Thermodynamics (ES 340) Quiz 1 May 25, 2004

Name: (1 Hour)

Problem 1: A piston/cylinder with cross sectional area of 0.01 m^2 has a piston mass of 100 kg resting on the stops, as shown in Figure 1. With outside atmospheric pressure of 100 kPa, what should the water pressure be to lift the piston? (Water density=1000 kg/m^3) (3 points)

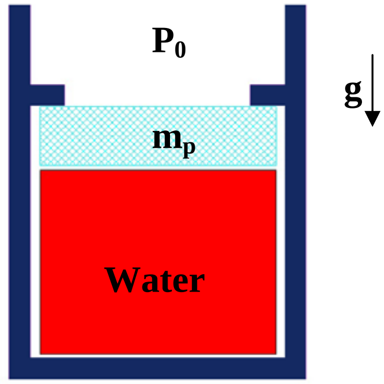


Figure 1

Problem 2: Consider a piston/cylinder arrangement as shown in Figure 2. Assume that the piston has the mass $\mathbf{m_p}$ and area $\mathbf{A_p}$ and the spring is a linear. The piston traps the gas inside the cylinder with a pressure **P**. If the outside atmospheric pressure is $\mathbf{P_0}$

(a) Derive the equation for work done by the system when the change in spring length is $\Delta x = x_1 - x_0$.

(b) If the surroundings temperature is $\mathbf{T_f}$, and the gas temperature is $\mathbf{T_b}$ where $\mathbf{T_b} > \mathbf{T_f}$ and only convection is the effective heat transfer mode, write the energy balance of the system and find the change in internal energy.

(c) Evaluate the total work, heat and the total energy if $\mathbf{m_p}$ = 1.3Kg, $\mathbf{A_p}$ = 1 m^2, **g** = 9.81 N/kg, $\mathbf{P_0}$= 1 bar, $\mathbf{T_f}$ =273.15 K, $\mathbf{T_b}$ =300K, h=0.171kW/m^2.K, $\Delta\forall$ =10 cm, and $\boldsymbol{K}$ = 0.2 N/m (spring constant).

Assume kinetic energy of the piston is negligible.
1m = 100 cm
1 bar = 10^5 Pa (7 points)

Hints: For a linear spring we have: $F_{spring} = K.\Delta x = K.\frac{\Delta\forall}{A} = K\frac{(V - V_0)}{A}$. In general, work done by compression is given as $W = \int_{\forall_1}^{\forall_2} P d\forall$.

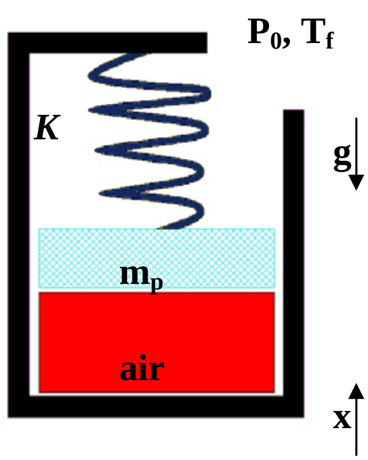
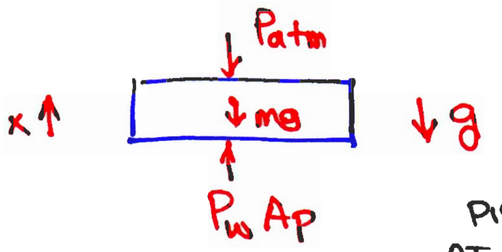


Figure 2



PISTON IS AT REST

$\sum F_y = 0$

$\rightarrow P_w A_p = P_{atm} A_p + m_p g$

$\rightarrow P_w = P_{atm} + \dfrac{m_p g}{A}$

$= 100\,\text{Kpa} + \dfrac{100 \times 9.81}{0.01}$

$= 198.1\ \text{Kpa}$

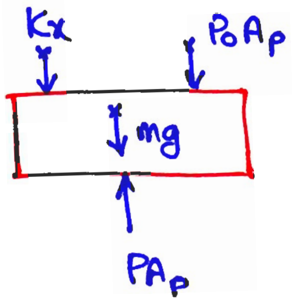
Chad's note:

$P_w \geqslant P_{atm} + \dfrac{m_p g}{A}$ or $P_w \geqslant 198.1\,\text{Kpa}$

that's the minimum pressure that water needs to have to move the piston.

PROBLEM TWO:

(a)



$$\sum F_x = 0 \Rightarrow kx + P_o A_p + mg = PA \quad , \quad \forall = A_p x \quad \text{(I)}$$

method ①

$$\rightarrow W = \int_{\forall_1}^{\forall_2} P \, d\forall = \int_{\forall_1}^{\forall_2} \left(\frac{k}{A_p^2} \forall + P_o + \frac{mg}{A_p} \right) d\forall$$

$$= \frac{1}{2} \frac{k}{A_p^2} \forall^2 + P_o \forall + \frac{mg}{A_p} \forall \Big|_{\forall_1}^{\forall_2}$$

$$= \frac{1}{2} \frac{k}{A_p^2} (\forall_2^2 - \forall_1^2) + P_o(\forall_2 - \forall_1) + \frac{mg}{A_p}(\forall_2 - \forall_1)$$

using (I) $\rightarrow$ $$\boxed{W = \frac{1}{2} k (x_2^2 - x_1^2) + P_o (x_2 - x_1) A_p + mg(x_2 - x_1)}$$

(a) Ans.

method ②

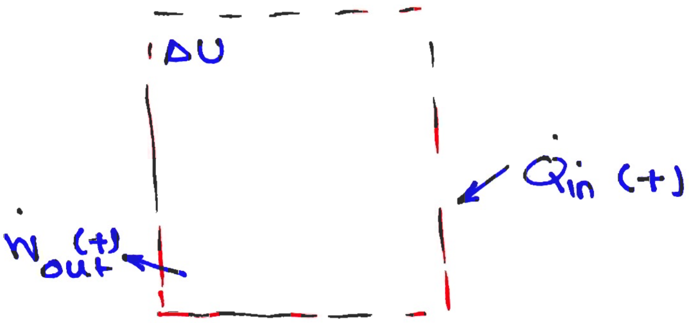
$$\rightarrow W = \int_{x_1}^{x_2} F \, dx = \int_{x_1}^{x_2} kx + P_o A + mg$$

$$= \frac{1}{2} k x^2 + P_o A x + mg x \Big|_{x_1}^{x_2}$$

$$= \frac{1}{2} k (x_2^2 - x_1^2) + P_o A (x_2 - x_1) + mg (x_2 - x_1)$$

(a) Ans.

(b)



Energy Balance:

$$\Delta E = \Delta KE + \Delta PE + \Delta U = Q - W$$

$$\boxed{dU = h A_p (T_f - T_b) - \dot{W}}$$

Zachary's Eq.:

$$W = \left(\frac{m_p \cdot g}{A_p} + P_o\right) \Delta \forall + \frac{K}{2A_p^2}\left(\Delta \forall^2 + 2\Delta \forall \forall_1\right)$$ (c)

$$W = \left(\frac{1.3 \times 9.81}{1} + 10^5\right)(0.1) + \frac{8 \times 10^3}{2(1)^2}\left(0.1^2 + 2 \times 0.1 \times 0.5\right)$$

$= 1.44$ KJ ✓ANS.

$$\dot{Q} = hA(T_f - T_b)$$

$$= \left(0.171 \times 10^3 \frac{W}{m2K}\right)(1)(273 - 300)$$

$= +4.617$ Kw ✓ANS.

$$\Delta U = Q - W$$

Approximate methods: (PROBlEM 2)

Drue & Adam's method:

$$W = P \Delta \forall$$

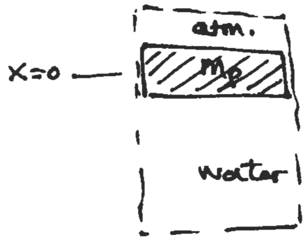
$$= \left(\frac{m_p g}{A_p} + \frac{K \Delta x}{A_p} + P_o \right) (A_p \Delta x)$$

$$\rightarrow \boxed{W = m_p g \Delta x + P_o A_p \Delta x + K (\Delta x)^2}$$

Using Energy Balance instead of Force Balance (PROBlEM 1)

David's method:

$$\Delta E = \cancelto{0}{\Delta KE} + \Delta PE + \cancelto{0}{\Delta U} = \cancelto{0}{Q} - \cancelto{0}{W}$$



$$\rightarrow \Delta PE = \Delta PE_{atm} + PE_{water} + PE_{piston}$$

$$\rightarrow \Delta E = 0 = \Delta PE = -m_p g h + P_{atm} \cdot A_p (-h) + P_W \cdot A_p (h)$$

$$\xrightarrow{\div h} \boxed{P_W = \frac{m_p g}{A_p} + P_{atm}}$$