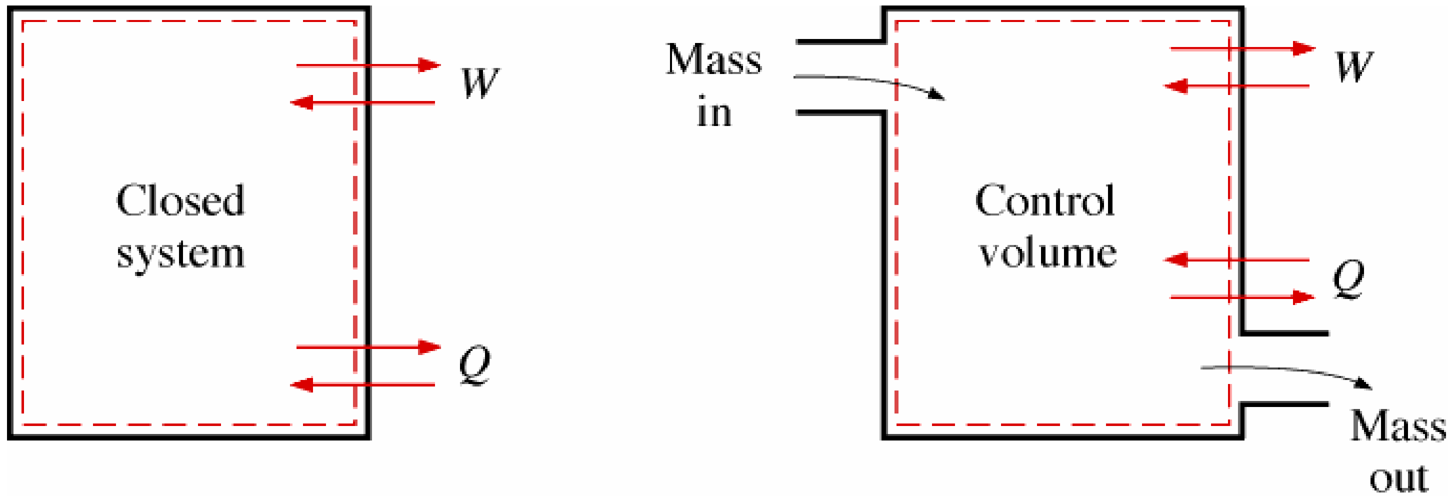
Control Volume Analysis Using Energy

Open vs. Closed Systems



Conservation of Mass

-In a closed system the mass is always constant

-In an open system (control volume) we need to consider mass balance

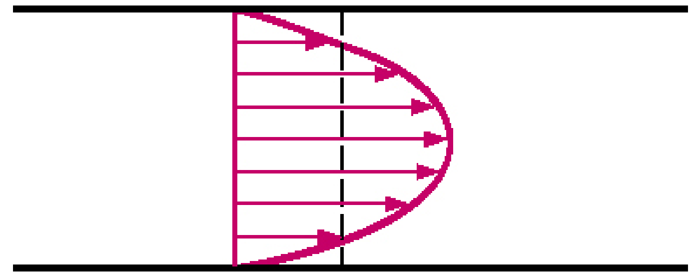
Sum of the rate of mass flowing into the control volume	-	Sum of the rate of mass flowing from the control volume	=	Time rate of change of the mass inside the control volume

$$\sum m_{in} - \sum m_{out} = \Delta m_{cv}$$

$$\sum \dot{m}_{in} - \sum \dot{m}_{out} = \frac{dm_{cv}}{dt}$$

Calculating The Flow Rate

$$\dot{m} = \rho \vec{V} \cdot \vec{A} = \frac{\vec{V} \cdot \vec{A}}{v}$$

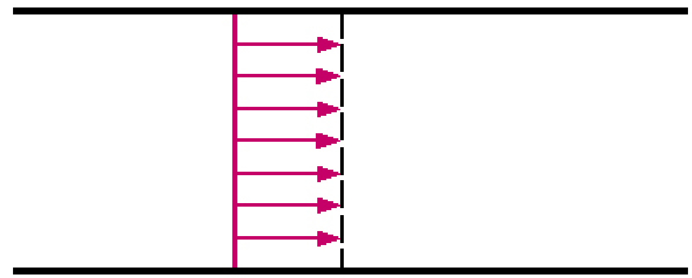


(a) Actual

Volumetric Flow Rate

$$\dot{\mathcal{V}} = \vec{V} \cdot \vec{A}$$

$$\dot{m} = \rho \dot{\mathcal{V}} = \frac{\dot{\mathcal{V}}}{v}$$



(b) Average

General Formula

$$\dot{m} = \int_{\vec{A}} \rho \vec{V} \cdot d\vec{A} = \int_{\mathcal{V}} \rho \, d\mathcal{V}$$

The total amount of mass in the control volume does not change with time

$$\sum m_i = \sum m_e$$

$$\sum \dot{m}_i = \sum \dot{m}_e$$

Energy Balance

First Law of Thermodynamics

$$\sum E_i - \sum E_e = \Delta E_{cv}$$

$$\sum \dot{E}_i - \sum \dot{E}_e = \frac{dE_{cv}}{dt}$$

Total Energy of a flowing fluid

$$e = u + ke + pe = u + \frac{V^2}{2} + gz$$

Energy Balance

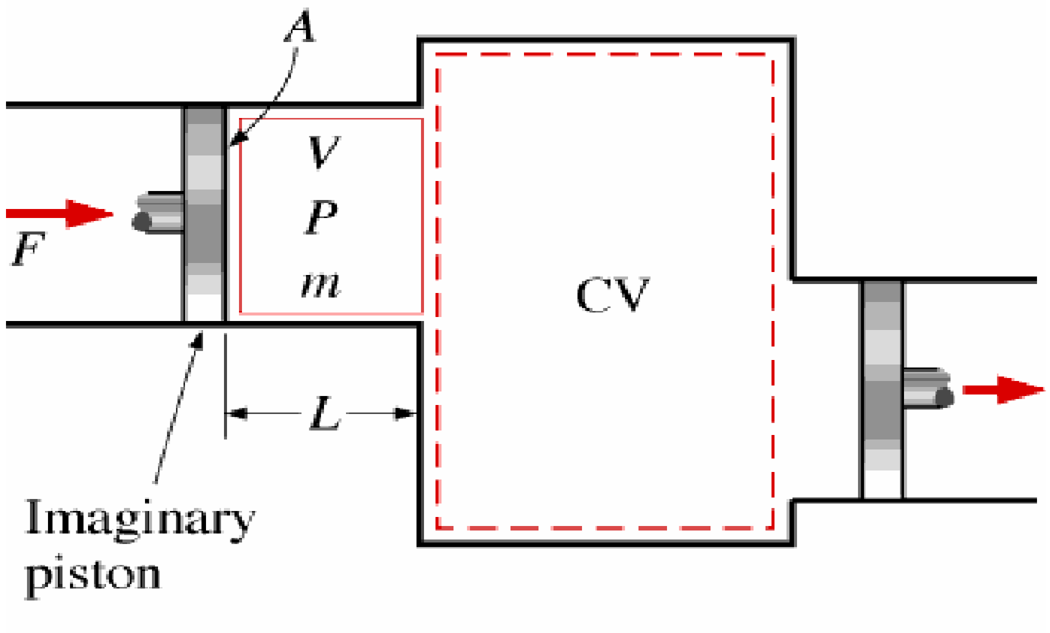
$$\frac{dE_{cv}}{dt} = \sum \dot{Q} - \sum \dot{W} + \sum \dot{m}_i \left(u_i + \frac{V_i^2}{2} + gz_i \right)$$

$$- \sum \dot{m}_e \left(u_e + \frac{V_e^2}{2} + gz_e \right)$$

Flow Work

It takes work to push a fluid into a system

It takes work to push a fluid out of a system



$$\dot{W}_{flow} = F.\frac{\Delta x}{t} = (pA).\frac{\Delta x}{t} = pAV$$

$$\dot{W} = \dot{W}_{cv} + \underbrace{(p_e A_e)V_e}_{\dot{m}_e(p_e v_e)} - (p_i A_i)V_i$$

Energy Balance

$$\frac{dE_{cv}}{dt} = \sum \dot{Q}_{cv} - \sum \dot{W}_{cv} + \sum \dot{m}_i \left(u_i + p_i v_i + \frac{V_i^2}{2} + gz_i \right)$$

$$- \sum \dot{m}_e \left(u_e + p_e v_e + \frac{V_e^2}{2} + gz_e \right)$$

or

$$\frac{dE_{cv}}{dt} = \sum \dot{Q}_{cv} - \sum \dot{W}_{cv} + \sum \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i \right)$$

$$- \sum \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e \right)$$

$\dot{W}_{cv}$ represents everything but the flow work

Steady Flow Process

Mass Balance(Continuity): $\dot{m}_{cv} = 0$

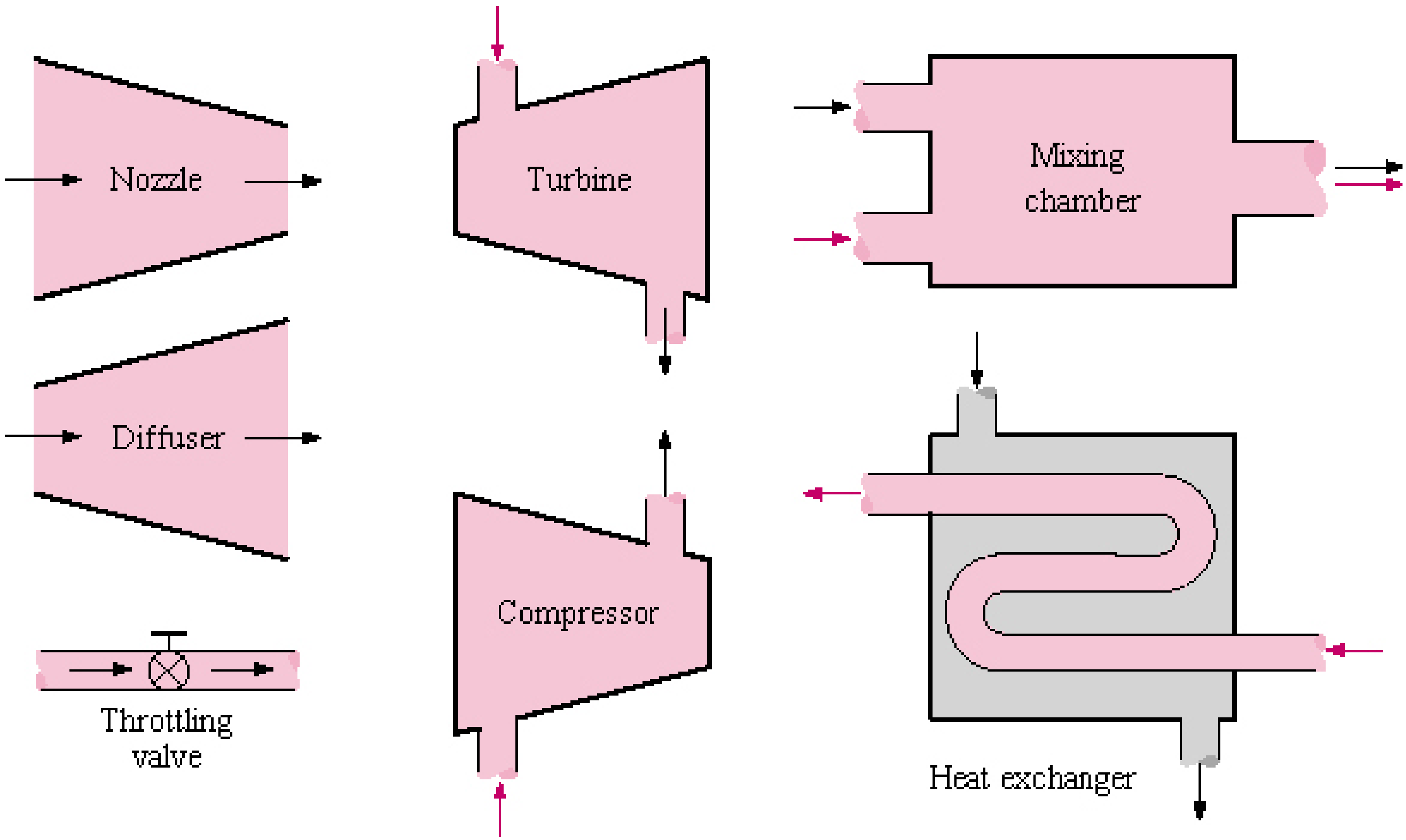
Energy Balance: $\frac{dE_{cv}}{dt} = 0$

or

$$\sum \dot{m}_i = \sum \dot{m}_e$$

$$\dot{Q}_{net} - \dot{W}_{net} - \sum \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e \right) + \sum \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i \right) = 0$$

Some Common Steady Flow Devices



Single Stream Steady Flow System

Nozzles

Diffusers

Turbines

Compressors

Throttling Valve

$$\dot{m}_i = \dot{m}_e = \dot{m}$$

$$\dot{Q} - \dot{W} = \dot{m}\left[h_e - h_i + \frac{V_e^2 - V_i^2}{2} + g\left(z_e - z_i\right) \right]$$