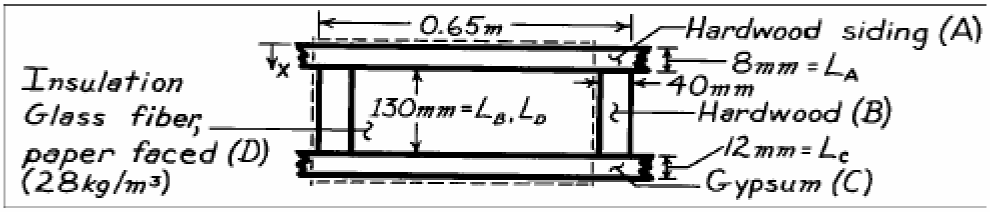
PROBLEM 3.15

KNOWN: Dimensions and materials associated with a composite wall (2.5m × 6.5m, 10 studs each 2.5m high).

FIND: Wall thermal resistance.

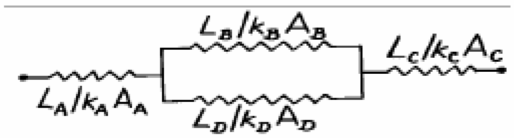
SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Temperature of composite depends only on x (surfaces normal to x are isothermal), (3) Constant properties, (4) Negligible contact resistance.

PROPERTIES: *Table A-3* (T ≈ 300K): Hardwood siding, $k_A = 0.094$ W/m·K; Hardwood, $k_B = 0.16$ W/m·K; Gypsum, $k_C = 0.17$ W/m·K; Insulation (glass fiber paper faced, 28 kg/m^3), $k_D = 0.038$ W/m·K.

ANALYSIS: Using the isothermal surface assumption, the thermal circuit associated with a single unit (enclosed by dashed lines) of the wall is



$$\left(L_A/k_AA_A\right)=\frac{0.008\text{m}}{0.094\text{ W/m}\cdot\text{K }\left(0.65\text{m}\times2.5\text{m}\right)}=0.0524\text{ K/W}$$

$$\left(L_B/k_BA_B\right)=\frac{0.13\text{m}}{0.16\text{ W/m}\cdot\text{K }\left(0.04\text{m}\times2.5\text{m}\right)}=8.125\text{ K/W}$$

$$\left(L_D/k_DA_D\right)=\frac{0.13\text{m}}{0.038\text{ W/m}\cdot\text{K}\left(0.61\text{m}\times2.5\text{m}\right)}=2.243\text{ K/W}$$

$$\left(L_C/k_CA_C\right)=\frac{0.012\text{m}}{0.17\text{ W/m}\cdot\text{K}\left(0.65\text{m}\times2.5\text{m}\right)}=0.0434\text{ K/W.}$$

The equivalent resistance of the core is

$$R_{eq}=\left(1/R_B+1/R_D\right)^{-1}=\left(1/8.125+1/2.243\right)^{-1}=1.758\text{ K/W}$$

and the total unit resistance is

$$R_{tot,1}=R_A+R_{eq}+R_C=1.854\text{ K/W.}$$

With 10 such units in parallel, the total wall resistance is

$$R_{tot}=\left(10\times1/R_{tot,1}\right)^{-1}=0.1854\text{ K/W.}$$ <

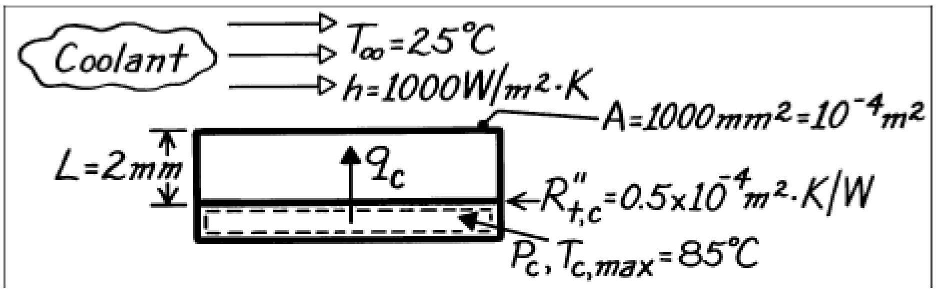
COMMENTS: If surfaces parallel to the heat flow direction are assumed adiabatic, the thermal circuit and the value of R_{tot} will differ.

PROBLEM 3.26

KNOWN: Surface area and maximum temperature of a chip. Thickness of aluminum cover and chip/cover contact resistance. Fluid convection conditions.

FIND: Maximum chip power.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer, (3) Negligible heat loss from sides and bottom, (4) Chip is isothermal.

PROPERTIES: *Table A.1*, Aluminum ($T \approx 325$ K): $k = 238$ W/m·K.

ANALYSIS: For a control surface about the chip, conservation of energy yields

$$\dot{E}_g - \dot{E}_{out} = 0$$

or

$$P_c - \frac{(T_c - T_\infty)A}{\left[(L/k) + R''_{t,c} + (1/h)\right]} = 0$$

$$P_{c,max} = \frac{(85-25)^\circ C\left(10^{-4} m^2\right)}{\left[(0.002/238) + 0.5\times10^{-4} + (1/1000)\right] m^2 \cdot K/W}$$

$$P_{c,max} = \frac{60\times10^{-4}\ {}^\circ C\cdot m^2}{\left(8.4\times10^{-6} + 0.5\times10^{-4} + 10^{-3}\right) m^2\cdot K/W}$$

$$P_{c,max} = 5.7\ W. \qquad <$$

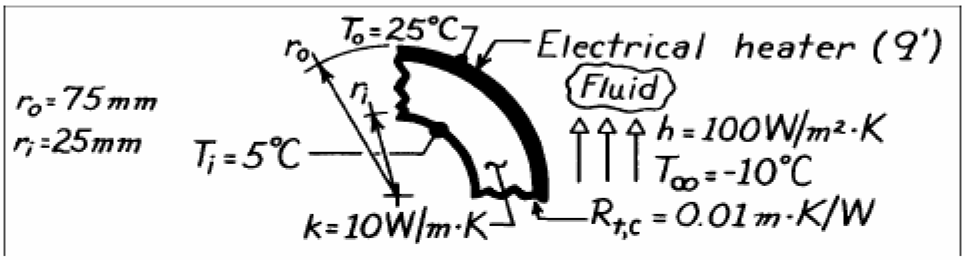
COMMENTS: The dominant resistance is that due to convection $\left(R_{conv} > R_{t,c} >> R_{cond}\right)$.

PROBLEM 3.37

KNOWN: Inner and outer radii of a tube wall which is heated electrically at its outer surface and is exposed to a fluid of prescribed h and T_∞. Thermal contact resistance between heater and tube wall and wall inner surface temperature.

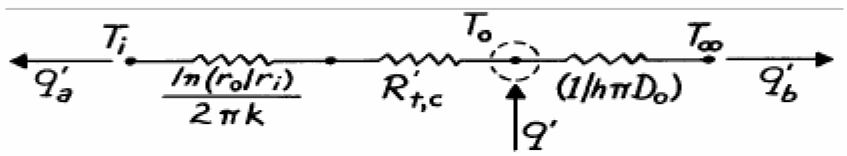
FIND: Heater power per unit length required to maintain a heater temperature of 25°C.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible temperature drop across heater.

ANALYSIS: The thermal circuit has the form



Applying an energy balance to a control surface about the heater,

$$q' = q'_a + q'_b$$

$$q' = \frac{T_o - T_i}{\frac{\ln(r_o/r_i)}{2\pi k} + R'_{t,c}} + \frac{T_o - T_\infty}{(1/h\pi D_o)}$$

$$q' = \frac{(25\text{-}5)^\circ \mathrm{C}}{\frac{\ln(75\mathrm{mm}/25\mathrm{mm})}{2\pi \times 10\ \mathrm{W/m\cdot K}} + 0.01\frac{\mathrm{m\cdot K}}{\mathrm{W}}} + \frac{\left[25-(-10)\right]^\circ \mathrm{C}}{\left[1/\left(100\ \mathrm{W/m^2\cdot K}\times\pi\times 0.15\mathrm{m}\right)\right]}$$

$$q' = (728 + 1649)\ \mathrm{W/m}$$

$$q' = 2377\ \mathrm{W/m}.$$ <

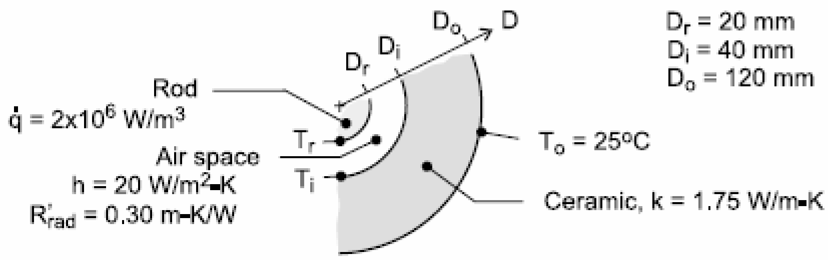
COMMENTS: The conduction, contact and convection resistances are 0.0175, 0.01 and 0.021 m ·K/W, respectively,

PROBLEM 3.44

KNOWN: Long rod experiencing uniform volumetric generation of thermal energy, $\dot{q}$, concentric with a hollow ceramic cylinder creating an enclosure filled with air. Thermal resistance per unit length due to radiation exchange between enclosure surfaces is R'_{rad}. The free convection coefficient for the enclosure surfaces is $h = 20\ W/m^2 \cdot K$.

FIND: (a) Thermal circuit of the system that can be used to calculate the surface temperature of the rod, T_r; label all temperatures, heat rates and thermal resistances; evaluate the thermal resistances; and (b) Calculate the surface temperature of the rod.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional, radial conduction through the hollow cylinder, (3) The enclosure surfaces experience free convection and radiation exchange.

ANALYSIS: (a) The thermal circuit is shown below. Note labels for the temperatures, thermal resistances and the relevant heat fluxes.

Enclosure, radiation exchange (given):

$$R'_{rad} = 0.30\ m \cdot K / W$$

Enclosure, free convection:

$$R'_{cv,rod} = \frac{1}{h \pi D_r} = \frac{1}{20\ W/m^2 \cdot K \times \pi \times 0.020m} = 0.80\ m \cdot K / W$$

$$R'_{cv,cer} = \frac{1}{h \pi D_i} = \frac{1}{20\ W/m^2 \cdot K \times \pi \times 0.040m} = 0.40\ m \cdot K / W$$

Ceramic cylinder, conduction:

$$R'_{cd} = \frac{\ell n(D_o / D_i)}{2\pi k} = \frac{\ell n(0.120/0.040)}{2\pi \times 1.75\ W/m \cdot K} = 0.10\ m \cdot K / W$$

The thermal resistance between the enclosure surfaces (r-i) due to convection and radiation exchange is

$$\frac{1}{R'_{enc}} = \frac{1}{R'_{rad}} + \frac{1}{R'_{cv,rod} + R'_{cv,cer}}$$

$$R'_{enc} = \left[\frac{1}{0.30} + \frac{1}{0.80 + 0.40}\right]^{-1} m \cdot K / W = 0.24\ m \cdot K / W$$

The total resistance between the rod surface (r) and the outer surface of the cylinder (o) is

$$R'_{tot} = R'_{enc} + R'_{cd} = (0.24 + 0.1) m \cdot K / W = 0.34\ m \cdot K / W$$

Continued …..

PROBLEM 3.44 (Cont.)

(b) From an energy balance on the rod (see schematic) find T_r.

$$\dot{E}'_{in} - \dot{E}'_{out} + \dot{E}'_{gen} = 0$$

$$-q + \dot{q}\forall = 0$$

$$-\left(T_r - T_i\right)/R'_{tot} + \dot{q}\left(\pi D_r^2/4\right) = 0$$

$$-\left(T_r - 25\right)\text{K}/0.34\ \text{m}\cdot\text{K/W} + 2\times10^6\ \text{W/m}^3\left(\pi\times0.020\text{m}^2/4\right) = 0$$

$$T_r = 239°\text{C}$$ <

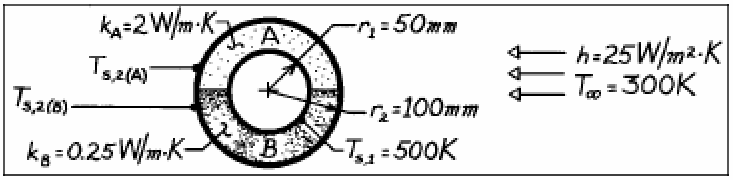
COMMENTS: In evaluating the convection resistance of the air space, it was necessary to define an average air temperature (T_∞) and consider the convection coefficients for each of the space surfaces. As you'll learn later in Chapter 9, correlations are available for directly estimating the convection coefficient (h_{enc}) for the enclosure so that $q_{cv} = h_{enc}(T_r - T_1)$.

PROBLEM 3.52

KNOWN: Inner surface temperature of insulation blanket comprised of two semi-cylindrical shells of different materials. Ambient air conditions.

FIND: (a) Equivalent thermal circuit, (b) Total heat loss and material outer surface temperatures.

SCHEMATIC:



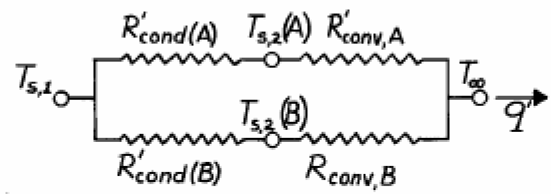
ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional, radial conduction, (3) Infinite contact resistance between materials, (4) Constant properties.

ANALYSIS: (a) The thermal circuit is,

$R'_{conv,A} = R'_{conv,B} = 1/\pi\, r_2 h$

$$R'_{cond(A)} = \frac{\ln(r_2/r_1)}{\pi\, k_A}$$

$$R'_{cond(B)} = \frac{\ln(r_2/r_1)}{\pi\, k_B}$$



<

The conduction resistances follow from Section 3.3.1 and Eq. 3.28. Each resistance is larger by a factor of 2 than the result of Eq. 3.28 due to the reduced area.

(b) Evaluating the thermal resistances and the heat rate $(q' = q'_A + q'_B)$,

$$R'_{conv} = \left(\pi \times 0.1\text{m} \times 25\ \text{W/m}^2\cdot\text{K}\right)^{-1} = 0.1273\ \text{m}\cdot\text{K/W}$$

$$R'_{cond(A)} = \frac{\ln(0.1\text{m}/0.05\text{m})}{\pi \times 2\ \text{W/m}\cdot\text{K}} = 0.1103\ \text{m}\cdot\text{K/W} \quad R'_{cond(B)} = 8\, R'_{cond(A)} = 0.8825\ \text{m}\cdot\text{K/W}$$

$$q' = \frac{T_{s,1} - T_\infty}{R'_{cond(A)} + R'_{conv}} + \frac{T_{s,1} - T_\infty}{R'_{cond(B)} + R'_{conv}}$$

$$q' = \frac{(500-300)\text{K}}{(0.1103+0.1273)\text{m}\cdot\text{K/W}} + \frac{(500-300)\text{K}}{(0.8825+0.1273)\text{m}\cdot\text{K/W}} = (842+198)\ \text{W/m} = 1040\ \text{W/m}.$$ <

Hence, the temperatures are

$$T_{s,2(A)} = T_{s,1} - q'_A R'_{cond(A)} = 500\text{K} - 842\frac{\text{W}}{\text{m}} \times 0.1103\frac{\text{m}\cdot\text{K}}{\text{W}} = 407\text{K}$$ <

$$T_{s,2(B)} = T_{s,1} - q'_B R'_{cond(B)} = 500\text{K} - 198\frac{\text{W}}{\text{m}} \times 0.8825\frac{\text{m}\cdot\text{K}}{\text{W}} = 325\text{K}.$$ <

COMMENTS: The total heat loss can also be computed from $q' = (T_{s,1} - T_\infty)/R_{equiv}$,

where $R_{equiv} = \left[\left(R'_{cond(A)} + R'_{conv,A}\right)^{-1} + \left(R'_{cond(B)} + R'_{conv,B}\right)^{-1}\right]^{-1} = 0.1923\ \text{m}\cdot\text{K/W}.$

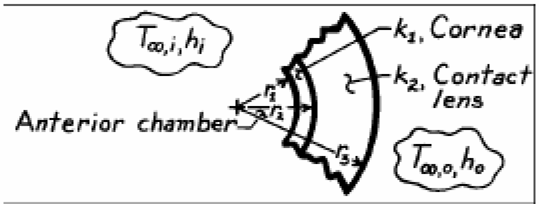
Hence $q' = (500-300)\text{K}/0.1923\ \text{m}\cdot\text{K/W} = 1040\ \text{W/m}.$

PROBLEM 3.64

KNOWN: Representation of the eye with a contact lens as a composite spherical system subjected to convection processes at the boundaries.

FIND: (a) Thermal circuits with and without contact lens in place, (b) Heat loss from anterior chamber for both cases, and (c) Implications of the heat loss calculations.

SCHEMATIC:



r_1=10.2mm k_1=0.35 W/m·K

r_2=12.7mm k_2=0.80 W/m·K

r_3=16.5mm

$T_{\infty,i}$=37°C h_i=12 W/m^2·K

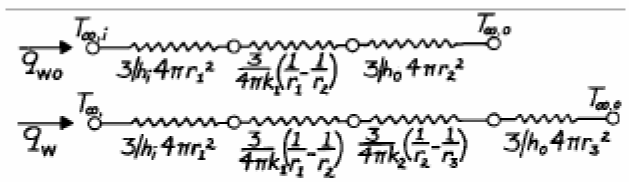
$T_{\infty,o}$=21°C h_o=6 W/m^2·K

ASSUMPTIONS: (1) Steady-state conditions, (2) Eye is represented as 1/3 sphere, (3) Convection coefficient, h_o, unchanged with or without lens present, (4) Negligible contact resistance.

ANALYSIS: (a) Using Eqs. 3.9 and 3.36 to express the resistance terms, the thermal circuits are:

Without lens: <

With lens: <



(b) The heat losses for both cases can be determined as $q = (T_{\infty,i} - T_{\infty,o})/R_t$, where R_t is the thermal resistance from the above circuits.

Without lens:
$$R_{t,wo} = \frac{3}{12\text{W/m}^2\cdot\text{K}4\pi\left(10.2\times10^{-3}\text{m}\right)^2} + \frac{3}{4\pi\times0.35\text{ W/m}\cdot\text{K}}\left[\frac{1}{10.2}-\frac{1}{12.7}\right]\frac{1}{10^{-3}}\text{m}$$

$$+\frac{3}{6\text{ W/m}^2\cdot\text{K}4\pi\left(12.7\times10^{-3}\text{m}\right)^2} = 191.2\text{ K/W}+13.2\text{ K/W}+246.7\text{ K/W}=451.1\text{ K/W}$$

With lens:
$$R_{t,w} = 191.2\text{ K/W}+13.2\text{ K/W}+\frac{3}{4\pi\times0.80\text{ W/m}\cdot\text{K}}\left[\frac{1}{12.7}-\frac{1}{16.5}\right]\frac{1}{10^{-3}}\text{m}$$

$$+\frac{3}{6\text{W/m}^2\cdot\text{K}4\pi\left(16.5\times10^{-3}\text{m}\right)^2} = 191.2\text{ K/W}+13.2\text{ K/W}+5.41\text{ K/W}+146.2\text{ K/W}=356.0\text{ K/W}$$

Hence the heat loss rates from the anterior chamber are

Without lens: $q_{wo} = (37-21)^\circ\text{ C}/451.1\text{ K/W}=35.5\text{mW}$ <

With lens: $q_w = (37-21)^\circ\text{ C}/356.0\text{ K/W}=44.9\text{mW}$ <

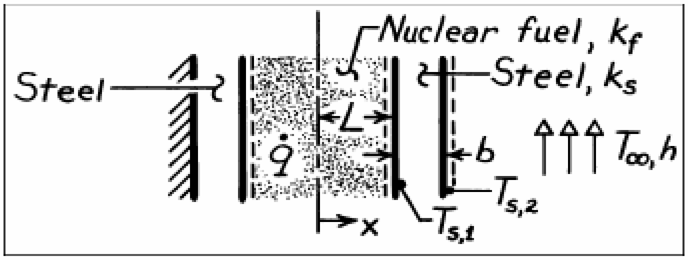
(c) The heat loss from the anterior chamber increases by approximately 20% when the contact lens is in place, implying that the outer radius, r_3, is less than the critical radius.

PROBLEM 3.77

KNOWN: Geometry and boundary conditions of a nuclear fuel element.

FIND: (a) Expression for the temperature distribution in the fuel, (b) Form of temperature distribution for the entire system.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional heat transfer, (2) Steady-state conditions, (3) Uniform generation, (4) Constant properties, (5) Negligible contact resistance between fuel and cladding.

ANALYSIS: (a) The general solution to the heat equation, Eq. 3.39,

$$\frac{d^2T}{dx^2}+\frac{\dot{q}}{k_f}=0 \qquad (-L \le x \le +L)$$

is $$T=-\frac{\dot{q}}{2k_f}x^2+C_1x+C_2.$$

The insulated wall at x = - (L+b) dictates that the heat flux at x = - L is zero (for an energy balance applied to a control volume about the wall, $\dot{E}_{in}=\dot{E}_{out}=0$). Hence

$$\left.\frac{dT}{dx}\right]_{x=-L}=-\frac{\dot{q}}{k_f}(-L)+C_1=0 \qquad \text{or} \qquad C_1=-\frac{\dot{q}L}{k_f}$$

$$T=-\frac{\dot{q}}{2k_f}x^2-\frac{\dot{q}L}{k_f}x+C_2.$$

The value of $T_{s,1}$ may be determined from the energy conservation requirement that $\dot{E}_g=q_{cond}=q_{conv}$, or on a unit area basis.

$$\dot{q}(2L)=\frac{k_s}{b}\left(T_{s,1}-T_{s,2}\right)=h\left(T_{s,2}-T_\infty\right).$$

Hence,

$$T_{s,1}=\frac{\dot{q}(2\,Lb)}{k_s}+T_{s,2} \qquad \text{where} \qquad T_{s,2}=\frac{\dot{q}(2L)}{h}+T_\infty$$

$$T_{s,1}=\frac{\dot{q}(2\,Lb)}{k_s}+\frac{\dot{q}(2L)}{h}+T_\infty.$$

Continued …..

Hence from Eq. (1),

$$T(L) = T_{s,1} = \frac{\dot{q}(2\,Lb)}{k_s} + \frac{\dot{q}(2\,L)}{h} + T_\infty = -\frac{3}{2}\ \frac{\dot{q}\left(L^2\right)}{k_f} + C_2$$

which yields

$$C_2 = T_\infty + \dot{q}L\left[\frac{2b}{k_s} + \frac{2}{h} + \frac{3}{2}\ \frac{L}{k_f}\right]$$

Hence, the temperature distribution for $(-L \le x \le +L)$ is

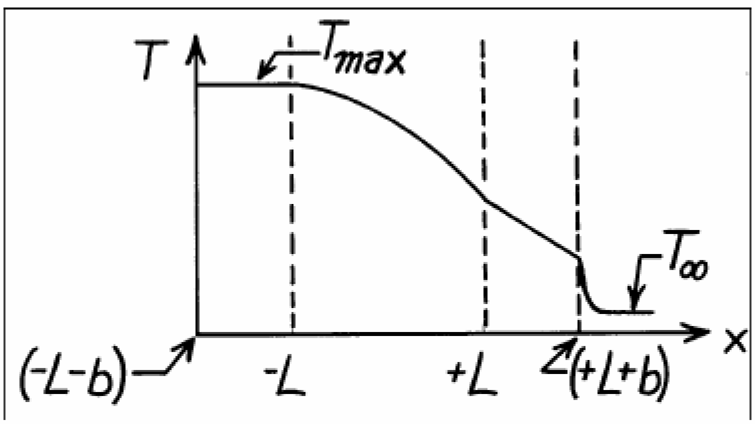
$$T = -\frac{\dot{q}}{2k_f}x^2 - \frac{\dot{q}L}{k_f}x + \dot{q}L\left[\frac{2b}{k_s} + \frac{2}{h} + \frac{3}{2}\ \frac{L}{k_f}\right] + T_\infty$$ <

(b) For the temperature distribution shown below,

$(-L-b) \le x \le -L$: $dT/dx = 0$, $T = T_{max}$

$-L \le x \le +L$: $|dT/dx| \uparrow$ with $\uparrow$ x

$+L \le x \le L+b$: (dT/dx) is const.

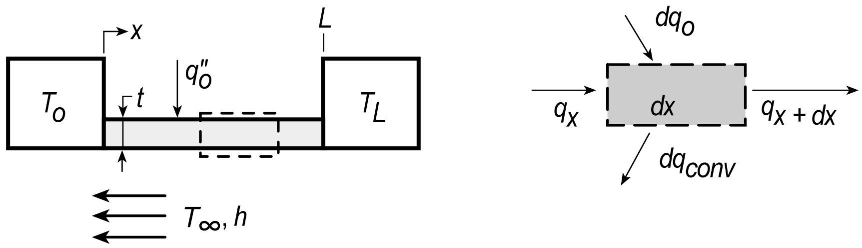


PROBLEM 3.102

KNOWN: Dimensions and surface conditions of a plate thermally joined at its ends to heat sinks at different temperatures.

FIND: (a) Differential equation which determines temperature distribution in plate, (b) Temperature distribution and an expression for the heat rate from the plate to the sinks, and (c) Compute and plot temperature distribution and heat rates corresponding to changes in different parameters.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in x (W,L >> t), (3) Constant properties, (4) Uniform surface heat flux and convection coefficient, (5) Negligible contact resistance.

ANALYSIS: (a) Applying conservation of energy to the differential control volume

$$q_x + dq_o = q_{x+dx} + dq_{conv}$$

where

$$q_{x+dx} = q_x + (dq_x/dx)dx \qquad\qquad dq_{conv} = h(T - T_\infty)(W \cdot dx)$$

Hence,

$$q_x + q''_o(W \cdot dx) = q_x + (dq_x/dx)dx + h(T - T_\infty)(W \cdot dx) \qquad\qquad \frac{dq_x}{dx} + hW(T - T_\infty) = q''_o W .$$

Using Fourier's law, $q_x = -k(t \cdot W)dT/dx$,

$$-ktW\frac{d^2T}{dx^2} + hW(T - T_\infty) = q''_o \qquad\qquad \frac{d^2T}{dx^2} - \frac{h}{kt}(T - T_\infty) + \frac{q''_o}{kt} = 0 .$$ <

(b) Introducing $\theta \equiv T - T_\infty$, the differential equation becomes

$$\frac{d^2\theta}{dx^2} - \frac{h}{kt}\theta + \frac{q''_o}{kt} = 0 .$$

This differential equation is of second order with constant coefficients and a source term. With $\lambda^2 \equiv h/kt$ and $S \equiv q''_o/kt$, it follows that the general solution is of the form

$$\theta = C_1e^{+\lambda x} + C_2e^{-\lambda x} + S/\lambda^2 . \tag{1}$$

Appropriate boundary conditions are: $\theta(0) = T_o - T_\infty \equiv \theta_o \qquad \theta(L) = T_L - T_\infty \equiv \theta_L$ (2,3)

Substituting the boundary conditions, Eqs. (2,3) into the general solution, Eq. (1),

$$\theta_o = C_1e^0 + C_2e^0 + S/\lambda^2 \qquad\qquad \theta_L = C_1e^{+\lambda L} + C_2e^{-\lambda L} + S/\lambda^2 \tag{4,5}$$

To solve for C_2, multiply Eq. (4) by $-e^{+\lambda L}$ and add the result to Eq. (5),

$$-\theta_o e^{+\lambda L} + \theta_L = C_2\left(-e^{+\lambda L} + e^{-\lambda L}\right) + S/\lambda^2\left(-e^{+\lambda L} + 1\right)$$

$$C_2 = \left[\left(\theta_L - \theta_o e^{+\lambda L}\right) - S/\lambda^2\left(-e^{+\lambda L} + 1\right)\right] \Big/ \left(-e^{+\lambda L} + e^{-\lambda L}\right) \tag{6}$$

Continued...

Substituting for C_2 from Eq. (6) into Eq. (4), find

$$C_1 = \theta_o - \left\{ \left[\left(\theta_L - \theta_o e^{+\lambda L} \right) - S/\lambda^2 \left(-e^{+\lambda L} + 1 \right) \right] \Big/ \left(-e^{+\lambda L} + e^{-\lambda L} \right) \right\} - S/\lambda^2 \qquad (7)$$

Using C_1 and C_2 from Eqs. (6,7) and Eq. (1), the temperature distribution can be expressed as

$$\theta(x) = \left[e^{+\lambda x} - \frac{\sinh(\lambda x)}{\sinh(\lambda L)} e^{+\lambda L} \right] \theta_o + \frac{\sinh(\lambda x)}{\sinh(\lambda L)} \theta_L + \left[-\left(1 - e^{+\lambda L}\right) \frac{\sinh(\lambda x)}{\sinh(\lambda L)} + \left(1 - e^{+\lambda L}\right) \right] \frac{S}{\lambda^2} \qquad (8)$$

<

The heat rate from the plate is $q_p = -q_x(0) + q_x(L)$ and using Fourier's law, the conduction heat rates, with $A_c = W \cdot t$, are

$$q_x(0) = -kA_c \left.\frac{d\theta}{dx}\right)_{x=0} = -kA_c \left\{ \left[\lambda e^0 - \frac{e^{\lambda L}}{\sinh(\lambda L)} \lambda \right] \theta_o + \frac{\lambda}{\sinh(\lambda L)} \theta_L + \left[-\frac{1 - e^{+\lambda L}}{\sinh(\lambda L)} \lambda - \lambda \right] \frac{S}{\lambda^2} \right\}$$

<

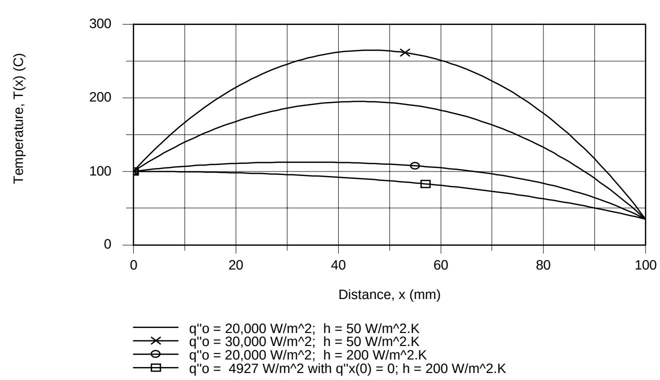
$$q_x(L) = -kA_c \left.\frac{d\theta}{dx}\right)_{x=L} = -kA_c \left\{ \left[\lambda e^{\lambda L} - \frac{e^{\lambda L}}{\sinh(\lambda L)} \lambda \cosh(\lambda L) \right] \theta_o + \frac{\lambda \cosh(\lambda L)}{\sinh(\lambda L)} \theta_L + \left[-\frac{1 - e^{+\lambda L}}{\sinh(\lambda L)} \lambda \cosh(\lambda L) - \lambda e^{+\lambda L} \right] \frac{S}{\lambda^2} \right\}$$

<

(c) For the prescribed base-case conditions listed below, the temperature distribution (solid line) is shown in the accompanying plot. As expected, the maximum temperature does not occur at the midpoint, but slightly toward the x-origin. The sink heat rates are

$$q_x''(0) = -17.22\,\text{W} \qquad q_x''(L) = 23.62\,\text{W}$$

<



The additional temperature distributions on the plot correspond to changes in the following parameters, with all the remaining parameters unchanged: (i) q_o'' = 30,000 W/m^2, (ii) h = 200 W/m$^2\cdot$K, (iii) the value of q_o'' for which $q_x''(0)$ = 0 with h = 200 W/m$^2\cdot$K. The condition for the last curve is q_o'' = 4927 W/m^2 for which the temperature gradient at x = 0 is zero.

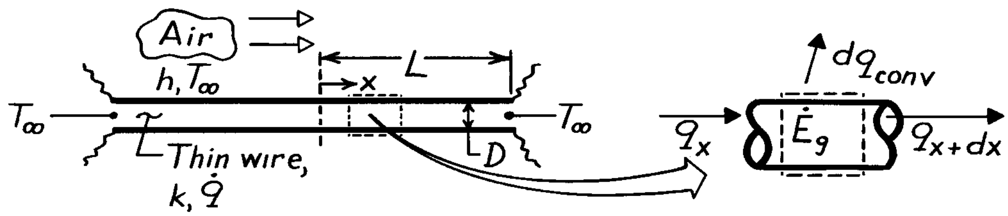
Base case conditions are: q_o'' = 20,000 W/m^2, T_o = 100°C, T_L = 35°C, T_∞ = 25°C, k = 25 W/m·K, h = 50 W/m$^2\cdot$K, L = 100 mm, t = 5 mm, W = 30 mm.

PROBLEM 3.104

KNOWN: Thermal conductivity, diameter and length of a wire which is annealed by passing an electrical current through the wire.

FIND: Steady-state temperature distribution along wire.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction along the wire, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient h.

ANALYSIS: Applying conservation of energy to a differential control volume,

$$q_x + \dot{E}_g - dq_{conv} - q_{x+dx} = 0$$

$$q_{x+dx} = q_x + \frac{dq_x}{dx}dx \qquad q_x = -k\left(\pi\ D^2/4\right)dT/dx$$

$$dq_{conv} = h\left(\pi\ D\ dx\right)\ \left(T - T_\infty\right) \qquad \dot{E}_g = \dot{q}\left(\pi\ D^2/4\right)dx.$$

Hence,

$$k\left(\pi\ D^2/4\right)\ \frac{d^2T}{dx^2}dx + \dot{q}\left(\pi\ D^2/4\right)dx - h\left(\pi\ Ddx\right)\ \left(T - T_\infty\right) = 0$$

or, with $\theta \equiv T - T_\infty$,
$$\frac{d^2\theta}{dx^2} - \frac{4h}{kD}\theta + \frac{\dot{q}}{k} = 0$$

The solution (general and particular) to this nonhomogeneous equation is of the form

$$\theta = C_1\ e^{mx} + C_2\ e^{-mx} + \frac{\dot{q}}{km^2}$$

where $m^2 = (4h/kD)$. The boundary conditions are:

$$\left.\frac{d\theta}{dx}\right]_{x=0} = 0 = m\ C_1\ e^0 - mC_2\ e^0 \rightarrow \qquad C_1 = C_2$$

$$\theta\left(L\right) = 0 = C_1\left(e^{mL} + e^{-mL}\right) + \frac{\dot{q}}{km^2} \rightarrow \qquad C_1 = \frac{-\dot{q}/km^2}{e^{mL} + e^{-mL}} = C_2$$

The temperature distribution has the form

$$T = T_\infty - \frac{\dot{q}}{km^2}\ \left[\frac{e^{mx} + e^{-mx}}{e^{mL} + e^{-mL}} - 1\right] = T_\infty - \frac{\dot{q}}{km^2}\ \left[\frac{\cosh mx}{\cosh mL} - 1\right].$$ <

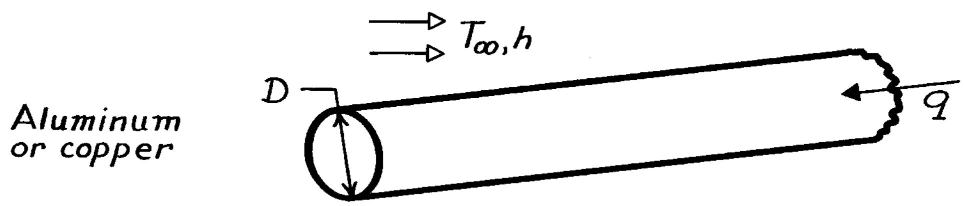
COMMENTS: This process is commonly used to anneal wire and spring products. To check the result, note that $T(L) = T(-L) = T_\infty$.

PROBLEM 3.119

KNOWN: Long, aluminum cylinder acts as an extended surface.

FIND: (a) Increase in heat transfer if diameter is tripled and (b) Increase in heat transfer if copper is used in place of aluminum.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Uniform convection coefficient, (5) Rod is infinitely long.

PROPERTIES: *Table A-1,* Aluminum (pure): k = 240 W/m·K; *Table A-1,* Copper (pure): k = 400 W/m·K.

ANALYSIS: (a) For an infinitely long fin, the fin heat rate from Table 3.4 is

$$q_f = M = (hPkA_c)^{1/2}\,\theta_b$$

$$q_f = \left(h\,\pi\,D\,k\,\pi\,D^2/4\right)^{1/2}\theta_b = \frac{\pi}{2}(hk)^{1/2}\,D^{3/2}\,\theta_b.$$

where $P = \pi D$ and $A_c = \pi D^2/4$ for the circular cross-section. Note that $q_f \propto D^{3/2}$. Hence, if the diameter is tripled,

$$\frac{q_f(3D)}{q_f(D)} = 3^{3/2} = 5.2$$

and there is a 420% increase in heat transfer. <

(b) In changing from aluminum to copper, since $q_f \propto k^{1/2}$, it follows that

$$\frac{q_f\,(Cu)}{q_f\,(A1)} = \left[\frac{k_{Cu}}{k_{A1}}\right]^{1/2} = \left[\frac{400}{240}\right]^{1/2} = 1.29$$

and there is a 29% increase in the heat transfer rate. <

COMMENTS: (1) Because fin effectiveness is enhanced by maximizing $P/A_c = 4/D$, the use of a larger number of small diameter fins is preferred to a single large diameter fin.

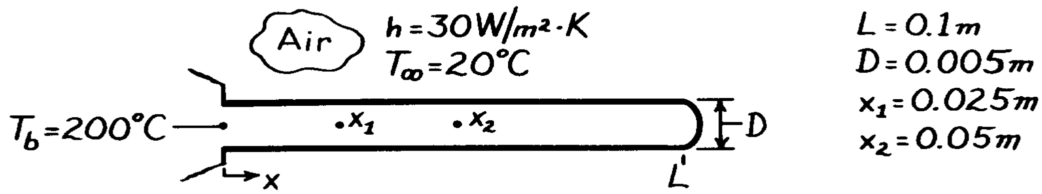
(2) From the standpoint of cost and weight, aluminum is preferred over copper.

PROBLEM 3.120

KNOWN: Length, diameter, base temperature and environmental conditions associated with a brass rod.

FIND: Temperature at specified distances along the rod.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient h.

PROPERTIES: *Table A-1,* Brass $\left(\overline{T} = 110^\circ C\right)$: $k = 133\ W/m \cdot K$.

ANALYSIS: Evaluate first the fin parameter

$$m = \left[\frac{hP}{kA_c}\right]^{1/2} = \left[\frac{h\pi\ D}{k\pi\ D^2/4}\right]^{1/2} = \left[\frac{4h}{kD}\right]^{1/2} = \left[\frac{4 \times 30\ W/m^2 \cdot K}{133\ W/m \cdot K \times 0.005m}\right]^{1/2}$$

$$m = 13.43\ m^{-1}.$$

Hence, $m L = (13.43) \times 0.1 = 1.34$ and from the results of Example 3.8, it is advisable not to make the infinite rod approximation. Thus from Table 3.4, the temperature distribution has the form

$$\theta = \frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}\theta_b$$

Evaluating the hyperbolic functions, cosh mL = 2.04 and sinh mL = 1.78, and the parameter

$$\frac{h}{mk} = \frac{30\ W/m^2 \cdot K}{13.43 m^{-1}\ (133\ W/m \cdot K)} = 0.0168,$$

with $\theta_b = 180°C$ the temperature distribution has the form

$$\theta = \frac{\cosh m(L-x) + 0.0168\ \sinh m(L-x)}{2.07}\left(180^\circ C\right).$$

The temperatures at the prescribed location are tabulated below.

x(m)	cosh m(L-x)	sinh m(L-x)	θ	T(°C)	
$x_1 = 0.025$	1.55	1.19	136.5	156.5	<
$x_2 = 0.05$	1.24	0.725	108.9	128.9	<
L = 0.10	1.00	0.00	87.0	107.0	<

COMMENTS: If the rod were approximated as infinitely long: $T(x_1) = 148.7°C$, $T(x_2) = 112.0°C$, and $T(L) = 67.0°C$. The assumption would therefore result in significant underestimates of the rod temperature.