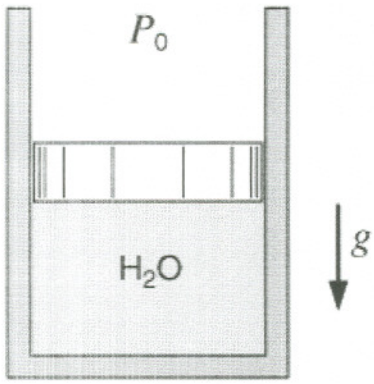
Thermodynamics (ES 340) Quiz 2 June 1, 2005
Name: (1 Hour)

Problem 1 (8 points): A piston/cylinder arrangement has the piston loaded with outside atmospheric pressure and the piston mass to a pressure of 150 kPa, shown here. It contains water at −2°C, which is then heated until the water becomes saturated vapor.

1. Draw the P-v and T-v diagrams. (2 points)
2. Find the final temperature. (2 points)
3. Find the specific work. (2 points)
4. Find the specific heat transfer using an energy balance of the system. (2 points)
5. **(Bonus)** Find the specific heat transfer without using the energy balance. (2 points)

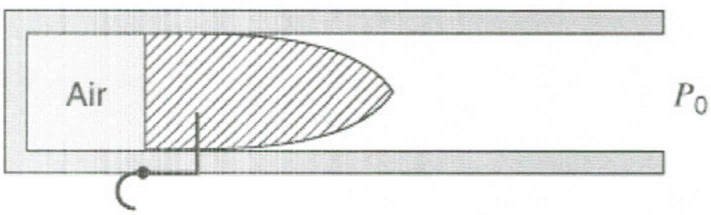


Problem 2 (12 points): An air pistol contains compressed air in a small cylinder, shown here. Assume that the volume is 1 cm3, pressure is 1 MPa, and the temperature is 27°C when armed. A bullet, m = 15 g, acts as a piston initially held by a pin (trigger); when released, the air expands in a polytropic process where n=*k* and *k* is the specific heat ratio of air. Assuming air is an ideal gas, if the air pressure is 0.1 MPa in the cylinder as the bullet leaves the gun,

1. Draw the P-v diagram. (2 points)
2. Find the final volume and the mass of air. (4 points)
3. Find the work done by the air and work done on the atmosphere. (6 points)
4. **(Bonus)** Find the work on the bullet and the bullet exit velocity. (2 points)

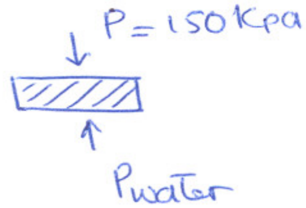
For Air M=30 and C_p can be found from the following formula

$$C_p = 1.05 - 0.365\theta + 0.85\theta^2 - 0.39\theta^3 \qquad , \theta = T(\text{Kelvin})/1000.$$



Problem 1

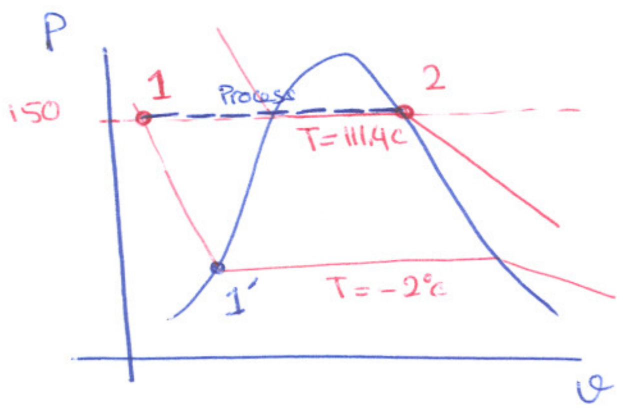
$\Sigma F_y = 0$

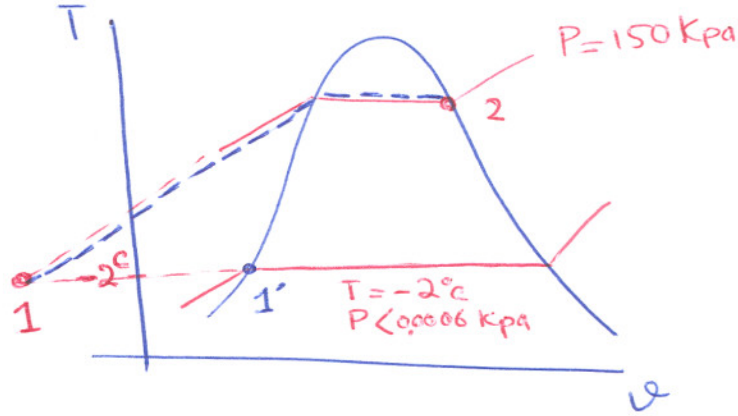


$\hookrightarrow P_{water} A_p = P A$

$\rightarrow P_{water} = 150$ Kpa

State ① $\begin{cases} P_1 = 150 \text{ Kpa} \\ T_1 = -2°C \end{cases} \xrightarrow{TA3} T_{sat} = 111.4\,C$





State ② $\rightarrow \begin{cases} P_2 = P_1 = 150 \text{ Kpa} & \text{(piston can freely move)} \\ x = 1 & \text{(saturated vapor)} \end{cases}$

So state 1 is compressed liquid, using the approx. For comp. liquid we can use saturated water table,

$$\text{Table A2} \rightarrow \begin{cases} T = 0.01 \rightarrow v_f = 1.0002 \times 10^{-3} \\ T = 4 \rightarrow v_f = 1.0001 \times 10^{-3} \\ T = -2 \xrightarrow[\text{-polation}]{\text{inter}} v_f = 1.00025 \times 10^{-3} \ m^3/kg \end{cases}, \begin{cases} u = 0 \\ u = 16.77 \\ u = -8.45 \ kJ/kg \end{cases}$$

For state ② use Table A2

$$\begin{cases} x = 1 \\ P_2 = P_1 = 150 \text{ Kpa} \end{cases} \rightarrow \underset{\text{Ans.}}{T = 111.4\,C} \rightarrow \begin{cases} u_2 = u_g = 2519.7 \ kJ/kg \\ v_2 = v_g = 1.159 \ m^3/kg \end{cases}$$

Work is

$$W = \int p\,dv = P\int_{v_1}^{v_2} dv = P_1(v_2 - v_1) = 150(1.159 - 1\times10^{-3})$$
$$= \underline{173.7 \text{ KJ/Kg}} \quad \text{Ans.}$$

Problem 1 (cont'd)

Energy Balance for a closed System

$\Delta E = Q - W$ or $\Delta e = q - w$

$\Delta e = \cancel{\Delta pe} + \cancel{\Delta ke} + \Delta u$

$\rightarrow \Delta u = q - w \rightarrow q = \Delta u + w = (u_2 - u_1) + w$

$= [2519 - (-8.45)] + 173.$

$\rightarrow \underline{q = 2700.45}$ KJ/Kg Ans

Now if we assume State 1 as Solid (which is the correct assumption) we need to use table A-6 instead of A2,

using T. A-6 and $T = -2^C \rightarrow \begin{cases} v_f = v_1 = 1.0904 \times 10^{-3} \text{ m}^3/\text{kg} \\ u_f = u_1 = -337.62 \text{ KJ/Kg} \end{cases}$

using these values

$\begin{cases} w = P_1(v_2 - v_1) = \underline{173.7} \text{ KJ/Kg} \text{ Ans.} \\ q = \underline{3031} \text{ KJ/Kg} \text{ Ans.} \end{cases}$

Bonus

$Q = mc_v \Delta T$ (Closed System)

$C_p = 4.238 \frac{KJ}{KgK}$ (Table A19 averege for $T = 387K$)

$C_v = C_p - R \rightarrow C_v = 3.776$ KJ/KgK

$\rightarrow Q = q \times m \rightarrow q = C_v \Delta T = 3.776 \times (111.4 - (-2)) = 428 \text{ KJ}$

Problem 2

air, ideal gas

① $\begin{cases} \forall_1 = 1\ cm^3 \\ P_1 = 1\ Mpa \\ T_1 = 27°c \Rightarrow T = 27 + 273 = 300K \end{cases}$

$m_b = 15g = 0.015\ Kg$

② $\{ P_2 = 0.1\ Mpa$

Polytropic process, $Pv^n = const.$, $n = K$ (isentropic process)

— ideal gas

$P\forall = mR_{air}T$

$R_{air} = \frac{\bar{R}}{M_{air}} = \frac{8.314}{30} = 0.277\ KJ/KgK$

$\xrightarrow{\text{for state ①}} m = \frac{P_1 \forall_1}{R_{air} T_1}$

$= \frac{1\ Mpa \times 1\ cm^3}{0.277 \times 10^3 \frac{J}{KgK} \times 300K}$

$\rightarrow m = 1.202 \times 10^{-5}\ Kg$

Ans.

— Polytropic process

$P\forall^K = const. \rightarrow P_1 \forall_1^K = P_2 \forall_2^K \rightarrow \frac{\forall_1}{\forall_2} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{K}}$ ①

for your info

for ideal gases, $P\forall = mRT$ ②

using ① and ② you can show $\frac{T_1}{T_2} = \left(\frac{\forall_2}{\forall_1}\right)^{K-1}$

using ① to find $\forall_2$ →, but we need to find K first.

Problem 2 (cont'd)

$R = C_p - C_v \rightarrow C_v = C_p - R$

$K = \frac{C_p}{C_v}$

$\rightarrow K = \frac{C_p}{C_p - R}$

$T = 300\,K \rightarrow \theta = \frac{300}{1000} = 0.3 \rightarrow C_p = 1.05 - 0.365(0.3) + 0.85(0.3)^2 - 0.39(0.3)^3$

$= 1.00647 \simeq 1\ KJ/kg\,K$

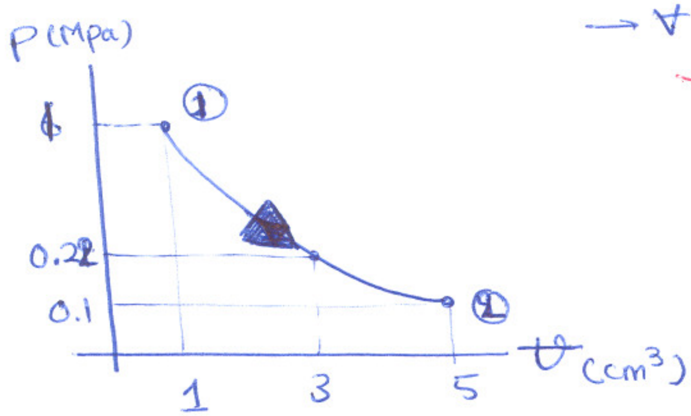
$\rightarrow K = \frac{1}{1 - 0.277} = 1.3797$

$\rightarrow P_1 \forall_1^K = P_2 \forall_2^K \rightarrow \frac{\forall_1}{\forall_2} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{K}}$

$\rightarrow \forall_2 = \forall_1 \left(\frac{P_1}{P_2}\right)^{1/K} = 1\,cm^3 \times \left(\frac{1\ Mpa}{0.1\ Mpa}\right)^{\frac{1}{1.3797}}$

$\rightarrow \forall_2 = 5.306\ cm^3$

Ans.



Comp. Work for a polytropic process $\rightarrow W = \int_{\forall_1}^{\forall_2} p\,d\forall$

$\rightarrow W_{c.v.} = \frac{1}{1-n}(P_1\forall_1 - P_2\forall_2)$ if $n = k$

For isentropic process $W_{c.v.} = \frac{1}{1-K}(P_1\forall_1 - P_2\forall_2)$

$= \frac{1}{1 - 1.3797}(1 \times 1 - 0.1 \times 5.306) = 1.238\ J$

Ans.

Problem 2 (con'd)

$$W_{atm} = \int p\, d\forall$$

$$P_{atm} = const.$$

$$\left.\right\} \rightarrow W_{atm} = p \int d\forall = p(\forall_2 - \forall_1)$$

$$= 0.1\,(5.306 - 1)$$

$$P_{atm} = P_2$$

$$W_{atm} = 0.4306 \text{ J}$$

Ans.

Bonus.

$$\Sigma W = W_{cv} - W_{atm} - W_{bullet} = 0$$

$$\rightarrow W_{bullet} = W_{cv} - W_{atm} = 1.238 - 0.4306 = 0.8074 \text{ J}$$

for the bullet we have

$$\rightarrow W_{bullet} = \cancel{\Delta PE}^{\,0} + \Delta KE + \cancel{\Delta U}^{\,0}$$

(bullet initially at rest) ① ②

$\Delta PE = 0$

$V_1 = 0$ $\quad V_2 = ?$

$\Delta U = 0$, $Q = 0$

$$\rightarrow W_{bullet} = \frac{1}{2} m_b (V_2^2 - \cancel{V_1^2}^{\,0})$$

$$\rightarrow V_2 = \sqrt{2 W_{bullet} / m_b} \Rightarrow V_2 = 10.38 \text{ m/s}$$

Ans.