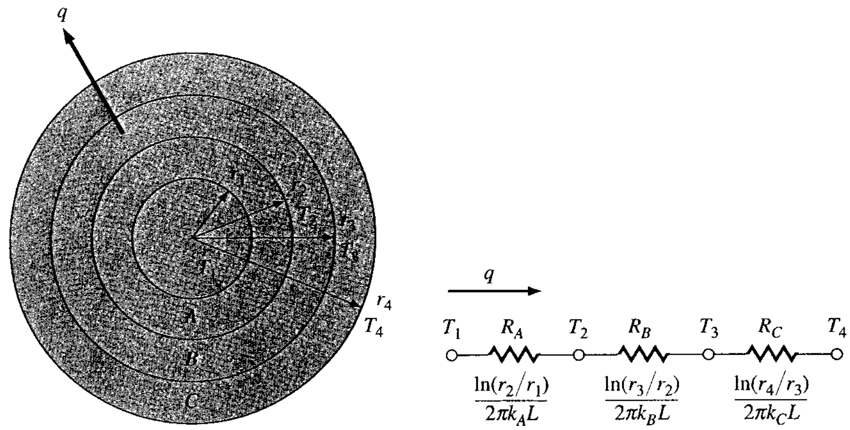
Figure 2-4 | One-dimensional heat flow through multiple cylindrical sections and electrical analog.



(*a*) (*b*)

with the boundary conditions

$$T = T_i \qquad \text{at } r = r_i$$
$$T = T_o \qquad \text{at } r = r_o$$

The solution to Equation (2-7) is

$$q = \frac{2\pi k L\,(T_i - T_o)}{\ln(r_o/r_i)} \qquad \text{[2-8]}$$

and the thermal resistance in this case is

$$R_{\text{th}} = \frac{\ln(r_o/r_i)}{2\pi k L}$$

The thermal-resistance concept may be used for multiple-layer cylindrical walls just as it was used for plane walls. For the three-layer system shown in Figure 2-4 the solution is

$$q = \frac{2\pi L\,(T_1 - T_4)}{\ln(r_2/r_1)/k_A + \ln(r_3/r_2)/k_B + \ln(r_4/r_3)/k_C} \qquad \text{[2-9]}$$

The thermal circuit is shown in Figure 2-4*b*.

Spheres

Spherical systems may also be treated as one-dimensional when the temperature is a function of radius only. The heat flow is then

$$q = \frac{4\pi k\,(T_i - T_o)}{1/r_i - 1/r_o} \qquad \text{[2-10]}$$

The derivation of Equation (2-10) is left as an exercise.

Multilayer Conduction — EXAMPLE 2-1

An exterior wall of a house may be approximated by a 4-in layer of common brick [$k = 0.7$ W/m·°C] followed by a 1.5-in layer of gypsum plaster [$k = 0.48$ W/m·°C]. What thickness of loosely packed rock-wool insulation [$k = 0.065$ W/m·°C] should be added to reduce the heat loss (or gain) through the wall by 80 percent?

T_1 — R_b — R_p — R_{RW} — T_2 or T_2

■ Solution

The overall heat loss will be given by

$$q = \frac{\Delta T}{\sum R_{\text{th}}}$$

Because the heat loss with the rock-wool insulation will be only 20 percent (80 percent reduction) of that before insulation

$$\frac{q \text{ with insulation}}{q \text{ without insulation}} = 0.2 = \frac{\sum R_{\text{th}} \text{ without insulation}}{\sum R_{\text{th}} \text{ with insulation}}$$

Q

We have for the brick and plaster, for unit area,

$$R_b = \frac{\Delta x}{k} = \frac{(4)(0.0254)}{0.7} = 0.145 \text{ m}^2\cdot{}^\circ\text{C/W}$$

$$R_p = \frac{\Delta x}{k} = \frac{(1.5)(0.0254)}{0.48} = 0.079 \text{ m}^2\cdot{}^\circ\text{C/W}$$

so that the thermal resistance without insulation is

$$R = 0.145 + 0.079 = 0.224 \text{ m}^2\cdot{}^\circ\text{C/W}$$

Then

$$R \text{ with insulation} = \frac{0.224}{0.2} = 1.122 \text{ m}^2\cdot{}^\circ\text{C/W}$$

use

and this represents the sum of our previous value and the resistance for the rock wool

$$1.122 = 0.224 + R_{rw}$$

$$R_{rw} = 0.898 = \frac{\Delta x}{k} = \frac{\Delta x}{0.065}$$

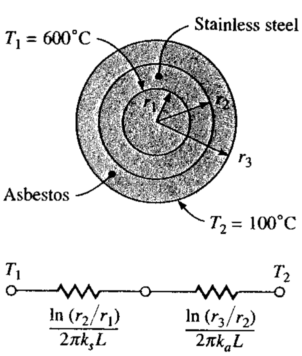
so that

$$\Delta x_{rw} = 0.0584 \text{ m} = 2.30 \text{ in}$$

EXAMPLE 2-2 Multilayer Cylindrical System

A thick-walled tube of stainless steel [18% Cr, 8% Ni, $k = 19$ W/m·°C] with 2-cm inner diameter (ID) and 4-cm outer diameter (OD) is covered with a 3-cm layer of asbestos insulation [$k = 0.2$ W/m·°C]. If the inside wall temperature of the pipe is maintained at 600°C, calculate the heat loss per meter of length. Also calculate the tube–insulation interface temperature.

Figure Example 2-2



■ Solution

Figure Example 2-2 shows the thermal network for this problem. The heat flow is given by

$$\frac{q}{L} = \frac{2\pi(T_1 - T_2)}{\ln(r_2/r_1)/k_s + \ln(r_3/r_2)/k_a} = \frac{2\pi(600 - 100)}{(\ln 2)/19 + (\ln \frac{5}{2})/0.2} = 680 \text{ W/m}$$

i

This heat flow may be used to calculate the interface temperature between the outside tube wall and the insulation. We have

$$\frac{q}{L} = \frac{T_a - T_2}{\ln(r_3/r_2)/2\pi k_a} = 680 \text{ W/m}$$

i

$\ln(\frac{r_2}{r_1})/2\pi kL$ $\quad \ln(\frac{r_3}{r_2})/2\pi kL$

T_1 — R_1 — T_a — R_2 — T_2

where T_a is the interface temperature, which may be obtained as

$$T_a = 595.8°\text{C} \quad = T_i$$

The largest thermal resistance clearly results from the insulation, and thus the major portion of the temperature drop is through that material.

Convection Boundary Conditions

We have already seen in Chapter 1 that convection heat transfer can be calculated from

$$q_{\text{conv}} = hA\,(T_w - T_\infty)$$

An electric-resistance analogy can also be drawn for the convection process by rewriting the equation as

$$q_{\text{conv}} = \frac{T_w - T_\infty}{1/hA} \qquad \textbf{[2-11]}$$

where now the $1/hA$ term becomes the convection resistance.

2-5 | THE OVERALL HEAT-TRANSFER COEFFICIENT

Consider the plane wall shown in Figure 2-5 exposed to a hot fluid A on one side and a cooler fluid B on the other side. The heat transfer is expressed by

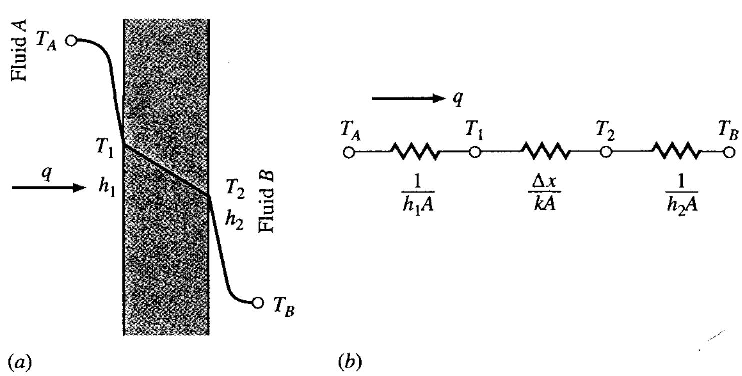
$$q = h_1 A\,(T_A - T_1) = \frac{kA}{\Delta x}\,(T_1 - T_2) = h_2 A\,(T_2 - T_B)$$

The heat-transfer process may be represented by the resistance network in Figure 2-5b, and the overall heat transfer is calculated as the ratio of the overall temperature difference to the sum of the thermal resistances:

$$q = \frac{T_A - T_B}{1/h_1 A + \Delta x/kA + 1/h_2 A} \qquad \textbf{[2-12]}$$

Observe that the value $1/hA$ is used to represent the convection resistance. The overall heat transfer by combined conduction and convection is frequently expressed in terms of

Figure 2-5 | Overall heat transfer through a plane wall.



Overall Heat-Transfer Coefficient for a Tube **EXAMPLE 2-4**

Water flows at 50°C inside a 2.5-cm-inside-diameter tube such that $h_i = 3500$ W/m²·°C. The tube has a wall thickness of 0.8 mm with a thermal conductivity of 16 W/m²·°C. The outside of the tube loses heat by free convection with $h_o = 7.6$ W/m²·°C. Calculate the overall heat-transfer coefficient and heat loss per unit length to surrounding air at 20°C.

■ **Solution**

There are three resistances in series for this problem, as illustrated in Equation (2-14). With $L = 1.0$ m, $d_i = 0.025$ m, and $d_o = 0.025 + (2)(0.0008) = 0.0266$ m, the resistances may be calculated as

$$R_i = \frac{1}{h_i A_i} = \frac{1}{(3500)\underbrace{\pi(0.025)(1.0)}_{\pi d_i L}} = 0.00364\text{°C/W}$$

$$R_t = \frac{\ln(d_o/d_i)}{2\pi k L}$$

$$= \frac{\ln(0.0266/0.025)}{2\pi(16)(1.0)} = 0.00062\text{°C/W}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(7.6)\underbrace{\pi(0.0266)(1.0)}_{\pi d_o L}} = 1.575\text{°C/W}$$

Clearly, the outside convection resistance is the largest, and *overwhelmingly so.* This means that it is the controlling resistance for the total heat transfer because the other resistances (in series) are negligible in comparison. We shall base the overall heat-transfer coefficient on the outside tube area and write

$$q = \frac{\Delta T}{\sum R} = U A_o \Delta T \qquad [a]$$

A_o is important #

$$U_o = \frac{1}{A_o \sum R} = \frac{1}{[\pi(0.0266)(1.0)](0.00364 + 0.00062 + 1.575)}$$

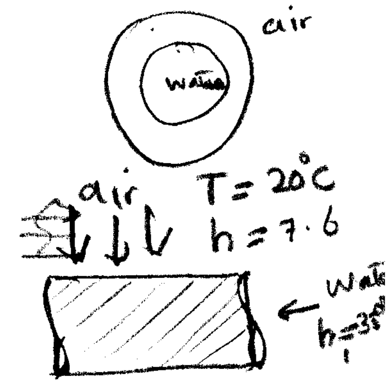
$$= 7.577 \text{ W/m}^2\cdot\text{°C}$$

or a value very close to the value of $h_o = 7.6$ for the outside convection coefficient. The heat transfer is obtained from Equation (*a*), with

$$q = U A_o \Delta T = (7.577)\pi(0.0266)(1.0)(50 - 20) = 19 \text{ W (for 1.0 m length)}$$

■ **Comment**

This example illustrates the important point that many practical heat-transfer problems involve multiple modes of heat transfer acting in combination; in this case, as a series of thermal resistances. It is not unusual for one mode of heat transfer to dominate the overall problem. In this example, the total heat transfer could have been computed very nearly by just calculating the free convection heat loss from the outside of the tube maintained at a temperature of 50°C. Because the inside convection and tube wall resistances are so small, there are correspondingly small temperature drops, and the outside temperature of the tube will be very nearly that of the liquid inside, or 50°C.



R_i — R_c — R_o

$\frac{1}{h_i A_i}$ $\frac{\ln(r_2/r_1)}{2\pi L K}$ $\frac{1}{h_o A_o}$

L = 1 m

$d_i = 0.025$ m

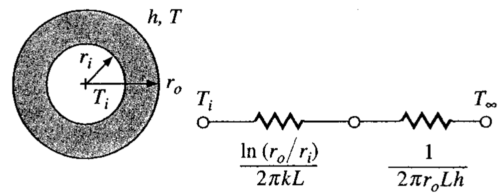
$d_o = 0.025 + 2(0.0008)$

$= 0.0266$ m

2-6 | CRITICAL THICKNESS OF INSULATION

Let us consider a layer of insulation which might be installed around a circular pipe, as shown in Figure 2-7. The inner temperature of the insulation is fixed at T_i, and the outer

Figure 2-7 | Critical insulation thickness.



surface is exposed to a convection environment at T_∞. From the thermal network the heat transfer is

$$q = \frac{2\pi L(T_i - T_\infty)}{\frac{\ln(r_o/r_i)}{k} + \frac{1}{r_o h}} \qquad [2\text{-}17]$$

Now let us manipulate this expression to determine the outer radius of insulation r_o which will maximize the heat transfer. The maximization condition is

$$\frac{dq}{dr_o} = 0 = \frac{-2\pi L(T_i - T_\infty)\left(\frac{1}{kr_o} - \frac{1}{hr_o^2}\right)}{\left[\frac{\ln(r_o/r_i)}{k} + \frac{1}{r_o h}\right]^2}$$

which gives the result

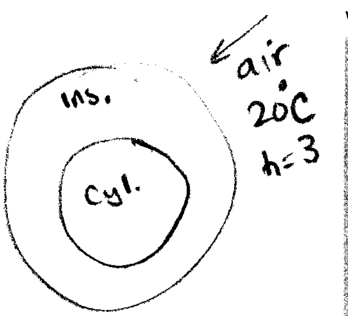
$$r_o = \frac{k}{h} \qquad [2\text{-}18]$$

Equation (2-18) expresses the critical-radius-of-insulation concept. If the outer radius is less than the value given by this equation, then the heat transfer will be *increased* by adding more insulation. For outer radii greater than the critical value an increase in insulation thickness will cause a decrease in heat transfer. The central concept is that for sufficiently small values of h the convection heat loss may actually increase with the addition of insulation because of increased surface area.

EXAMPLE 2-5

Critical Insulation Thickness

Calculate the critical radius of insulation for a material of ~~asbestos~~ [$k = 0.17$ W/m·°C] surrounding a pipe and exposed to room air at 20°C with $h = 3.0$ W/m^2·°C. Calculate the heat loss from a 200°C, 5.0-cm-diameter pipe when covered with the critical radius of insulation and without insulation.



R = Rconv + Rcond.

■ Solution

From Equation (2-18) we calculate r_o as K

$$r_o = \frac{k}{h} = \frac{0.17}{3.0} = 0.0567 \text{ m} = 5.67 \text{ cm}$$ ✓

The inside radius of the insulation is 5.0/2 = 2.5 cm, so the heat transfer is calculated from Equation (2-17) as

$$\frac{q}{L} = \frac{2\pi(200 - 20)}{\frac{\ln(5.67/2.5)}{0.17} + \frac{1}{(0.0567)(3.0)}} = 105.7 \text{ W/m}$$

Without insulation the convection from the outer surface of the pipe is

$$\frac{q}{L} = h(2\pi r)(T_i - T_o) = (3.0)(2\pi)(0.025)(200 - 20) = 84.8 \text{ W/m}$$

So, the addition of 3.17 cm (5.67 − 2.5) of insulation actually *increases* the heat transfer by 25 percent.

As an alternative, fiberglass having a thermal conductivity of 0.04 W/m · °C might be employed as the insulation material. Then, the critical radius would be

$$r_o = \frac{k}{h} = \frac{0.04}{3.0} = 0.0133 \text{ m} = 1.33 \text{ cm}$$

Now, the value of the critical radius is less than the outside radius of the pipe (2.5 cm), so addition of *any* fiberglass insulation would cause a *decrease* in the heat transfer. In a practical pipe insulation problem, the total heat loss will also be influenced by radiation as well as convection from the outer surface of the insulation.

2-7 | HEAT-SOURCE SYSTEMS

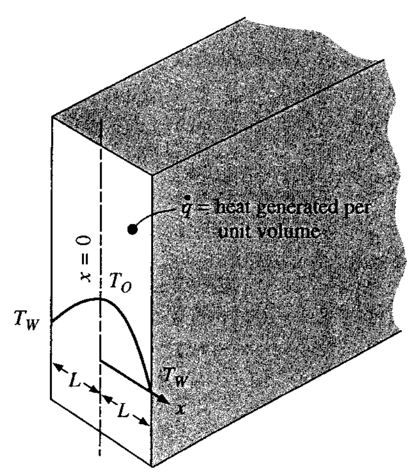
A number of interesting applications of the principles of heat transfer are concerned with systems in which heat may be generated internally. Nuclear reactors are one example; electrical conductors and chemically reacting systems are others. At this point we shall confine our discussion to one-dimensional systems, or, more specifically, systems where the temperature is a function of only one space coordinate.

Plane Wall with Heat Sources

Consider the plane wall with uniformly distributed heat sources shown in Figure 2-8. The thickness of the wall in the x direction is $2L$, and it is assumed that the dimensions in the other directions are sufficiently large that the heat flow may be considered as one-dimensional. The heat generated per unit volume is $\dot{q}$, and we assume that the thermal conductivity does not vary with temperature. This situation might be produced in a practical situation by passing a current through an electrically conducting material. From Chapter 1, the differential equation which governs the heat flow is

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0 \tag{2-19}$$

Figure 2-8 | Sketch illustrating one-dimensional conduction problem with heat generation.



or, in dimensionless form,

$$\frac{T - T_w}{T_0 - T_w} = 1 - \left(\frac{r}{R}\right)^2 \qquad \text{[2-25b]}$$

where T_0 is the temperature at $r = 0$ and is given by

$$T_0 = \frac{\dot{q}R^2}{4k} + T_w \qquad \text{[2-26]}$$

It is left as an exercise to show that the temperature gradient at $r = 0$ is zero.

For a hollow cylinder with uniformly distributed heat sources the appropriate boundary conditions would be

$$T = T_i \qquad \text{at } r = r_i \text{ (inside surface)}$$
$$T = T_o \qquad \text{at } r = r_o \text{ (outside surface)}$$

The general solution is still

$$T = -\frac{\dot{q}r^2}{4k} + C_1 \ln r + C_2$$

Application of the new boundary conditions yields

$$T - T_o = \frac{\dot{q}}{4k}\left(r_o^2 - r^2\right) + C_1 \ln \frac{r}{r_o} \qquad \text{[2-27]}$$

where the constant C_1 is given by

$$C_1 = \frac{T_i - T_o + \dot{q}\left(r_i^2 - r_o^2\right)/4k}{\ln\left(r_i/r_o\right)} \qquad \text{[2-28]}$$

EXAMPLE 2-6

Heat Source with Convection

A current of 200 A is passed through a stainless-steel wire [$k = 19$ W/m·°C] 3 mm in diameter. The resistivity of the steel may be taken as 70 $\mu\Omega$·cm, and the length of the wire is 1 m. The wire is submerged in a liquid at 110 °C and experiences a convection heat-transfer coefficient of 4 kW/m²·°C. Calculate the center temperature of the wire.

■ Solution

All the power generated in the wire must be dissipated by convection to the liquid:

Energy Balance: $$P = I^2R = q = hA(T_w - T_\infty) \qquad [a]$$

The resistance of the wire is calculated from

$$R = \rho\frac{L}{A} = \frac{(70 \times 10^{-6})(100)}{\pi(0.15)^2} = 0.099\ \Omega$$

where ρ is the resistivity of the wire. The surface area of the wire is πdL, so from Equation (a),

$$(200)^2(0.099) = 4000\pi(3 \times 10^{-3})(1)(T_w - 110) = 3960\text{ W}$$

and

$$T_w = 215^\circ\text{C} \quad [419^\circ\text{F}]$$

The heat generated per unit volume $\dot{q}$ is calculated from

$$P = \dot{q}V = \dot{q}\pi r^2 L$$

so that

$$\dot{q} = \frac{3960}{\pi(1.5\times10^{-3})^2(1)} = 560.2\ \text{MW/m}^3 \quad [5.41\times10^7\ \text{Btu/h ft}^3]$$

Finally, the center temperature of the wire is calculated from Equation (2-26):

$$T_0 = \frac{\dot{q} r_o^2}{4k} + T_w = \frac{(5.602\times10^8)(1.5\times10^{-3})^2}{(4)(19)} + 215 = 231.6^\circ\text{C} \quad [449^\circ\text{F}]$$

2-9 | CONDUCTION-CONVECTION SYSTEMS

The heat that is conducted through a body must frequently be removed (or delivered) by some convection process. For example, the heat lost by conduction through a furnace wall must be dissipated to the surroundings through convection. In heat-exchanger applications a finned-tube arrangement might be used to remove heat from a hot liquid. The heat transfer from the liquid to the finned tube is by convection. The heat is conducted through the material and finally dissipated to the surroundings by convection. Obviously, an analysis of combined conduction-convection systems is very important from a practical standpoint.

We shall defer part of our analysis of conduction-convection systems to Chapter 10 on heat exchangers. For the present we wish to examine some simple extended-surface problems. Consider the one-dimensional fin exposed to a surrounding fluid at a temperature T_∞ as shown in Figure 2-9. The temperature of the base of the fin is T_0. We approach the problem by making an energy balance on an element of the fin of thickness dx as shown in the figure. Thus

Energy in left face = energy out right face + energy lost by convection

The defining equation for the convection heat-transfer coefficient is recalled as

$$q = hA\,(T_w - T_\infty) \qquad \textbf{[2-29]}$$

Figure 2-9 | Sketch illustrating one-dimensional conduction and convection through a rectangular fin.