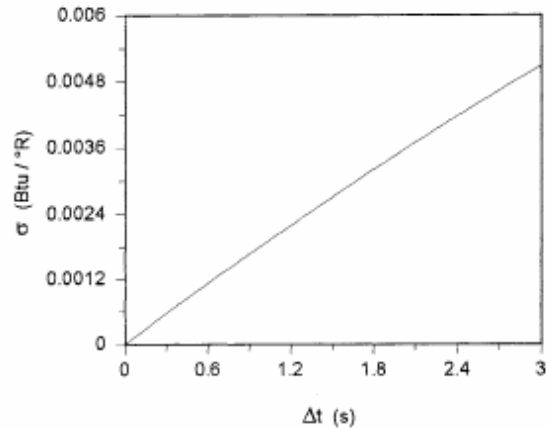
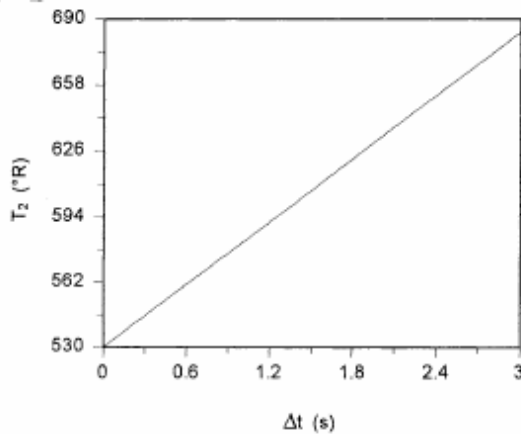
$$= (530 + 51.2 \Delta t)^\circ\text{R} \quad (1)$$

An entropy balance reduces to give

$$\Delta S = \int_1^2 \left( \frac{\delta Q}{T} \right)_b + \sigma \Rightarrow \sigma = \Delta S = mc \ln \frac{T_2}{T_1} \quad (\text{Eq. 6.24})$$

Introducing Eq.(1) the expression for  $\sigma$  becomes

$$\sigma = (0.1 \text{ lb})(0.2 \frac{\text{Btu}}{\text{lb}\cdot^\circ\text{R}}) \ln \left( 1 + \frac{51.2 \Delta t}{530} \right) = 0.02 \frac{\text{Btu}}{^\circ\text{R}} \ln \left( 1 + \frac{51.2 \Delta t}{530} \right) \quad (2)$$



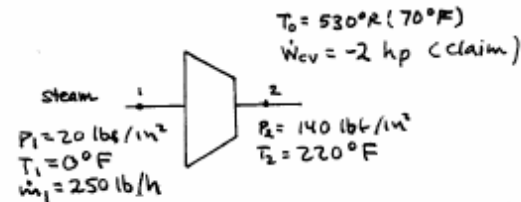
### PROBLEM 6.94\*

**KNOWN:** R134a enters and exits a compressor operating at steady state at specified temperatures and pressures. The ambient temperature is also specified.

The required power input is claimed to be 2 hp.

**FIND:** Determine whether this claim can be correct.

**SCHEMATIC & GIVEN DATA:**



**ASSUMPTIONS:** (1) Two control volumes at steady state are considered. One encloses the compressor. The other encloses the compressor plus a portion of the nearby surroundings so heat transfer occurs at the ambient temperature  $T_0$ . (2) Changes in kinetic and potential energy from inlet to exit can be ignored.

**ANALYSIS:** An entropy rate balance is used to determine if the claimed value for the power input can be correct. But first consider a control volume enclosing the compressor, and apply an energy rate balance to obtain the heat transfer. Thus with  $\dot{m}_1 = \dot{m}_2 = \dot{m}$

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_1 - h_2 + v_1^2 - v_2^2 + g(z_1 - z_2))$$

With assumption 2 and data from Table A-1E

$$\frac{\dot{Q}_{cv}}{\dot{m}} = \frac{\dot{W}_{cv}}{\dot{m}} + h_2 - h_1 \Rightarrow \frac{\dot{Q}_{cv}}{\dot{m}} = \frac{(-2 \text{ hp}) \left| \frac{2545 \text{ Btu/h}}{1 \text{ hp}} \right|}{(250 \text{ lb/h})} + 146.18 - 101.88 = 23.94 \text{ Btu/lb}$$

As the variation of temperature over the surface of this control volume is not specified, the entropy transfer accompanying heat transfer cannot be evaluated without further information. Accordingly, consider an entropy rate balance for an enlarged control volume enclosing the compressor plus a portion of the nearby surroundings so heat transfer occurs at the ambient temperature  $T_0$ . That is

$$0 = \frac{\dot{Q}_{cv}}{T_0} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$$

where  $\dot{\sigma}_{cv}$  represents the rate of entropy production within the enlarged control volume. With data from Table A-1E

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = -\frac{\dot{Q}_{cv}/\dot{m}}{T_0} + s_2 - s_1 = \frac{-(23.94 \text{ Btu/lb})}{530^\circ\text{R}} + 0.2667 \frac{\text{Btu}}{\text{lb}^\circ\text{R}} - 0.2238 \frac{\text{Btu}}{\text{lb}^\circ\text{R}}$$

$$\textcircled{1} \quad = -0.00227 \text{ Btu/lb}^\circ\text{R}$$

As  $\dot{\sigma}_{cv} \geq 0$ , the claimed value for the work input cannot be correct. ←

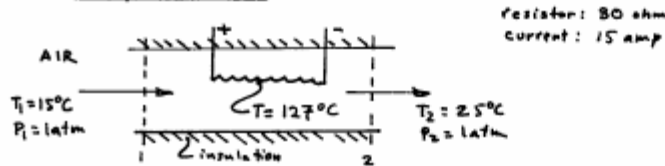
1. Note that the calculated value for  $\dot{\sigma}_{cv}/\dot{m}$  is not for the compressor alone, but for an enlarged control volume of compressor plus nearby surroundings.

### PROBLEM 6.104

**KNOWN:** Operating data are provided for an electrical resistor located in an insulated duct carrying air.

**FIND:** (a) For the resistor as the system, determine the rate of entropy production.  
(b) For a control volume enclosing the air in the duct and the resistor, determine the volumetric flow rate and the rate of entropy production.

**SCHEMATIC & GIVEN DATA:**



**ASSUMPTIONS:** (1) For part (a), the closed system consists of the resistor only. Heat transfer from this system takes place at  $T_b = 400\text{ K}$  ( $127^\circ\text{C}$ ). The system is at steady state. (2) For part (b), the control volume encloses the air in the duct and the resistor.  $\dot{Q}_{cv} = 0$  and kinetic and potential energy effects are negligible. The control volume is at steady state. Air is modeled as an ideal gas.

**ANALYSIS:** (a) An energy rate balance for a closed system at steady state reduces to give  $\dot{Q}_{cv} = \dot{W}_{cv}$ . In this case, electrical work is done on the system. With given data

$$\dot{W}_{cv} = -(\dot{I})^2 R = -(15\text{ amp})^2 (30\text{ ohm}) \left[ \frac{\text{KW}}{10^3 (\text{amp})^2 (\text{ohm})} \right] = -6.75\text{ KW}$$

At steady state an entropy rate balance becomes for heat transfer at  $T_b$  only

$$\frac{dS^0}{dt} = \frac{\dot{Q}_{cv}}{T_b} + \dot{\sigma} \Rightarrow \dot{\sigma} = -\frac{\dot{Q}_{cv}}{T_b}$$

Finally

$$\dot{\sigma} = \frac{-(-6.75\text{ KW})}{400\text{ K}} = 0.0169 \frac{\text{KW}}{\text{K}} \quad \leftarrow \dot{\sigma}$$

(b) For the control volume at steady state the energy rate balance reduces with listed assumptions to

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} (h_1 - h_2 + \frac{V_1^2 - V_2^2}{2}) + \dot{g} (z_1 - z_2)$$

Solving for  $\dot{m}$  and using enthalpies from Table A-22

$$\dot{m} = \frac{-\dot{W}_{cv}}{h_2 - h_1} = \frac{-(-6.75\text{ KW}) (1\text{ kJ/s/kW})}{[299.2 - 281.2] \text{ kJ/kg}} = 0.675 \frac{\text{kg}}{\text{s}} \quad \leftarrow \dot{m}$$

The volumetric flow rate entering the duct is then

$$(\dot{A}V)_1 = \dot{m} v_1 \Rightarrow (\dot{A}V)_1 = \dot{m} \left( \frac{RT_1}{P_1} \right) = (0.675 \frac{\text{kg}}{\text{s}}) \left( \frac{8314 \text{ N}\cdot\text{m}}{28.97 \text{ kg}\cdot\text{K}} \right) (288\text{ K}) \left( \frac{1}{1.01325 \times 10^5 \text{ N/m}^2} \right) \\ = 0.55 \text{ m}^3/\text{s} \quad \leftarrow (\dot{A}V)_1$$

An entropy rate balance reduces at steady state to

$$0 = \sum \frac{\dot{Q}_j}{T_j} + \dot{m} (s_1 - s_2) + \dot{\sigma}_{cv} \Rightarrow \dot{\sigma}_{cv} = \dot{m} (s_2 - s_1)$$

With Eq. 6.21a

$$s_2 - s_1 = s^0(T_2) - s^0(T_1) - R \ln \frac{P_2}{P_1}$$

Thus with data from Table A-22

$$\dot{\sigma}_{cv} = \dot{m} (s^0(T_2) - s^0(T_1)) = (0.675 \frac{\text{kg}}{\text{s}}) (1.69528 - 1.66103) \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \\ = 0.0238 \text{ KW/K} \quad \leftarrow \dot{\sigma}_{cv}$$

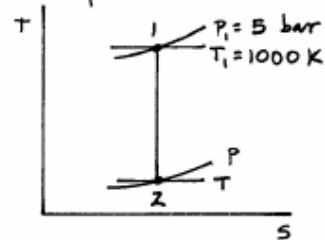
**COMMENT:** In part (a),  $\dot{\sigma}$  accounts for the irreversibility of electrical current flow through a resistance. In part (b),  $\dot{\sigma}_{cv}$  includes both the electrical irreversibility and the irreversibility of heat transfer from the resistor to the air.

PROBLEM 6.133

**KNOWN:** Methane gas undergoes an isentropic expansion from a given initial state to a final state at  $T, p$ .

**FIND:** Using the ideal gas model, and  $c_p(T)$  data from Table A-21, determine (a)  $p$  when  $T=500\text{K}$  and (b)  $T$  when  $p=1\text{bar}$ . Check using IT.

**SCHEMATIC AND GIVEN DATA:**



**ASSUMPTIONS:** (1) The methane can be modeled as an ideal gas. (2)  $T_1$  and  $T_2$  are in the range for which the  $c_p(T)$  data in Table A-22 are applicable.

**ANALYSIS:** When expressed on a molar basis, Eq. 6.19 becomes for  $\Delta s = 0$

$$0 = \int_{T_1}^T \frac{\bar{c}_p(T)}{T} dT - \bar{R} \ln \frac{P}{P_1}$$

Where the integration is between state 1 ( $T_1, P_1$ ) and a second state expressed as ( $T, P$ ). With  $\bar{c}_p$  from Table A-21:  $\bar{c}_p = \bar{R} [\alpha + \beta T + \gamma T^2 + \delta T^3 + \epsilon T^4]$  this expression becomes

$$\ln \frac{P}{P_1} = \int_{T_1}^T \left[ \frac{\alpha}{T} + \beta + \gamma T + \delta T^2 + \epsilon T^3 \right] dT$$

or

$$\ln \frac{P}{P_1} = \alpha \ln \left( \frac{T}{T_1} \right) + \beta (T - T_1) + \frac{\gamma}{2} (T^2 - T_1^2) + \frac{\delta}{3} (T^3 - T_1^3) + \frac{\epsilon}{4} (T^4 - T_1^4) \quad (1)$$

(a) and (b): Eq. (1) can be solved for  $p$  when  $T$  is given or for  $T$  when  $p$  is given. The following IT code illustrates this, using data for the coefficients  $\alpha, \beta, \dots$  from Table A-21:

```
p1 = 5 // bar
T1 = 1000 // K
```

```
ln(p/p1) = alpha*ln(T/T1) + beta*(T-T1) + (gamma/2)*(T^2-T1^2) +
(delta/3)*(T^3-T1^3) + (epsilon/4)*(T^4-T1^4)
alpha = 3.826
beta = -3.979E-03
gamma = 24.558E-06
delta = -22.733E-09
epsilon = 6.962E-12
```

```
Results: (a) T = 500 K
           // p = 0.03825 bar
          (b) p = 1 bar
           // T = 821.1 K
```

← (a) P  
← (b) T

(c) To check these results, we use the  $s = s_{TP}(\text{"CH4"}, T, P)$  functions of it, as follows:

```
s = s1
s1 = s_TP("CH4", T1, p1)
s = s_TP("CH4", T, p)
```

① Results: (c) T = 500 K; p = 0.03818 bar ← (c)  
p = 1 bar; T = 821.2 K

1. Note that the results using the internal functions of IT and using data from Table A-21 agree quite well in this case.

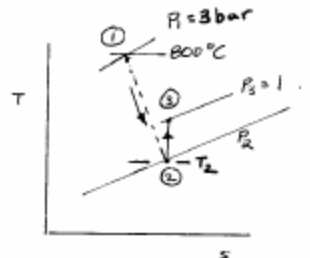
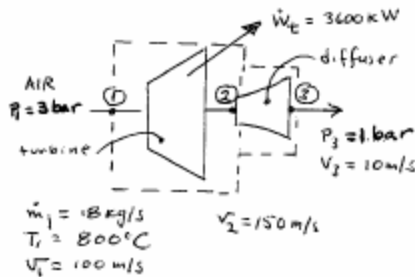


**PROBLEM 6.140**

**KNOWN:** Steady state operating data are provided for an air turbine fitted with a diffuser at its exit.

**FIND:** Determine the pressure and temperature at the turbine exit and the rate of entropy production in the turbine. Show the processes on a T-s diagram.

**SCHEMATIC & GIVEN DATA:**



**ASSUMPTIONS:** (1) The overall control volume consists of two sub-control volumes, each at steady state. (2) The turbine operates adiabatically. (3) Flow through the diffuser is isentropic. (4) Potential energy effects can be ignored. (5) Air is modeled as an ideal gas.

**ANALYSIS:** (a) For the isentropic process of air from 2 to 3,  $P_2/P_3 = P_r(T_2)/P_r(T_3)$ . Accordingly, to find  $P_2$  requires  $P_r(T_2)$  and  $P_r(T_3)$ .

Mass and energy rate balances for the turbine reduce to give

$$0 = \dot{Q}_{cv} - \dot{W}_t + \dot{m} (h_1 - h_2 + \frac{V_1^2 - V_2^2}{2}) \Rightarrow h_2 = \frac{-\dot{W}_t}{\dot{m}} + h_1 + \frac{V_1^2 - V_2^2}{2}$$

with  $h_1$  from Table A-22

$$h_2 = \frac{-3600 \text{ kJ/s} + 1129.8 \text{ kJ/kg} + \frac{(100)^2 - (10)^2}{2} \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right)}{18 \text{ kg/s}} = 923.6 \frac{\text{kJ}}{\text{kg}}$$

Then, interpolation in Table A-22 gives  $P_{r2} = 72.66$ ,  $T_2 = 892 \text{ K} (619^\circ \text{C})$ . ←  $T_2$

Mass and energy rate balances for the diffuser reduce to give

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} (h_2 - h_3 + \frac{V_2^2 - V_3^2}{2}) \Rightarrow h_3 = h_2 + \frac{V_2^2 - V_3^2}{2}$$

$$\text{or } h_3 = 923.6 + \frac{(10)^2 - (150)^2}{2000} = 934.8 \text{ kJ/kg.}$$

Interpolating in Table A-22,  $P_{r3} = 75.85$ . Then

$$P_2 = P_3 \frac{P_{r2}}{P_{r3}} = (1 \text{ bar}) \left( \frac{72.66}{75.85} \right) = 0.958 \text{ bar} \quad \leftarrow P_2$$

(b) Mass and entropy rate balances reduce to give for the turbine

$$\dot{Q}_T = \dot{m} (s_2 - s_1)$$

With Eq. 6.21a this becomes

$$\dot{Q}_T = \dot{m} [s^0(T_2) - s^0(T_1) - R \ln P_2/P_1]$$

$$= (18 \text{ kg/s}) \left[ 2.8381 - 3.0485 - \frac{8.314}{28.97} \ln \left( \frac{0.958}{3} \right) \right] \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

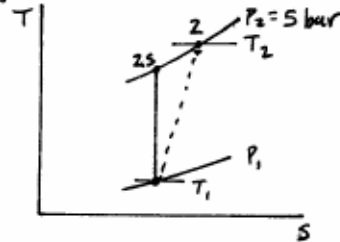
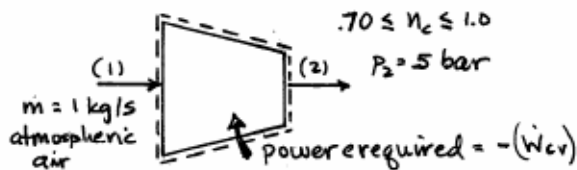
$$= 2.11 \frac{\text{kW}}{\text{K}} \quad \leftarrow \dot{Q}_T$$

PROBLEM 6.157

**KNOWN:** A compressor operating at steady state takes in atmospheric air at a known mass flow rate and discharges air at a given pressure.

**FIND:** Plot the power required and the compressor exit temperature, each versus compressor isentropic efficiency ranging from 70% to 100%.

**SCHEMATIC & GIVEN DATA:**



**ASSUMPTIONS:** (1) The control volume shown is at steady state. (2) For the control volume,  $\dot{Q}_{cv} = 0$ . (3) Kinetic and potential energy changes from inlet to exit can be ignored. (4) Since the incoming air is atmospheric, we assume  $P_1 = 1 \text{ bar}$  and  $T_1 = 20^\circ\text{C}$ .

**ANALYSIS:** The mass and energy rate balances reduce to give

$$\text{power required} = -(\dot{W}_{cv}) = \dot{m}(h_2 - h_1) \quad (1)$$

the enthalpy  $h_1$  can be determined since  $P_1$  and  $T_1$  are known from assumption 4. The enthalpy  $h_2$  is found using compressor efficiency  $\eta_c$ , as follows:

$$h_2 = h_1 + \frac{(h_{2s} - h_1)}{\eta_c} \quad (2)$$

where  $h_{2s}$  corresponds to an isentropic compression from state 1 to  $P_2$ .

**Sample Calculations**

A sample calculation will be performed using data from Table A-22. From the table;  $h_1 = 293.2 \text{ kJ/kg}$  and  $P_{r1} = 1.27652$ . Then, from Eq. 6.43

$$P_{r(T_{2s})} = 1.27652 \left( \frac{P_2}{P_1} \right) = 6.3826$$

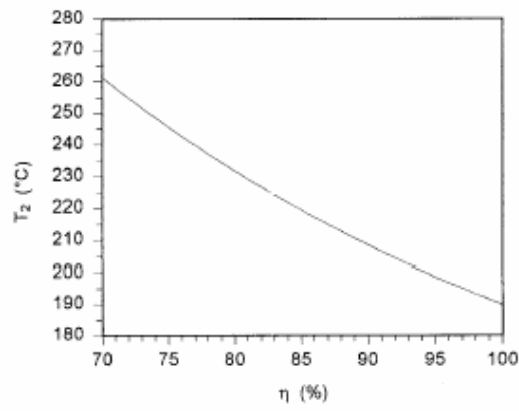
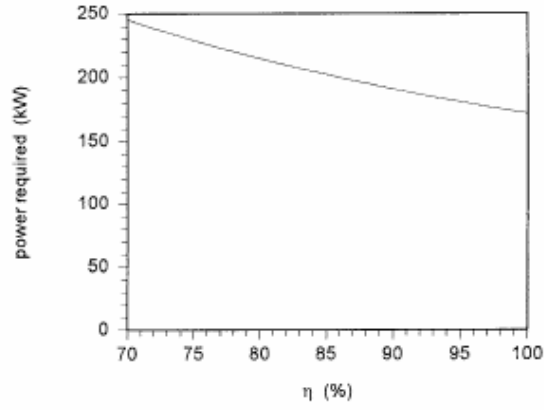
Interpolating in Table A-22;  $h_{2s} = 464.8 \text{ kJ/kg}$ . Accordingly, for  $\eta_c = 0.7$

$$h_2 = 293.2 + \frac{(464.8 - 293.2)}{0.7} = 538.3 \text{ kJ/kg}$$

and

$$\left( \text{power required} \right) = \left( 1 \frac{\text{kg}}{\text{s}} \right) (538.3 - 293.2) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 245.1 \text{ kW}$$

Plots:



Note: As  $\eta_c$  decreases, the required power increases as does the exit temperature. This is expected.