DIFFERENTIATION

This page contains a list of commonly used differentiation formulas. Applications of each formula can be found on the pages that follow.

1. Derivative of a Constant

$$\frac{d}{dx}k = 0$$

where k is a constant

2. Power Rule

$$\frac{d}{dx}x^r = rx^{r-1}$$

3. Constant-Multiple Rule

$$\frac{d}{dx}k \cdot f(x) = kf'(x)$$

where k is a constant

4. Sum Rule

$$\frac{d}{dx}[f(x)+g(x)] = f'(x)+g'(x)$$

5. General Power Rule

$$\frac{d}{dx}[f(x)]^r = r[f(x)]^{r-1} \cdot f'(x)$$

6. $\frac{d}{dx}e^x = e^x$

7. $\frac{d}{dx}e^{f(x)} = e^{f(x)} \cdot f'(x)$

8. $\frac{d}{dx}\ln(x) = \frac{1}{x},\ x > 0$

9. $\frac{d}{dx}\ln f(x) = \frac{f'(x)}{f(x)}$

10. Product Rule

$$\frac{d}{dx}f(x)g(x) = f(x)g'(x) + g(x)f'(x)$$

11. Quotient Rule

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{f'(x)g(x)-g'(x)f(x)}{[g(x)]^2}$$

11A. $\frac{d}{dx}\frac{1}{g(x)} = \frac{-g'(x)}{[g(x)]^2}$

12. $\frac{d}{dx}\sin x = \cos x$

13. $\frac{d}{dx}\sin f(x) = [\cos f(x)] f'(x)$

14. $\frac{d}{dx}\cos x = -\sin x$

15. $\frac{d}{dx}\cos f(x) = [-\sin f(x)] f'(x)$

16. $\frac{d}{dx}\tan x = \sec^2 x$

17. $\frac{d}{dx}\tan f(x) = [\sec^2 f(x)] f'(x)$

18. $\frac{d}{dx}\csc x = -(\csc x \cot x)$

19. $\frac{d}{dx}\csc f(x) = -(\csc x \cot x) f'(x)$

20. $\frac{d}{dx}\sec x = \sec x \tan x$

21. $\frac{d}{dx}\sec f(x) = (\sec x \tan x) f'(x)$

22. $\frac{d}{dx}\cot x = -\csc^2 x$

23. $\frac{d}{dx}\cot f(x) = -(\csc^2 x) f'(x)$

1. Derivative of a Constant $\frac{d}{dx}k = 0$ where k is a constant	Ex. $\frac{d}{dx}7 = 0$
2. Power Rule $\frac{d}{dx}x^r = rx^{r-1}$	Ex. $\frac{d}{dx}x^5 = 5x^4$ Ex. $\frac{d}{dx}\frac{1}{x^3} = \frac{d}{dx}x^{-3} = -3x^{-4}$ Ex. $\frac{d}{dx}x = 1$
3. Constant-Multiple Rule $\frac{d}{dx}k \cdot f(x) = kf'(x)$ where k is a constant	Ex. $\frac{d}{dx}2x^3 = 2 \cdot \frac{d}{dx}x^3 = 2 \cdot 3x^2 = 6x^2$
4. Sum Rule $\frac{d}{dx}[f(x)+g(x)] = f'(x)+g'(x)$	Ex. $\frac{d}{dx}(x^4 + 3x^2)$ $= \frac{d}{dx}\mathrm{x}^4 + \frac{d}{dx}3\mathrm{x}^2$ $= 4x^3 + 6x$
5. General Power Rule $\frac{d}{dx}[f(x)]^r = r[f(x)]^{r-1} \cdot f'(x)$	Ex. $\frac{d}{dx}(x^3+2x^2+1)^8$ $= 8\left(x^3+2x^2+1\right)^7 \cdot \frac{d}{dx}(x^3+2x^2+1)^8$ $= 8\left(x^3+2x^2+1\right)^7 \left(3x^2+4x\right)$
6. $\frac{d}{dx}e^x = e^x$	Ex. $\frac{d}{dx}3e^x = 3 \cdot \frac{d}{dx}e^x = 3e^x$

7. $\frac{d}{dx}e^{f(x)} \; = \; e^{f(x)} \cdot f'(x)$	Ex. $\frac{d}{dx}e^{5x} = e^{5x}\cdot \frac{d}{dx}5x = e^{5x}\cdot 5 = 5e^{5x}$ $\frac{d}{dx}e^{(x^2+2)^3} = e^{(x^2+2)^3}\cdot \frac{d}{dx}(x^2+2)^3$ Ex. $\quad = \; e^{(x^2+2)^3}\cdot 3(x^2+2)^2\cdot \frac{d}{dx}(x^2+2)$ $= \; e(x^2+2)^3\cdot 3(x^2+2)^2\cdot 2x$ $= \; 6x(x^2+2)^2 e(x^2+2)^3$ Note: The General Power Rule was also utilized in this problem.
8. $\frac{d}{dx}\ln(x) \; = \; \frac{1}{x}, \; x \; > \; 0$	Ex. $\frac{d}{dx}2\ln x = 2 \cdot \frac{d}{dx}\ln x = \frac{2}{x}$
9. $\frac{d}{dx}\ln \; f(x) \; = \; \frac{f'(x)}{f(x)}$	Ex. $\frac{d}{dx}\ln(x^2+7x)^3$ **<u>Method 1</u>**: $= \frac{\frac{d}{dx}(x^2+7x)^3}{(x^2+7x)^3}$ $= \frac{3(x^2+7x)^2(2x+7)}{(x^2+7x)^3}$ $= \frac{3(2x+7)}{x^2+7x}$ **<u>Method 2</u>**: This method utilizes the law of logarithms that states ln (x^a) = a ln x. $\frac{d}{dx}\ln(x^2+7x)^3 = \frac{d}{dx}\ln 3(x^2+7x)$ $= 3\frac{d}{dx}\ln(x^2+7x)$ $= \frac{\frac{d}{dx}(x^2+7x)}{x^2+7x}$ $= \frac{3(2x+7)}{x^2+7x}$

10. Product Rule

$$\frac{d}{dx} f(x)g(x) = f(x)g'(x) + g(x)f'(x)$$

Ex.

$$\frac{d}{dx}(2x^2+3x)(x^3-7x^2+4)$$

$$= (2x^2+3x)\cdot \frac{d}{dx}(x^3-7x^2+4)$$

$$+ (x^3-7x^2+4)\cdot \frac{d}{dx}(2x^2+3x)$$

$$= (2x^2+3x)(3x^2-14x)$$

$$+ (x^3-7x^2+4)(4x+3)$$

Ex.

$$\frac{d}{dx} x^3 e^{-5x+2}$$

$$= x^3 \frac{d}{dx} e^{-5x+2} + e^{-5x+2} \cdot \frac{d}{dx} x^3$$

$$= x^3 e^{-5x+2} \frac{d}{dx}(-5x+2) + e^{-5x+2} \cdot 3x^2$$

$$= -5x^3 e^{-5x+2} + 3x^2 e^{-5x+2}$$

$$= x^2 e^{-5x+2}(-5x+3)$$

$$= x^2(-5x+3)e^{-5x+2}$$

11. Quotient Rule

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

Ex. $\frac{d}{dx}\frac{3x^2+1}{4x^2-2}$

$$= \frac{(4x^3-2)\frac{d}{dx}(3x^2+1)(3x^2+1)\frac{d}{dx}(3x^2-2)}{(4x^3-2)^2}$$

$$= \frac{(4x^3-2)(6x)-(3x^2+1)(12x^2)}{(4x^3-2)^2}$$

$$= \frac{24x^4-12x-36x^4-12x^2}{(4x^3-2)^2}$$

$$= \frac{-12x^4-12x^2-12x}{(4x^3-2)^2}$$

Ex. $\frac{d}{dx}\frac{1-lnx}{x^2}$

$$= \frac{x^2\frac{d}{dx}(1-lnx)-(1-lnx)\frac{d}{dx}x^2}{(x^2)^2}$$

$$= \frac{x^2\cdot\frac{-1}{x}-2x(1-lnx)}{x^4}$$

$$= \frac{-x-2x+2xlnx}{x^4}$$

$$= \frac{-3x+2xlnx}{x^4}$$

$$= \frac{x(-3+2lnx)}{x^4}$$

$$= \frac{-3+2lnx}{x^3}$$

11A. $\frac{d}{dx}\frac{1}{g(x)} = \frac{-g'(x)}{[g(x)]^2}$

Ex. $\frac{d}{dx}\frac{1}{3x^2-2x} = \frac{-\frac{d}{dx}3x^2-2x}{(3x^2-2x)^2}$

$$= \frac{-(6x-2)}{(3x^2-2x)^2}$$

$$= \frac{-6x+2}{(3x^2-2x)^2}$$

12. $\frac{d}{dx}\sin x \;=\; \cos x$	Ex. $\frac{d}{dx}\text{-}3\sin x = \text{-}3\frac{d}{dx}\sin x = \text{-}3\cos x$
13. $\frac{d}{dx}\sin\ f(x) \;=\; [\cos f(x)]\,f\,'(x)$	Ex. $\frac{d}{dx}\sin 4x = (\cos 4x)\frac{d}{dx}4x$ $= (\cos 4x)4$ $= 4\cos 4x$
14. $\frac{d}{dx}\cos x \;=\; \text{-}\sin x$	Ex. $\frac{d}{dx}\frac{\cos x}{2} = \frac{1}{2}\cdot\frac{d}{dx}\cos x = \frac{-\sin x}{2}$
15. $\frac{d}{dx}\cos f(x) \;=\; [\text{-}\sin f(x)]\,f\,'(x)$	Ex. $\frac{d}{dx}\cos(x^2+1)$ $= [\text{-}\sin(x^2+1)]\ \frac{d}{dx}(x^2+1)$ $= [\text{-}\sin(x^2+1)]2x$ $= \text{-}2x\sin(x^2+1)$
16. $\frac{d}{dx}\tan x \;=\; \sec^2 x$	Ex. $\frac{d}{dx}6\tan x = 6\frac{d}{dx}\tan x$ $= 6\sec^2 x$
17. $\frac{d}{dx}\tan f(x) \;=\; [\sec^2 f(x)]\,f\,'(x)$	Ex. $\frac{d}{dx}\tan(x^3+1)$ $= [\sec^2(x^3+1)]\frac{d}{dx}(x^3+1)$ $= [\sec^2(x^3+1)]\ 3x^2$ $= 3x^2\ \sec^2(x^3+1)$

INTEGRATION

This page contains a list of commonly used integration formulas. Applications of each formula can be found on the following pages.

1. $\int x^r dx = \frac{x^{r+1}}{r+1} + C$

1A. $\int k dx = kx + C$
where k is a constant

2. $\int kf(x)dx = k\int f(x)dx$
where k is a constant

3. $\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$

4. $\int e^x dx = e^x + C$

5. $\int e^{f(x)} f'(x)dx = e^{f(x)} + C$

6. $\int \frac{1}{x} dx = \ln|x| + C$

7. $\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + C$

8. Integration by Substitution
$\int f(u)du = F(u) + C$
where $u = g(x)$ and
$du = g'(x)dx$

9. Integration by Parts
$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$

10. $\int \cos x dx = \sin x + C$

11. $\int [\cos f(x)] f'(x)dx = \sin f(x) + C$

12. $\int \sin x dx = -\cos x + C$

13. $\int [\sin f(x)] f'(x)dx = -\cos f(x) + C$

14. $\int \sec^2 x dx = \tan x + C$

15. $\int [\sec^2 f(x)] f'(x)dx = \tan f(x) + C$

$\int x^r dx = \frac{x^{r+1}}{r+1} + C$	Ex. $\int x^7 dx = \frac{x^8}{8} + C$
1A. $\int k dx = kx + C$ where k is a constant	Ex. $\int 5 dx = 5x + C$
$\int kf(x)dx = k\int f(x)dx$ where k is a constant	Ex. $\int 5x^6 dx = 5\int x^6 dx$ $= \frac{5x^7}{7} + C$
$\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$	Ex. $\int (2x^2 + 3x + 2)dx$ $= \int 2x^2 dx + \int 3x dx + \int 2 dx$ $= \frac{2x^3}{3} + \frac{3x^2}{2} + 2x + C$
$\int e^x dx = e^x + C$	Ex. $\int 6e^x dx$ $= 6\int e^x dx$ $= 6e^x + C$
$\int e^{f(x)} f'(x) dx = e^{f(x)} + C$	Ex. $\int e^{2x^2+1} 4x dx = e^{2x^2+1} + C$ Ex. $\int e^{3x} dx$ $= \int \frac{1}{3} e^{3x} 3 dx$ $= \frac{1}{3}\int e^{3x} 3 dx$ $= \frac{e^{3x}}{3} + C$

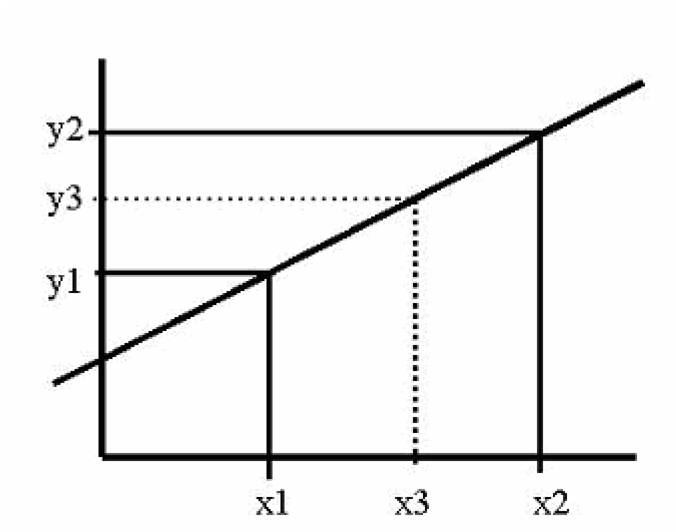
$\int \frac{1}{x}dx = \ln\lvert x\rvert + C$	Ex. $\int \frac{5dx}{x}$ $= 5\int \frac{dx}{x}$ $= 5\ln\lvert x\rvert + C$
$\int \frac{f'(x)}{f(x)}dx = \ln\lvert f(x)\rvert + C$	Ex. $\int \frac{2x}{x^2+1}dx = \ln\lvert x^2+1\rvert + C$ Ex. $\int \frac{x^2+1}{x^3+3x+2}dx$ $= \int \frac{\frac{1}{3}(3x^2+3)}{x^3+3x+2}dx$ $= \frac{1}{3}\int \frac{3x^2+3}{x^3+3x+2}dx$ $= \frac{1}{3}\ln\lvert x^3+3x+2\rvert + C$
Integration by Parts $\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$ Also written as: $\int udv = uv - \int vdu$ where $u = f(x) \qquad dv = g(x)dx$ $du = f'(x)dx \qquad v = G(x)$	Ex. $\int x^2 \ln x dx$ $f(x) = \ln x \qquad g(x) = x^2$ $f'(x) = \frac{1}{x} \qquad \mathrm{G}(x) = \frac{x^3}{3}$ $\int x^2 \ln x dx = \ln x \cdot \frac{x^3}{3} - \int \frac{1}{x} \cdot \frac{x^3}{3}dx$ $= \frac{x^3}{3}\ln x - \frac{1}{3}\int x^2 dx$ $= \frac{x^3}{3}\ln x - \frac{x^3}{9} + C$
$\int \cos x dx = \sin x + C$	Ex. $\int \frac{\cos x dx}{2} = \frac{1}{2}\int \cos x dx$ $= \frac{\sin x}{2} + C$

$\int \left[\cos f(x)\right] f'(x)\,dx = \sin f(x) + C$	Ex. $\int \cos 4x dx$ $= \int \frac{1}{4} \cos 4x 4dx$ $= \frac{1}{4} \int \cos 4x 4dx$ $= \frac{1}{4} \sin 4x + C$
$\int \sin x dx = -\cos x + C$	Ex. $\int -2 \sin x dx$ $= -2 \int \sin x dx$ $= -2(-\cos x) + C$ $= 2 \cos x + C$

LINEAR INTERPOLATION

If we assume a linear trend between two points (x_1, y_1) and (x_2, y_2) then for any "x value" say x_3, we can use "similar triangles" to estimate the missing y value (y_3), algebraically…

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_3 - y_1}{x_3 - x_1}$$



Example:

$$\begin{vmatrix} x_1 \\ y_1 \end{vmatrix} = \begin{vmatrix} 46.56 \\ 0 \end{vmatrix}, \quad \begin{vmatrix} x_2 \\ y_2 \end{vmatrix} = \begin{vmatrix} 50.95 \\ 25 \end{vmatrix}, \quad x_3 = 50, \; y_3 = ?$$

then

$$\frac{25 - 0}{50.95 - 46.56} = \frac{y_3 - 0}{50 - 46.56}$$

Solving for y_3 gives

y_3 = 19.6