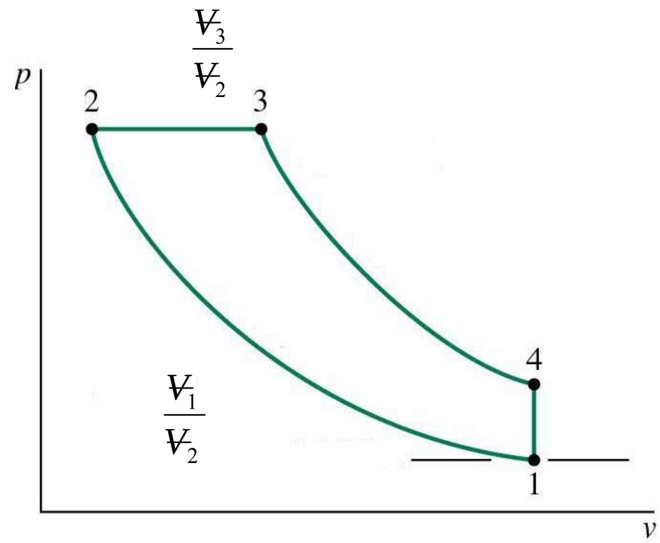
Name:

Answer ONLY two of the following problems. Good Luck!

Problem 1 (25 points)

h_3=1855.5 kJ/kg
u_3=1373.24 kJ/kg
h_4=895.11 kJ/kg
u_4=646.8 kJ/kg
h_1=295.17 kJ/kg
u_1=210.49 kJ/kg
$r_c=V_3/V_2=2$
$r= V_1/V_2=15.5$



T_1=295 K
P_1= 1 bar = 10^5 N/m^2
V_1= 0.02 m^3

In the air-standard Diesel cycle above, the compression ratio is r = $\frac{V_1}{V_2}$ = 15.5, the cutoff ratio is r_c = $\frac{V_3}{V_2}$ = 2, and the initial (state 1) temperature, pressure and volume are T_1 = 295 K, p_1 = 1 bar and V_1=0.02 m^3. Additional specific enthalpy and specific heat data are provided above (note that not all this data is needed to answer the questions below).

(a) **(8 points)** Assuming process 1-2 is isentropic, find the temperature at state 2 using
  (i) air tables
  (ii) isentropic process relations

(b) **(7 points)** Compute the heat added to the system.

(c) **(7 points)** What is the net work of the cycle?

(d) **(3 points)** Find the thermal efficiency of the cycle.

Problem 2 (25 points)

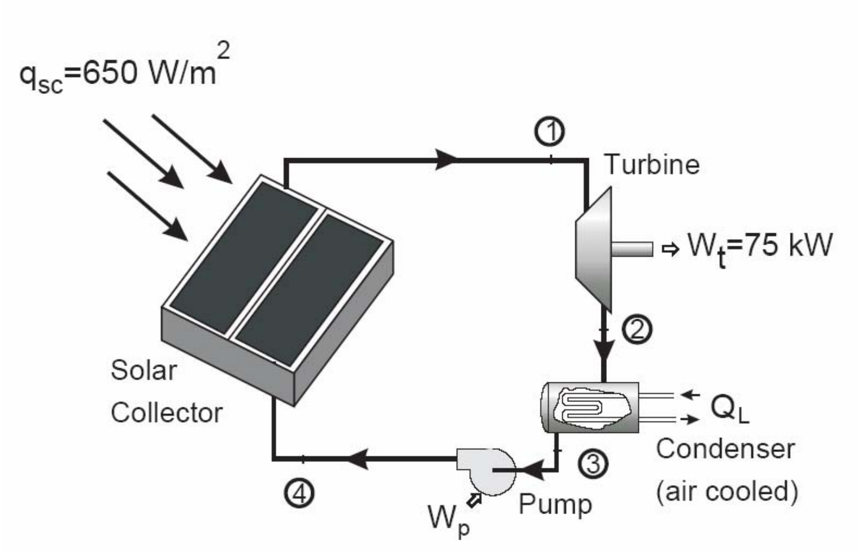
A solar driven Rankine-cycle, power plant uses water as the working fluid. During daytime operation, the turbine inlet state is 100ºC at 50kPa. The air cooled condenser operates at a pressure of 5 kPa. The pump is assumed to be isentropic and the turbine has an efficiency of 0.80. The work output is 75kW when the solar collector input is 650W/ m^2. The water leaves the condenser as a saturated liquid.

a) **(10 points)** Determine the thermal efficiency of the cycle.

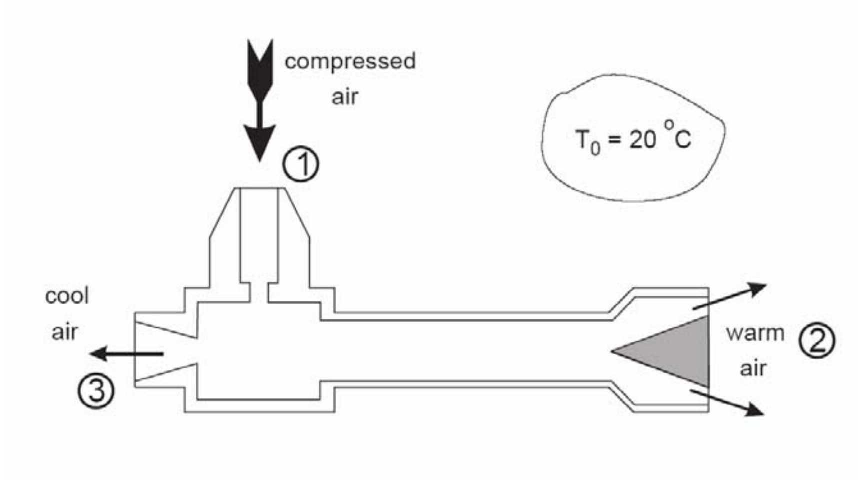
b) **(6 points)** Determine the mass flow rate of the working fluid, [kg/s].

c) **(6 points)** Determine the required collector surface area, [m^2].

d) **(3 points)** Give three thermodynamic advantages and disadvantages of this type of system.



Problem 3 (25 points)
A vortex tube is a steady-state device that splits a high pressure gas stream into two streams, one warm and one cool. During a test, air entered the tube at 19.3ºC and 0.52MPa. The warm air left the tube at 26.5ºC and the cool air left at −21.8ºC. Both exit streams were at 0.10135MPa. The ratio of the mass flow rate of the warm air to that of the cool air was 5.39. There was no work transfer. The temperature of the surroundings was 20ºC. Is this process viable? (Hint: does the process violate the second law) Assume air is an ideal gas with Cp = 1.005 kJ/kg · K.



Problem 1

Solution:

a) **If the process is isentropic, then Eq. 6.44 applies and** $\frac{v_{r1}}{v_{r2}} = \frac{v_1}{v_2} = \frac{V_1}{V_2} = r$.

From the supplied table at 295 K, v_{r1} = 647.9, so $v_{r2} = \frac{v_{r1}}{r} = \frac{647.9}{15.5} = 41.8$ **, so from the supplied table, T_2 = 840 K.**

b) **In the Diesel cycle, heat is added during the constant pressure process 2-3. From the first law,** $\Delta U = Q - W$ **, but since work is being done at constant pressure,** $W_{23} = \int_2^3 pdV = p(V_3 - V_2) = m \cdot p(v_3 - v_2)$

So $Q_{23} = m(u_3 - u_2) + m \cdot p(v_3 - v_2) = m[(u_3 + pv_3) - (u_2 + pv_2)]$**, but h = u + pv by definition, so**

$$Q_{23} = m(h_3 - h_2)$$

From the ideal gas equation of state,

$$m = \frac{p_1 V_1}{RT} = \frac{\left(1x10^5 \frac{N}{m^2}\right)(0.02m^3)}{\left(\frac{8314}{28.97} \frac{J}{kg \cdot K}\right)(295K)} = 0.0236kg$$

Therefore $Q_{in} = Q_{23} = 0.0236kg(1855.5 - 866.08)kJ/kg = 23.35kJ$

c) **The net work of any cycle is equal to the net heat transfer, so** $W_{cycle} = Q_{in} - Q_{out} = Q_{23} - Q_{41}$ **. From a first law analysis with no work, since process 4-1 is at constant volume,** $Q_{41} = m(u_4 - u_1)$**.**
Therefore $W_{cycle} = 23.35kJ - 0.0236kg(646.48 - 210.49)kJ/kg = 13.06kJ$

d) **The thermal efficiency of the cycle is** $\eta = \frac{W_{cycle}}{Q_{in}} = \frac{13.06}{23.35} = 0.559 = 55.9\%$

Problem 2.

Check to see is state point $2s$ is inside the dome. We know that

$$s_{2s} = s_1 = 7.6947\ kJ/kg \cdot K$$

and at a pressure of $5\ kPa$ the entropy of the saturated vapour is $s_g = 8.3951\ kJ/kg \cdot K$. Therefore state point $2s$ is inside the dome.

We can find the quality of state point $2s$ as follows:

$$\begin{aligned} s_{2s} &= s_f(1-x) + s_g \cdot x \\ 7.6947 &= 0.4764(1-x) + 8.3951x \end{aligned}$$

Therefore $x = 0.912$.
The enthalpy at $2s$ is given as

$$h_{2s} = 137.82(1-0.912) + 2561.5 \cdot (0.912) = 2348.2\ kJ/kg$$

We can now find the enthalpy at state point 2 using the isentropic efficiency of the turbine.

$$0.8 = \frac{h_1 - h_2}{h_1 - h_{2s}} = \frac{2682.5 - h_2}{2682.5 - 2348.2}$$

Therefore $h_2 = 2415.1\ kJ/kg$.

Part a)

Performing an energy balance over the turbine we get

$$\frac{\dot{W}_t}{\dot{m}} = h_1 - h_2 = 2682.5 - 2415.1 = 267.4\ kJ/kg$$

The work input to the pump is

The work input of the pump can be determined as

$$\begin{aligned} \frac{\dot{W}_p}{\dot{m}} = v_3(P_4 - P_3) &= (0.001005\ m^3/kg)(50-5)\ kPa \cdot \frac{1\ kJ/m^3}{1\ kPa} \\ &= 0.045\ kJ/kg \end{aligned}$$

Therefore

$$h_{4s} = 137.87\ kJ/kg$$

The heat input at the collector is given as

$$\frac{\dot{Q}_{sc}}{\dot{m}} = h_1 - h_{4s} = 2682.5 - 137.87 = 2544.63\ kJ/kg$$

The first law efficiency is given as

$$\eta_1 = \frac{\dot{W}_t/\dot{m} - \dot{W}_p/\dot{m}}{\dot{Q}_{sc}/\dot{m}} = \frac{267.4 - 0.045}{2544.63} = 0.105$$

Part b)

The mass flow rate of the working fluid is given as

$$\dot{m} = \frac{\dot{W}_t}{\dot{W}_t/\dot{m}} = \frac{75\ kW}{267.4\ kJ/kg} = 0.28\ kg/s$$

Part c) The required collector area is

$$A_{sc} = \frac{\dot{m} \times \dot{Q}_{sc}/\dot{m}}{q_{sc}} = \frac{0.28\ kg/s \times 2544.63\ kJ/kg}{0.65\ kW/m^2} = 1096.1\ m^2$$

Part d)

Advantages include:

1. low cost source of energy input
2. low operating temperatures, especially at the collector, mean heat is transferred across a small ΔT and therefore the production of entropy is low
3. since heat losses in the system are proportional to $\dot{m} c_p \Delta T$, the low ΔT between the system and the surroundings means heat losses are low (but this is offset somewhat by the high c_p of water)

Disadvantages include:

1. day time operation only, means restricted operation
2. the low efficiency of the energy conversion process means a large surface area of collectors is required to give a moderate energy output at the turbine
3. the low system operating pressure (less than the ambient) means that every device in the system must be sealed very tight to prevent air entrainment and loss of efficiency
4. since $\eta \propto 1 - \dfrac{T_L}{T_H}$, the small temperature difference between the collectors and the condenser means the efficiency will never be high

Problem 3.

Since we do not know the actual mass flow rate, we need to find the relative flow rates normalized with respect to the total mass flow rate in the system. Therefore, from a mass balance we can write

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3$$

or normalized with respect to $\dot{m}_1$

$$\frac{\dot{m}_1}{\dot{m}_1} = \frac{\dot{m}_2}{\dot{m}_1} + \frac{\dot{m}_3}{\dot{m}_1} = 1$$

But we know that $\dot{m}_2 = 5.39\ \dot{m}_3$, therefore

$$\begin{aligned} \frac{\dot{m}_2}{\dot{m}_1} &= 1 - \frac{1}{6.39} \\ \frac{\dot{m}_3}{\dot{m}_1} &= \frac{1}{6.39} \end{aligned}$$

Given the normalized mass flow rates, we can now set up an energy balance over the vortex tube to determine the heat transfer to or from the tube. If we assume the heat transfer is out of the tube we get

$$\dot{Q} = \dot{m}_1 \cdot h_1 - \dot{m}_2 \cdot h_2 - \dot{m}_3 \cdot h_3$$

or normalized with respect to $\dot{m}_1$

$$\begin{aligned} \frac{\dot{Q}}{\dot{m}_1} &= h_1 - \frac{\dot{m}_2}{\dot{m}_1} \times h_2 - \frac{\dot{m}_3}{\dot{m}_1} \times h_3 \\ &= h_1 - h_2 + \frac{1}{6.39}\ h_2 - \frac{1}{6.39}\ h_3 \\ &= (h_1 - h_2) - \frac{1}{6.39}(h_3 - h_2) \\ &= c_p(T_1 - T_2) - \frac{1}{6.39} c_p(T_3 - T_2) \\ &= 1.005\ \frac{kJ}{kg \cdot {}^\circ C}(19.3 - 26.5)\ {}^\circ C - \frac{1}{6.39}\left(1.005\ \frac{kJ}{kg \cdot {}^\circ C}\right)((-21.8) - 26.5)\ {}^\circ C \\ &= 0.361\ kJ/kg \end{aligned}$$

Now that we know we have heat transfer out of the vortex tube, we can set up an entropy balance equation.

$$\dot{m}_1 s_1 + \dot{\mathcal{P}}_s = \dot{m}_2 s_2 + \dot{m}_3 s_3 + \frac{\dot{Q}}{T_0}$$

Again we can normalize with respect to $\dot{m}_1$.

$$
\begin{aligned}
\frac{\dot{\mathcal{P}}_s}{\dot{m}_1} &= \frac{\dot{m}_2}{\dot{m}_1}s_2 + \frac{\dot{m}_3}{\dot{m}_1}s_3 - s_1 + \frac{\dot{Q}/\dot{m}_1}{T_0} \\
&= s_2 - \frac{1}{6.39}s_2 + \frac{1}{6.39}s_3 - s_1 + \frac{\dot{Q}/\dot{m}_1}{T_0} \\
&= (s_2 - s_1) - \frac{1}{6.39}(s_2 - s_3) + \frac{\dot{Q}/\dot{m}_1}{T_0}
\end{aligned}
$$

Since we know

$$
\begin{aligned}
s_2 - s_1 &= c_p \ln(T_2/T_1) - R\ln(P_2/P_1) \\
&= 1.005\ \frac{kJ}{kg \cdot K}\ln(299.65/292.45) - 0.286\ \frac{kJ}{kg \cdot K}\ln(0.10135/0.52) \\
&= 0.492\ \frac{kJ}{kg \cdot K} \\
s_2 - s_3 &= c_p \ln(T_2/T_3) - R\ln(P_2/P_3) \\
&= 1.005\ \frac{kJ}{kg \cdot K}\ln(299.65/251.35) - 0.286\ \frac{kJ}{kg \cdot K}\ln(0.10135/0.10135) \\
&= 0.177\ \frac{kJ}{kg \cdot K}
\end{aligned}
$$

Therefore the entropy production can be written as

$$
\frac{\dot{\mathcal{P}}_s}{\dot{m}_1} = 0.492\ \frac{kJ}{kg \cdot K} - \frac{1}{6.39} \times 0.177\ \frac{kJ}{kg \cdot K} + \frac{0.361\ kJ/kg}{293.15\ K} = 0.466\ \frac{kJ}{kg \cdot K}
$$

Since production of entropy is positive, the second law is not violated and the process is possible.