Polytropic Process of an Ideal Gas

Polytropic Process of an Ideal Gas

- **The relationship between the pressure and volume during compression or expansion of an ideal gas can be described analytically. One form of this relationship is given by the equation**

$$p\forall^{n} = \text{constant}$$

- **where n is a constant for the particular process.**

- **A thermodynamic process described by the above equation is called a Polytropic process.**
- **For a Polytropic process between two states 1-2**

$$p_1\forall_1^{n} = p_2\forall_2^{n} = \text{constant}$$

Remarks

$$p_1 V_1^n = p_2 V_2^n = \text{constant}$$

- When n=0, p = constant, and the process is a constant pressure or an ***isobaric*** process.
- When n=1, pV = constant, and the process is a constant temperature or an ***isothermal*** process.
- When n→∞, it is called an ***isometric*** process.
- When n=k, it is an called ***isentropic*** process.

Adiabatic Process

A thermodynamic process in which there is no heat into or out of a system is called an *adiabatic* process. To perform an ideal adiabatic process it is necessary, that the system be surrounded by a perfect heat insulator. If a compression or expansion of a gas takes place in a short time, it would be nearly adiabatic, such as the compression stroke of a gasoline or a diesel engine.

Adiabatic-Polytropic (Isentropic) Process

Let an ideal gas undergo an infinitesimal adiabatic process:

$$dQ = 0$$

$$dU = nC_v dT, \text{ and } dW = Pd\forall.$$

From the first law:

$$dU = dQ - dW$$

$$nC_v dT = -Pd\forall$$

Taking the derivative of the ideal gas law:

$$P\forall = nRT$$

results in

$$Pd\forall + \forall dP = n\bar{R}dT$$

Eliminating dT between these two equations and using

$$Cp - Cv = \bar{R}$$

results in:

$$\frac{dp}{p} + \frac{C_p d\forall}{C_v \forall} = 0$$

Adiabatic-Polytropic (Isentropic) Process

Denote C_p/C_v = k, the ratio of specific heat capacities of the gas. Then

$$\frac{dp}{p} + k\frac{d\mathcal{V}}{\mathcal{V}} = 0$$

Integration gives

$$\ln(\mathrm{P}) + \mathrm{k}\ \ln(\mathcal{V}) = \ln(\mathrm{constant})$$

So

$$\mathrm{p}\mathcal{V}^{\mathrm{k}} = \mathrm{constant}$$

Remarks

For an adiabatic process

$$p_1V_1^k = p_2V_2^k$$

Work done during an adiabatic process:

$$W_{12} = (p_1V_1 - p_2V_2)/(k-1)$$

Alternate expression: $W_{12} = nC_v(T_1-T_2)$, for constant C_v

Ideal Gas Polytropic Process

From $$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^n$$

$p_1V_1=nRT_1$ and $p_2V_2=nRT_2$

we get $$\frac{p_2}{p_1} = \left(\frac{nRT_1/p_1}{nRT_2/p_2}\right)^n = \left(\frac{T_1 p_2}{T_2 p_1}\right)^n = \left(\frac{p_2}{p_1}\right)^n \left(\frac{T_1}{T_2}\right)^n$$

$$\text{or } \frac{T_1}{T_2} = \left(\frac{p_2}{p_1}\right)^{\frac{1-n}{n}} \quad \text{or } \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

Similarly: $$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1}$$