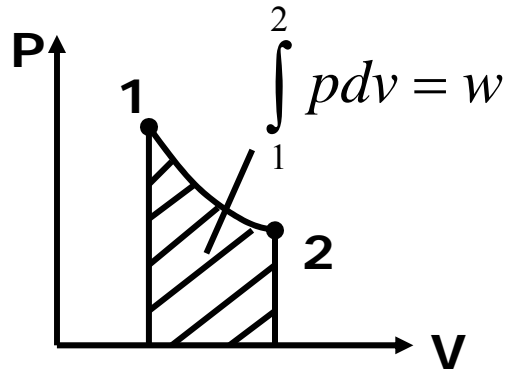

The Second Law of Thermodynamics

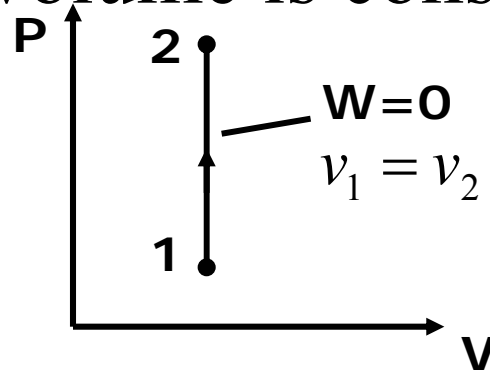
Note:

for a reversible process, on p-v diagram

$$\int_1^2 P dv = W_{rev}$$



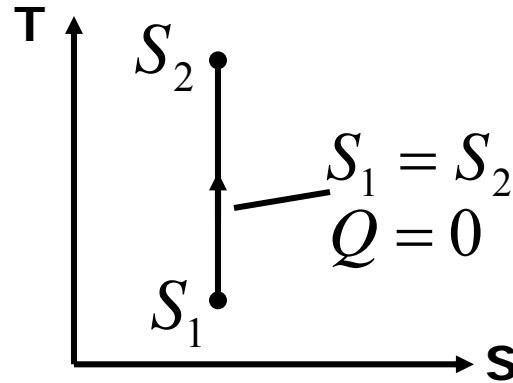
and If the volume is constant,



$$\int_1^2 P dv = 0$$

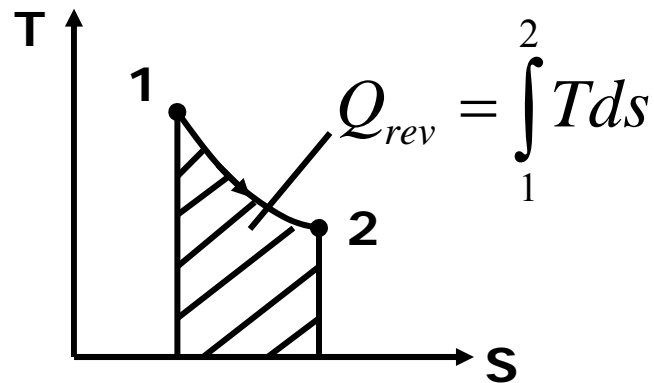
-
- Similarly, for any reversible adiabatic process, $Q=0$

$$Q_{rev} = \int Tds = 0$$



- For any other reversible process

$$Q_{rev} = \int_1^2 Tds$$



-
- Clausius Inequality:

$$\oint \frac{\delta Q}{T} \leq 0$$

- Clausius inequality is valid for all thermodynamic cycles, reversible or irreversible.

$$\oint \left(\frac{\delta Q}{T} \right)_{rev} = 0 \quad \text{for internally reversible cycle}$$

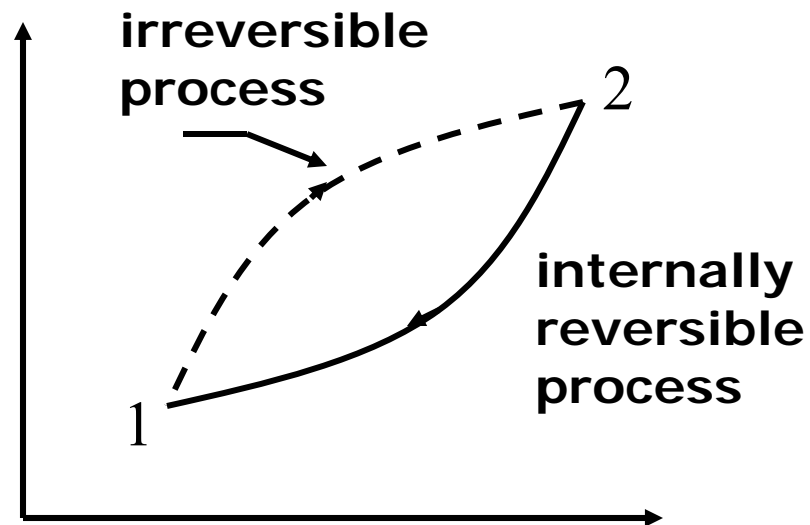
$$\oint \left(\frac{\delta Q}{T} \right) < 0 \quad \text{for irreversible cycle}$$

$\left(\frac{\delta Q}{T}\right)_{\text{rev}}$ represents a property change

$$dS = \left(\frac{\delta Q}{T}\right)_{\text{rev}}$$

$$\Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_{\text{rev}}$$

-
- Consider a cycle composed of a reversible and an irreversible process:



-
- The quantity $\Delta s = s_2 - s_1$ represents the entropy change of the system.

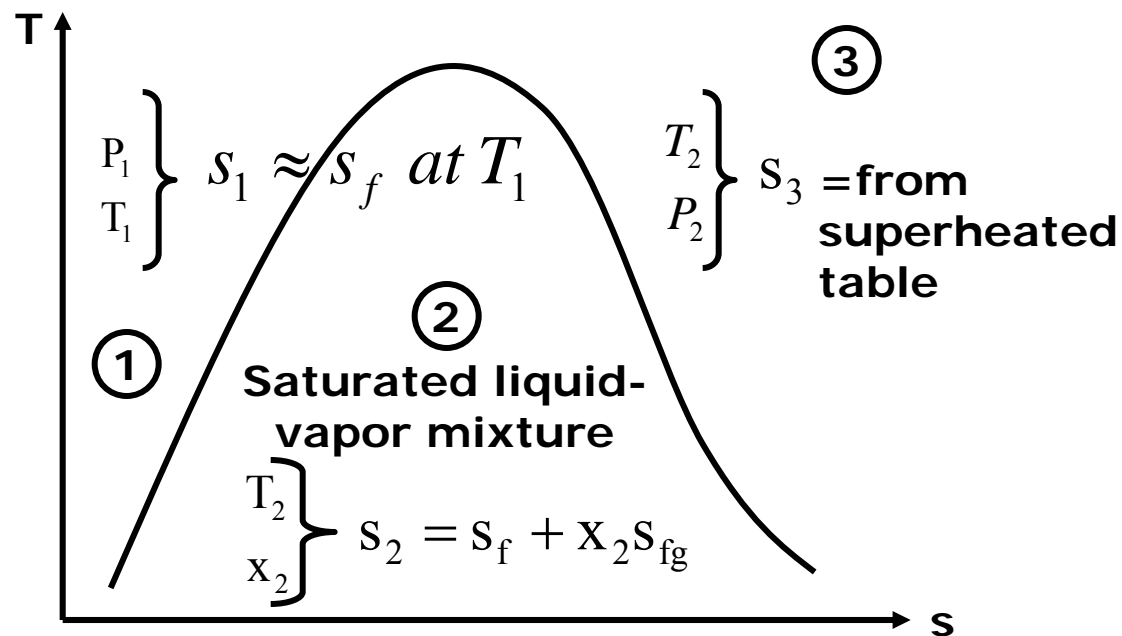
- For a reversible process, entropy transfer with heat
$$\Delta s = \int_1^2 \frac{\delta Q}{T}$$
- For an irreversible process

$$\Delta s_{sys} = \int_1^2 \frac{\delta Q}{T} + s_{gen}$$

- Some entropy is generated during an irreversible process and is always positive quantity. Its value depends on the process, and thus it is not a property of the system.

Entropy Change for Pure Substances

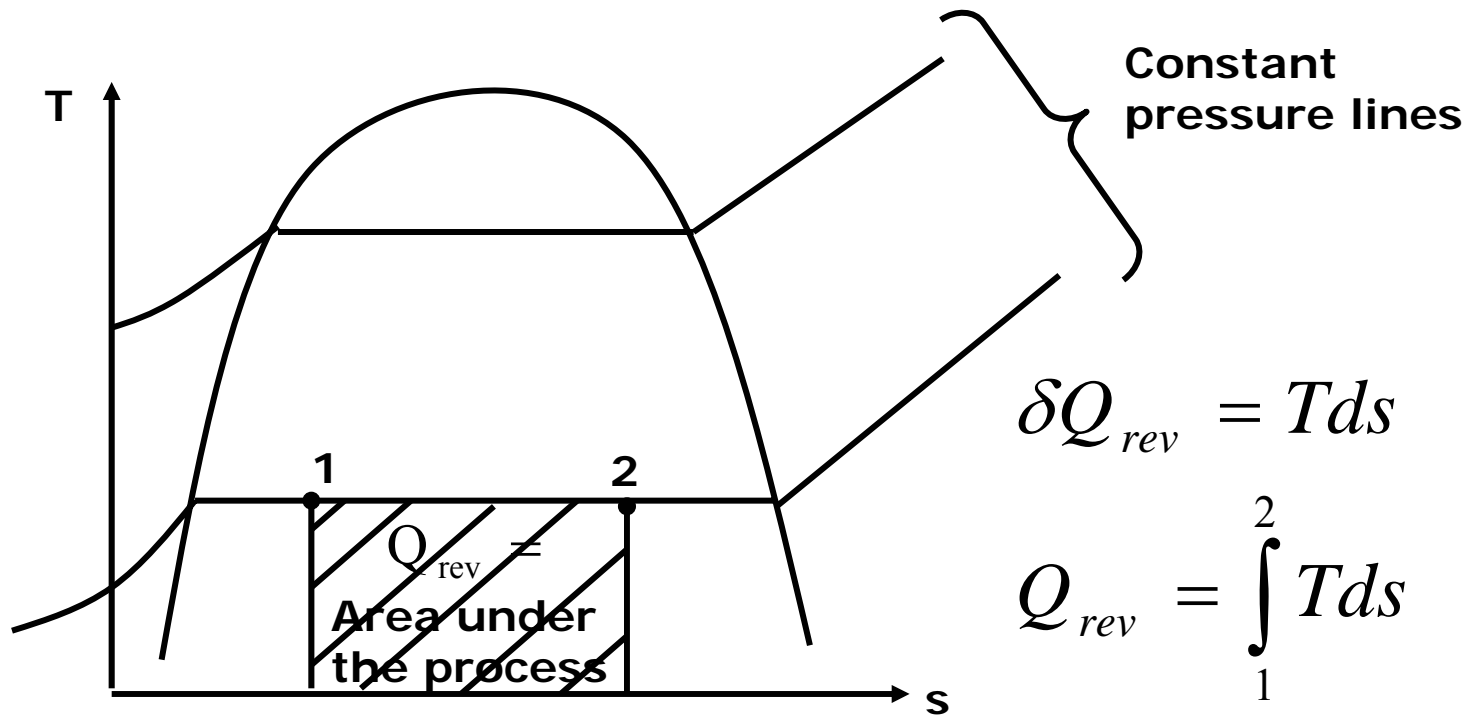
- The entropy of a pure substance is determined from the tables, just as for any other property.



$$s = s_f + x s_{fg}$$

$$\Delta S = m(s_2 - s_1)$$

- T-s diagram of properties:



$$\delta Q_{rev} = T ds$$

$$Q_{rev} = \int_1^2 T ds$$

$$Q_{rev} = T (S_2 - S_1)$$

- Entropy of a fixed mass can be changed by:
 - 1) Heat transfer
 - 2) Irreversibilities

-
- The entropy of a fixed mass will not change during a process that is internally reversible and adiabatic.
 - For a reversible, adiabatic process (called Isentropic process)

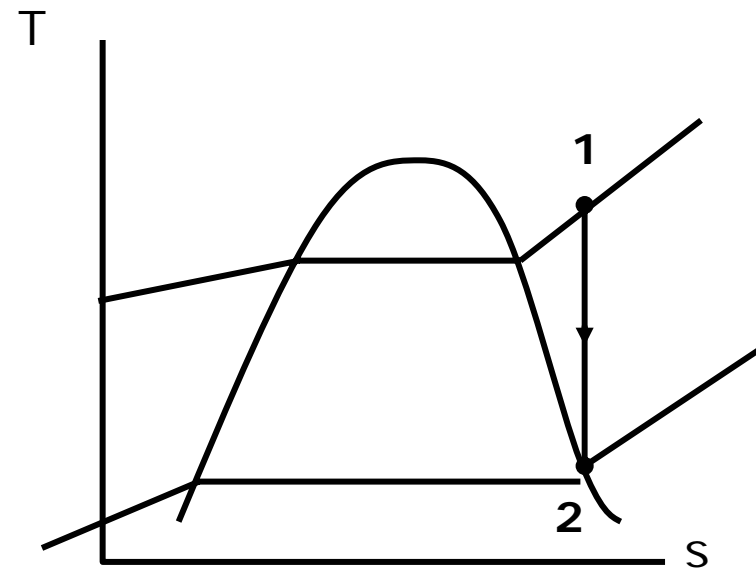
$$\Delta s = 0 \implies s_1 = s_2$$

-
- If a steam turbine is reversible, and the turbine is insulated (thus the process is reversible and adiabatic)

then $\Delta s = 0$

or

$$s_1 = s_2$$



-
- The Tds relations: differential form of the first law:

$$\delta Q_{rev} - \delta W_{rev} = dU$$

second law: $\delta Q_{rev} = Tds$

$$TdS = PdV + dU$$

$$Tds = Pdv + du \quad \leftarrow \quad \text{First Tds relation}$$

$$Tds = du + Pdv$$

$$h = u + Pv$$

$$dh = du + Pdv + vdP$$

$$Tds = dh - vdP \quad \leftarrow \quad \text{Second Tds relation}$$

-
- Entropy changes for Liquids and solids:

$$Tds=du+pdv$$

$$ds= \frac{du}{T} + \frac{pdv}{T}$$

- Liquids and solids are incompressible substances

$$dv \cong 0$$

$$ds = \frac{du}{T} = \frac{cdT}{T} \quad (c_p = c_v = c)$$

$$s_2 - s_1 = \int_1^2 c \frac{dT}{T} = c \ln \frac{T_2}{T_1} \quad \frac{kJ}{kgK}$$

Isentropic process for liquids and solids:

$$s_2 - s_1 = 0 = c \ln \frac{T_2}{T_1} \Rightarrow T_2 = T_1$$

the isentropic process of an incompressible substance is also isothermal.

Note: *Tds equations are derived by considering an internally reversible process. An entropy change obtained by integrating these equations is the change for any process.*

“Entropy is a property and the change in entropy between any two states is independent of the details of the process linking the states.”

Entropy change of an Ideal gas

Tds equations are used to evaluate the entropy change between two states of an ideal gas.

$$ds = \frac{du}{T} + \frac{P}{T} dv$$

$$ds = \frac{dh}{T} - \frac{v}{T} dv$$

For an ideal gas:

$$du = c_v dT$$

$$dh = c_p dT$$

$$Pv = RT, \quad c_p = c_v + R$$

$$ds = c_v \frac{dT}{T} + R \frac{dv}{v}$$

$$ds = c_p \frac{dT}{T} - R \frac{dP}{P}$$

Now integrating

$$s_2 - s_1 = \int_{T_1}^{T_2} c_v \frac{dT}{T} + R \ln \frac{v_2}{v_1}$$

$$s_2 - s_1 = \int_{T_1}^{T_2} c_P \frac{dT}{T} - R \ln \frac{P_2}{P_1}$$

To integrate these relations, we must know the temperature dependence of the specific heats.

Using Ideal gas Tables:

Define $s^o(T) = \int_0^T \frac{c_p(T)}{T} dT$

Where $s^o(T)$ is the specific entropy at a temperature T and a pressure of 1 atm.

Note: *The specific entropy is set to zero at the state where the temperature is 0 K and the pressure is 1 atm.*

Note: Because S^o depends only on the temperature, it can be tabulated versus temperature, like h and u .

Also,

$$\int_{T_1}^{T_2} c_p \frac{dT}{T} = \int_0^{T_2} c_p \frac{dT}{T} - \int_0^{T_1} c_p \frac{dT}{T} = s_2^o - s_1^o$$

then ;

$$s_2 - s_1 = (s_2^o - s_1^o) - R \ln \frac{P_2}{P_1} \quad \frac{\text{KJ}}{\text{Kg.K}}$$

$$\text{or } \bar{s}_2 - \bar{s}_1 = (\bar{s}_2^o - \bar{s}_1^o) - R_u \ln \frac{P_2}{P_1} \quad \frac{\text{KJ}}{\text{K mole.K}}$$

Entropy change of an
Incompressible substance:
for incompressible
substance, specific heat
depends solely on
temperature, and

$$du = C(T)dT$$

$$ds = \frac{du}{T} + \frac{P}{T} dv$$

$$ds = \frac{C(T)dT}{T} + \frac{P}{T} \cancel{dv} = C(T) \frac{dT}{T}$$

$$S_2 - S_1 = \int_{T_1}^{T_2} C(T) \frac{dT}{T} \quad \leftarrow \text{incompressible}$$

When the specific
heat is constant:

$$s_2 - s_1 = C \ln \frac{T_2}{T_1} \quad \leftarrow \text{incompressible, constant C}$$

Isentropic Processes of an Ideal Gas

For internally reversible process:

$$s_2 - s_1 = \int_1^2 c_p(T) \frac{dT}{T} - R \ln \frac{P_2}{P_1}$$

$$s_2 - s_1 = \int_1^2 c_v(T) \frac{dT}{T} - R \ln \frac{V_2}{V_1}$$

For Isentropic process(Reversible, adiabatic process):

$$s_2 - s_1 = 0 \quad 0 = \int_1^2 c_p(T) \frac{dT}{T} - R \ln \frac{P_2}{P_1}$$

$$0 = \int_1^2 c_v(T) \frac{dT}{T} - R \ln \frac{V_2}{V_1}$$

$$0 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

$$0 = c_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}$$

$$\Rightarrow \ln \frac{T_2}{T_1} = -\frac{R}{c_v} \ln \frac{V_2}{V_1} = \ln \left(\frac{V_1}{V_2} \right)^{\frac{R}{c_v}}$$

$$\ln \frac{T_2}{T_1} = \ln \left(\frac{V_1}{V_2} \right)^{k-1}$$

$$\left(\frac{T_2}{T_1} \right)_{s=\text{constant}} = \left(\frac{V_1}{V_2} \right)^{k-1}$$

similarly,

$$\left(\frac{T_2}{T_1} \right)_{s=\text{constant}} = \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}}$$

$$Tds = du + Pdv$$

Ideal gas, Isentropic Process

($PV=RT$, $ds=0$)

$$ds = \frac{du}{T} + P \frac{dv}{T}$$

$$0 = c_v \frac{dT}{T} + R \frac{dv}{v}$$

Integrate:

$$\text{Constant} = c_v \ln T + R \ln v$$

$$\text{use } PV = RT \Rightarrow \ln \frac{Pv}{R} + \frac{R}{c_v} \ln v = \text{Constant}$$

$$\ln Pv + \ln v^{k-1} = \text{Constant} - \ln R = \text{Constant}$$

$$\ln(Pv \times v^{k-1}) = \text{Constant}$$

$$\Rightarrow Pv^k = \text{Constant} \leftarrow \text{Isentropic process ideal gas, with constant specific heats}$$

$$s_2 - s_1 = \int_{T_1}^{T_2} c_p \frac{dT}{T} - R \ln \frac{P_2}{P_1}$$

$$s_2 - s_1 = s_2^o - s_1^o - R \ln \frac{P_2}{P_1}$$

Isentropic process: $s_2 - s_1 = 0$

$$s_2^o - s_1^o = R \ln \frac{P_2}{P_1}$$

$$\ln \frac{P_2}{P_1} = \left(\frac{s_2^o - s_1^o}{R} \right)$$

$$\frac{P_2}{P_1} = \exp \frac{s_2^o - s_1^o}{R}$$

Relative pressure and relative volume:

$$\frac{P_2}{P_1} = \frac{\exp(s_2^o / R)}{\exp(s_1^o / R)}$$

The quantity $\exp(s^o / R)$ is a function of temperature only and is given the symbol P_r , define as the relative pressure, and is tabulated for air and other ideal gases

$$\frac{P_2}{P_1} = \frac{P_{r_2}}{P_{r_1}}$$

**I sentropic Process
($s_1=s_2$) Ideal gas
variable specific heats**

where $P_{r_1} = P_r(T_1)$ and $P_{r_2} = P_r(T_2)$

Note: P_r is not truly pressure, and also P_r should not be confused with the reduced pressure of compressibility chart.

Similarly a relation between specific volumes and temperatures and for two states having the same entropy can also be developed

$$\frac{v_2}{v_1} = \frac{\left(\frac{RT_2}{P_2}\right)}{\left(\frac{P_1}{RT_1}\right)} \quad \text{or} \quad \frac{v_2}{v_1} = \frac{\left(\frac{RT_2}{P_{r_2}}\right)}{\left(\frac{P_{r_1}}{RT_1}\right)}$$

The ratio $\frac{RT}{P_r} = v_r$ is the relative volume

$$\frac{v_2}{v_1} = \frac{v_{r_2}}{v_{r_1}}$$



**Isentropic process,
Ideal gas variable
specific heats**

In general Isentropic process for ideal gas: ($s_2=s_1$)

$$Pv^k = C \quad \leftarrow \quad \text{constant specific heat}$$

$$\left. \begin{array}{l} \frac{P_{r_2}}{P_{r_1}} = \frac{P_2}{P_1} \\ \frac{v_{r_2}}{v_{r_1}} = \frac{v_2}{v_1} \end{array} \right\} \leftarrow \quad \text{specific heat is a function of temperature}$$

General, reversible (Polytropic) process for an Ideal gas:

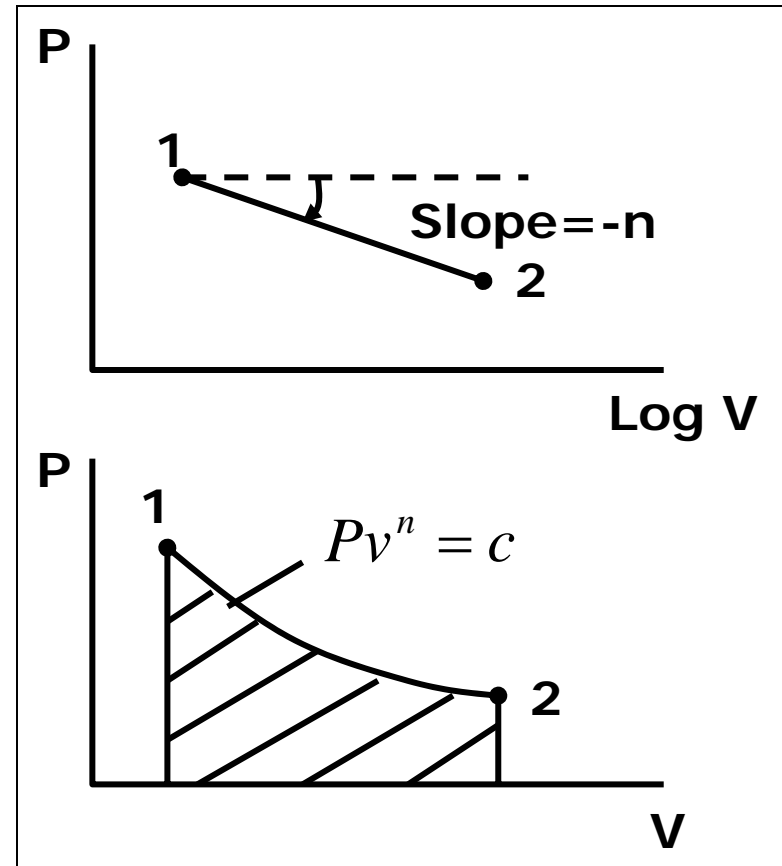
$Pv^n=c$, where n is constant

$$P_1 v_1^n = P_2 v_2^n$$

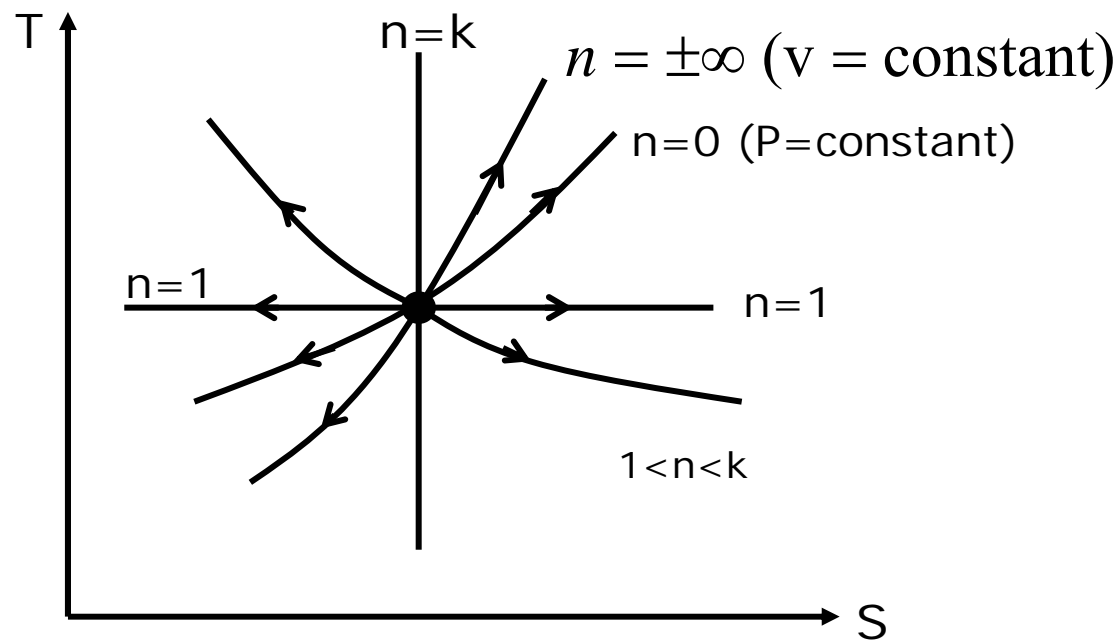
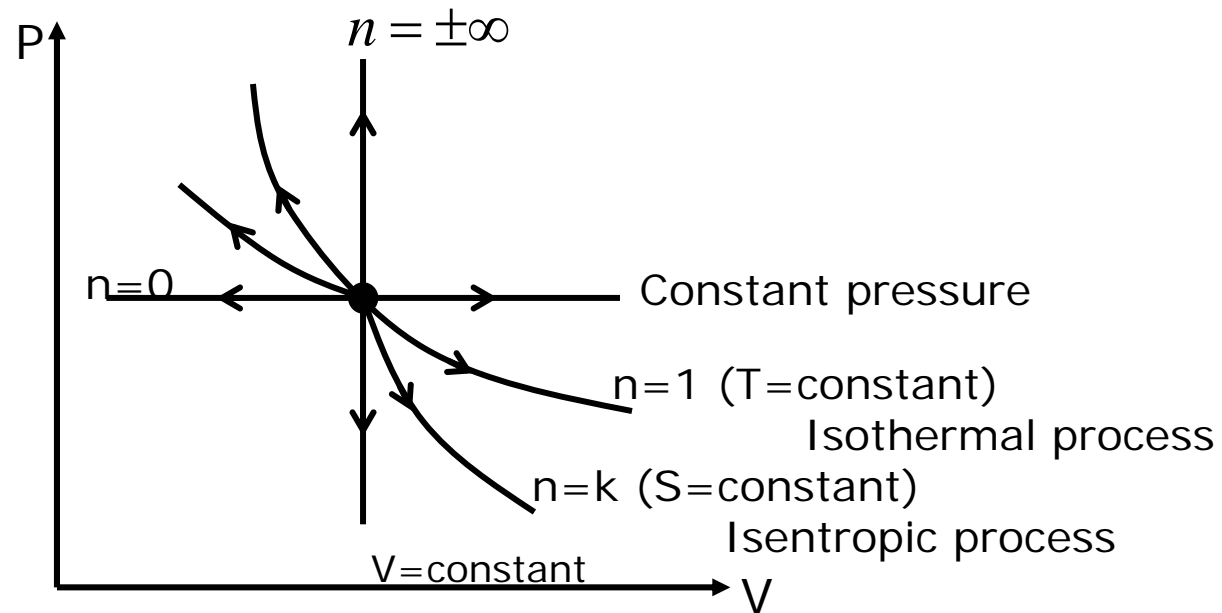
$$\frac{P_2}{P_1} = \left(\frac{v_1}{v_2} \right)^n$$

$$W = \int_1^2 P dv$$

$$= \int_1^2 \frac{c}{v^n} dv = \frac{P_1 v_1 - P_2 v_2}{n - 1}$$



Polytropic process on P-v and T-s diagrams



Isentropic Efficiencies of Steady-Flow Devices

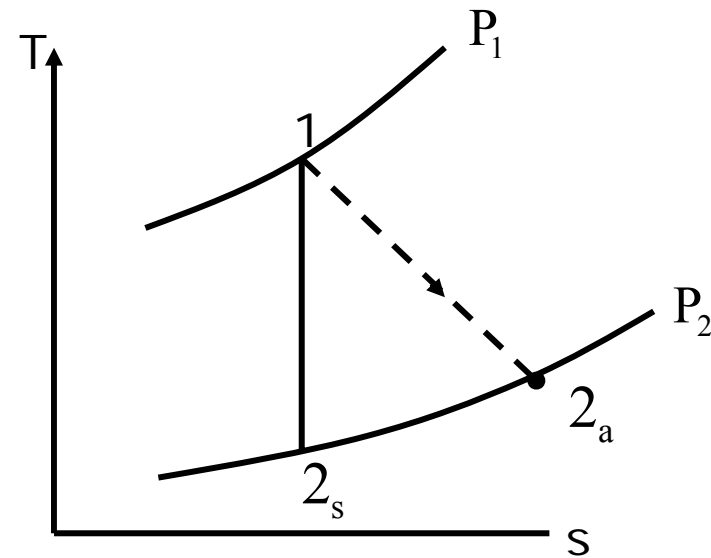
- The Isentropic process involves no irreversibilities and serves as the ideal process for adiabatic devices.
- The actual process is irreversible and the actual device performance is less than the ideal case.
- The more closely the actual process approximates the idealized isentropic process, the better the device will perform.
- We define the efficiency (isentropic efficiency) of these devices as a measure of deviation of actual processes from the idealized one.

1) Isentropic efficiency of Turbines

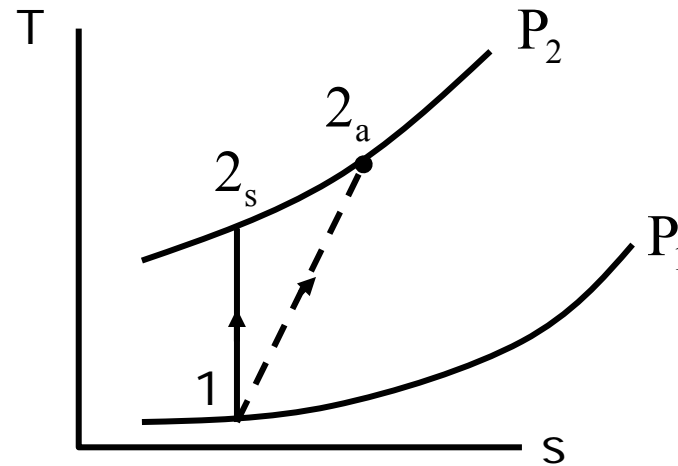
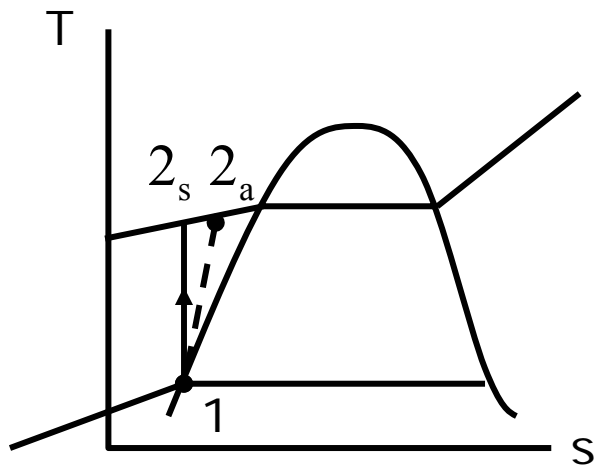
$$\dot{w} = h_1 - h_2$$

$$\eta_t = \frac{\text{Actual turbine work}}{\text{Isentropic work}}$$

$$\eta_t = \frac{w_a}{w_s} = \frac{h_1 - h_{2a}}{h_1 - h_{2s}} < 100\%$$



2) Isentropic efficiency of Compressors and Pumps



$$\eta_c \text{ or } \eta_p = \frac{W_s}{W_a}$$

$$\eta_c \text{ or } \eta_p = \frac{h_{2s} - h_1}{h_{2a} - h_1}$$

3) Isentropic efficiency of Nozzles

$$\eta_N = \frac{\text{Actual K. E. at Nozzle exit}}{\text{Isentropic K.E. at Nozzle exit}}$$

$$\eta_N = \frac{V_{2a}^2}{V_{2s}^2}$$

$$\text{Note : } h_1 = h_{2a} + \frac{V_{2a}^2}{2}$$

$$\eta_N \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

