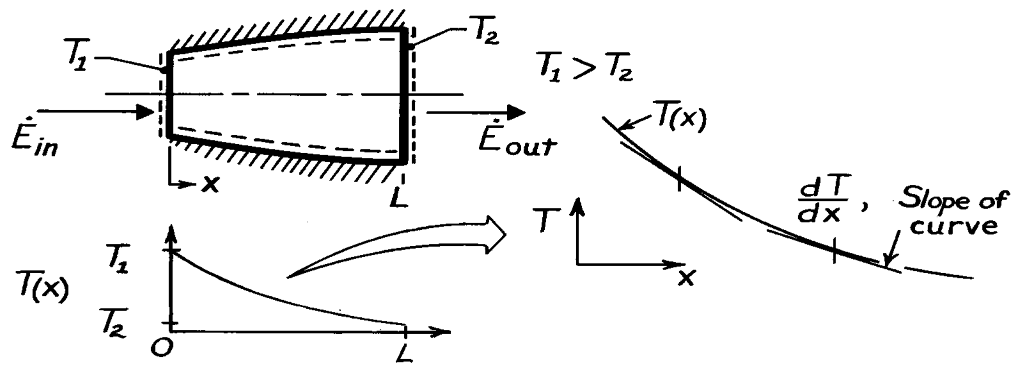
PROBLEM 2.1

KNOWN: Steady-state, one-dimensional heat conduction through an axisymmetric shape.

FIND: Sketch temperature distribution and explain shape of curve.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, one-dimensional conduction, (2) Constant properties, (3) No internal heat generation.

ANALYSIS: Performing an energy balance on the object according to Eq. 1.11a, $\dot{E}_{in} - \dot{E}_{out} = 0$, it follows that

$$\dot{E}_{in} - \dot{E}_{out} = q_x$$

and that $q_x \neq q_x(x)$. That is, the heat rate within the object is everywhere constant. From Fourier's law,

$$q_x = -kA_x \frac{dT}{dx},$$

and since q_x and k are both constants, it follows that

$$A_x \frac{dT}{dx} = \text{Constant.}$$

That is, the product of the cross-sectional area normal to the heat rate and temperature gradient remains a constant and independent of distance x. It follows that since A_x increases with x, then dT/dx must decrease with increasing x. Hence, the temperature distribution appears as shown above.

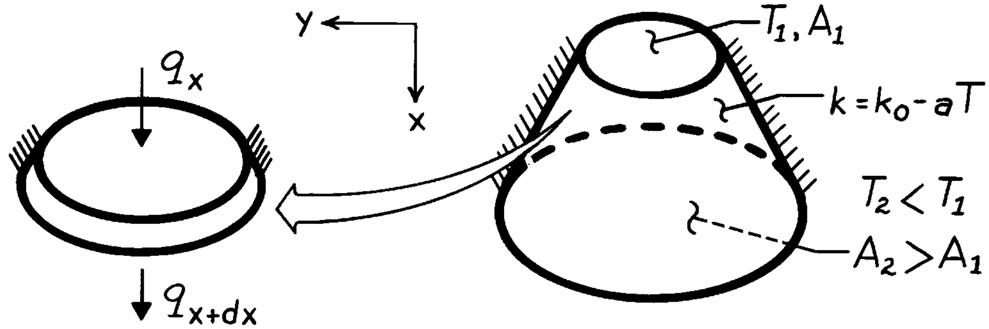
COMMENTS: (1) Be sure to recognize that dT/dx is the slope of the temperature distribution. (2) What would the distribution be when $T_2 > T_1$? (3) How does the heat flux, q''_x, vary with distance?

PROBLEM 2.5

KNOWN: End-face temperatures and temperature dependence of k for a truncated cone.

FIND: Variation with axial distance along the cone of q_x, q''_x, k, and dT/dx.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in x (negligible temperature gradients along y), (2) Steady-state conditions, (3) Adiabatic sides, (4) No internal heat generation.

ANALYSIS: For the prescribed conditions, it follows from conservation of energy, Eq. 1.11a, that for a differential control volume, $\dot{E}_{in} = \dot{E}_{out}$ or $q_x = q_{x+dx}$. Hence

q_x is independent of x.

Since A(x) *increases* with *increasing* x, it follows that $q''_x = q_x / A(x)$ *decreases* with *increasing* x. Since T *decreases* with *increasing* x, k *increases* with *increasing* x. Hence, from Fourier's law, Eq. 2.2,

$$q''_x = -k \frac{dT}{dx},$$

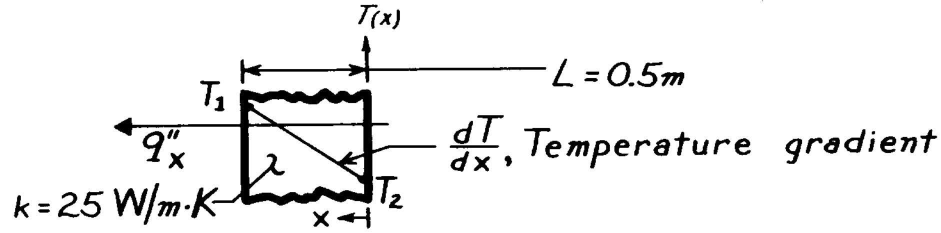
it follows that | dT/dx | *decreases* with increasing x.

PROBLEM 2.7

KNOWN: Thermal conductivity and thickness of a one-dimensional system with no internal heat generation and steady-state conditions.

FIND: Unknown surface temperatures, temperature gradient or heat flux.

SCHEMATIC:

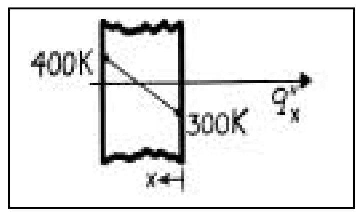


ASSUMPTIONS: (1) One-dimensional heat flow, (2) No internal heat generation, (3) Steady-state conditions, (4) Constant properties.

ANALYSIS: The rate equation and temperature gradient for this system are

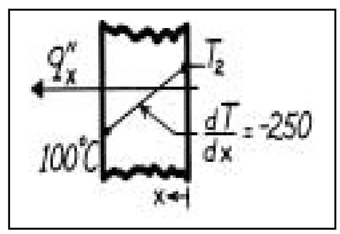
$$q''_x = -k\frac{dT}{dx} \qquad \text{and} \qquad \frac{dT}{dx} = \frac{T_1 - T_2}{L}. \qquad (1,2)$$

Using Eqs. (1) and (2), the unknown quantities can be determined.



(a) $\frac{dT}{dx} = \frac{(400-300)\,K}{0.5m} = 200 \text{ K/m}$

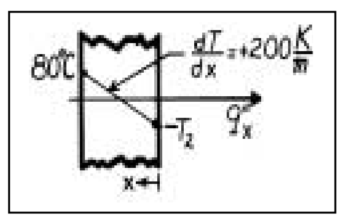
$$q''_x = -25\frac{W}{m\cdot K}\times 200\frac{K}{m} = -5000 \text{ W/m}^2.$$ <



(b) $q''_x = -25\frac{W}{m\cdot K}\times\left[-250\frac{K}{m}\right] = 6250 \text{ W/m}^2$

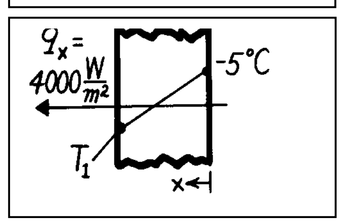
$$T_2 = T_1 - L\left[\frac{dT}{dx}\right] = 1000^\circ\text{C-0.5m}\left[\text{-250}\frac{K}{m}\right]$$

$$T_2 = 225^\circ\text{C}.$$ <



(c) $q''_x = -25\frac{W}{m\cdot K}\times 200\frac{K}{m} = -5000 \text{ W/m}^2$

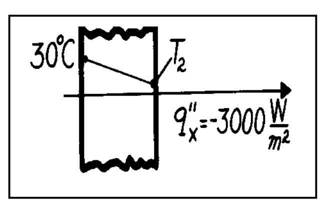
$$T_2 = 80^\circ\text{C-0.5m}\left[200\frac{K}{m}\right] = -20^\circ\text{C}.$$ <



(d) $\frac{dT}{dx} = -\frac{q''_x}{k} = -\frac{4000 \text{ W/m}^2}{25 \text{ W/m}\cdot\text{K}} = -160\frac{K}{m}$

$$T_1 = L\left[\frac{dT}{dx}\right] + T_2 = 0.5m\left[\text{-160}\frac{K}{m}\right] + \left(-5^\circ C\right)\sqrt{a^2+b^2}$$

$$T_1 = -85^\circ\text{C}.$$ <



(e) $\frac{dT}{dx} = -\frac{q''_x}{k} = -\frac{\left(-3000 \text{ W/m}^2\right)}{25 \text{ W/m}\cdot\text{K}} = 120\frac{K}{m}$

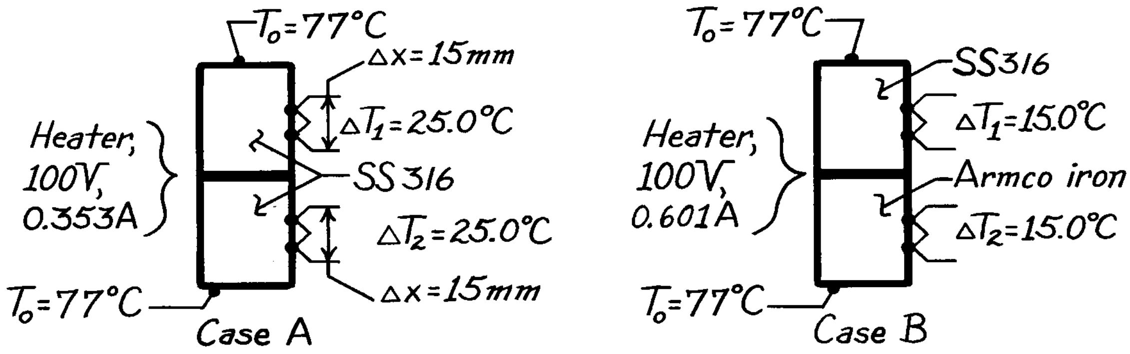
$$T_2 = 30^\circ\text{C-0.5m}\left[120\frac{K}{m}\right] = -30^\circ\text{C}.$$

PROBLEM 2.17

KNOWN: Electrical heater sandwiched between two identical cylindrical (30 mm dia. × 60 mm length) samples whose opposite ends contact plates maintained at T_o.

FIND: (a) Thermal conductivity of SS316 samples for the prescribed conditions (A) and their average temperature, (b) Thermal conductivity of Armco iron sample for the prescribed conditions (B), (c) Comment on advantages of experimental arrangement, lateral heat losses, and conditions for which $\Delta T_1 \neq \Delta T_2$.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional heat transfer in samples, (2) Steady-state conditions, (3) Negligible contact resistance between materials.

PROPERTIES: *Table A.2*, Stainless steel 316 $(\bar{T} = 400\text{ K})$: $k_{ss} = 15.2\text{ W/m}\cdot\text{K}$; Armco iron $(\bar{T} = 380\text{ K})$: $k_{iron} = 71.6\text{ W/m}\cdot\text{K}$.

ANALYSIS: (a) For Case A recognize that half the heater power will pass through each of the samples which are presumed identical. Apply Fourier's law to a sample

$$q = kA_c\frac{\Delta T}{\Delta x}$$

$$k = \frac{q\Delta x}{A_c\Delta T} = \frac{0.5(100\text{V}\times 0.353\text{A})\times 0.015\text{ m}}{\pi(0.030\text{ m})^2/4\times 25.0^\circ\text{C}} = 15.0\text{ W/m}\cdot\text{K}.$$ <

The total temperature drop across the length of the sample is $\Delta T_1(L/\Delta x)$ = 25°C (60 mm/15 mm) = 100°C. Hence, the heater temperature is T_h = 177°C. Thus the average temperature of the sample is

$$\bar{T} = (T_o + T_h)/2 = 127^\circ\text{C} = 400\text{ K}$$ <

.

We compare the calculated value of k with the tabulated value (see above) at 400 K and note the good agreement.

(b) For Case B, we assume that the thermal conductivity of the SS316 sample is the same as that found in Part (a). The heat rate through the Armco iron sample is

Continued …..

PROBLEM 2.17 (CONT.)

$$q_{iron} = q_{heater} - q_{ss} = 100\text{V} \times 0.601\text{A} - 15.0\ \text{W/m}\cdot\text{K} \times \frac{\pi(0.030\ \text{m})^2}{4} \times \frac{15.0°\text{C}}{0.015\ \text{m}}$$

$$q_{iron} = (60.1 - 10.6)\text{W} = 49.5\ \text{W}$$

where

$$q_{ss} = k_{ss} A_c \Delta T_2 / \Delta x_2 .$$

Applying Fourier's law to the iron sample,

$$k_{iron} = \frac{q_{iron}\Delta x_2}{A_c \Delta T_2} = \frac{49.5\ \text{W} \times 0.015\ \text{m}}{\pi(0.030\ \text{m})^2/4 \times 15.0°\text{C}} = 70.0\ \text{W/m}\cdot\text{K}. \qquad <$$

The total drop across the iron sample is 15°C(60/15) = 60°C; the heater temperature is (77 + 60)°C = 137°C. Hence the average temperature of the iron sample is

$$\overline{T} = (137 + 77)°\text{C}/2 = 107°\text{C} = 380\ \text{K}. \qquad <$$

We compare the computed value of k with the tabulated value (see above) at 380 K and note the good agreement.

(c) The principal advantage of having two identical samples is the assurance that all the electrical power dissipated in the heater will appear as equivalent heat flows through the samples. With only one sample, heat can flow from the backside of the heater even though insulated.

Heat leakage out the lateral surfaces of the cylindrically shaped samples will become significant when the sample thermal conductivity is comparable to that of the insulating material. Hence, the method is suitable for metallics, but must be used with caution on nonmetallic materials.

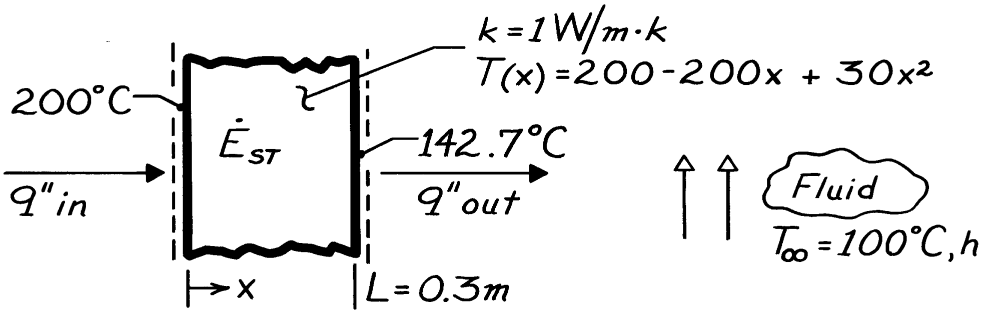
For any combination of materials in the upper and lower position, we expect $\Delta T_1 = \Delta T_2$. However, if the insulation were improperly applied along the lateral surfaces, it is possible that heat leakage will occur, causing $\Delta T_1 \neq \Delta T_2$.

PROBLEM 2.24

KNOWN: Wall thickness, thermal conductivity, temperature distribution, and fluid temperature.

FIND: (a) Surface heat rates and rate of change of wall energy storage per unit area, and (b) Convection coefficient.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in x, (2) Constant k.

ANALYSIS: (a) From Fourier's law,

$$q''_x = -k\frac{\partial T}{\partial x} = (200 - 60x)\cdot k$$

$$q''_{in} = q''_{x=0} = 200\frac{°C}{m}\times 1\frac{W}{m\cdot K} = 200\ W/m^2 \qquad <$$

$$q''_{out} = q''_{x=L} = (200 - 60\times 0.3)°C/m\ \times\ 1\ W/m\cdot K = 182\ W/m^2. \qquad <$$

Applying an energy balance to a control volume about the wall, Eq. 1.11a,

$$\dot{E}''_{in} - \dot{E}''_{out} = \dot{E}''_{st}$$

$$\dot{E}''_{st} = q''_{in} - q''_{out} = 18\ W/m^2. \qquad <$$

(b) Applying a surface energy balance at x = L,

$$q''_{out} = h\left[T(L) - T_\infty\right]$$

$$h = \frac{q''_{out}}{T(L) - T_\infty} = \frac{182\ W/m^2}{(142.7 - 100)°C}$$

$$h = 4.3\ W/m^2\cdot K. \qquad <$$

COMMENTS: (1) From the heat equation,

$$(\partial T/\partial t) = (k/\rho c_p)\ \partial^2 T/\partial x^2 = 60(k/\rho c_p),$$

it follows that the temperature is increasing with time at every point in the wall.

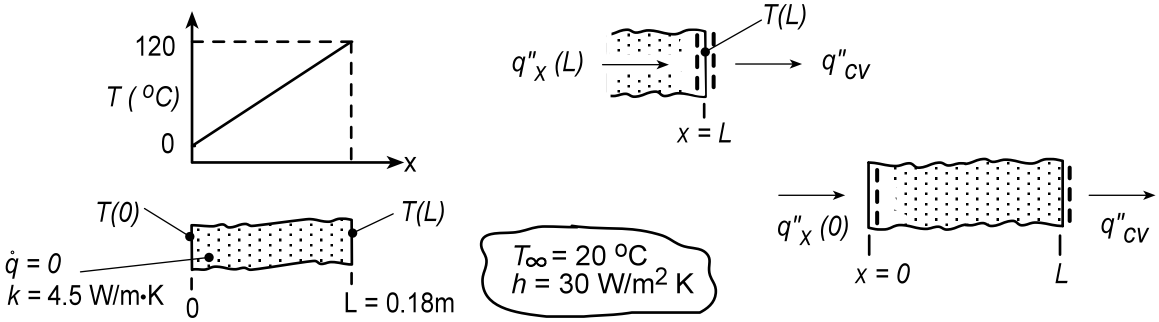
(2) The value of h is small and is typical of free convection in a gas.

PROBLEM 2.30

KNOWN: Plane wall with no internal energy generation.

FIND: Determine whether the prescribed temperature distribution is possible; explain your reasoning. With the temperatures T(0) = 0°C and T_∞ = 20°C fixed, compute and plot the temperature T(L) as a function of the convection coefficient for the range $10 \le h \le 100$ W/m^2·K.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) No internal energy generation, (3) Constant properties, (4) No radiation exchange at the surface x = L, and (5) Steady-state conditions.

ANALYSIS: (a) Is the prescribed temperature distribution possible? If so, the energy balance at the surface x = L as shown above in the Schematic, must be satisfied.

$$\dot{E}_{in} - \dot{E}_{out} \; ? = ? \; 0 \qquad\qquad q''_x(L) - q''_{cv} \; ? = ? \; 0 \qquad\qquad (1,2)$$

where the conduction and convection heat fluxes are, respectively,

$$q''_x(L) = -k\left.\frac{dT}{dx}\right)_{x=L} = -k\frac{T(L)-T(0)}{L} = -4.5\,\mathrm{W/m\cdot K}\times(120-0)^\circ\mathrm{C}\big/0.18\,\mathrm{m} = -3000\,\mathrm{W/m^2}$$

$$q''_{cv} = h\left[T(L)-T_\infty\right] = 30\,\mathrm{W/m^2\cdot K}\times(120-20)^\circ\mathrm{C} = 3000\,\mathrm{W/m^2}$$

Substituting the heat flux values into Eq. (2), find (-3000) - (3000) ≠ 0 and therefore, the temperature distribution is not possible.

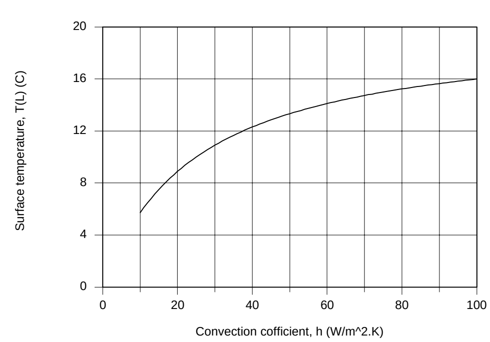
(b) With T(0) = 0°C and T_∞ = 20°C, the temperature at the surface x = L, T(L), can be determined from an overall energy balance on the wall as shown above in the Schematic,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q''_x(0) - q''_{cv} = 0 \qquad -k\frac{T(L)-T(0)}{L} - h\left[T(L)-T_\infty\right] = 0$$

$$-4.5\,\mathrm{W/m\cdot K}\left[T(L)-0^\circ\mathrm{C}\right]\big/0.18\,\mathrm{m} - 30\,\mathrm{W/m^2\cdot K}\left[T(L)-20^\circ\mathrm{C}\right] = 0$$

T(L) = 10.9°C <

Using this same analysis, T(L) as a function of the convection coefficient can be determined and plotted. We don't expect T(L) to be linearly dependent upon h. Note that as h increases to larger values, T(L) approaches T_∞. To what value will T(L) approach as h decreases?

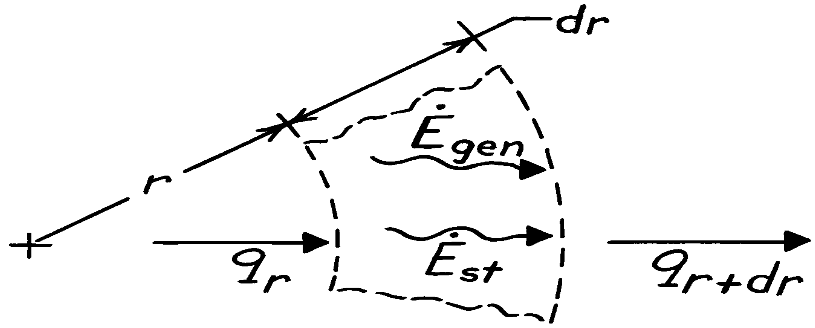


PROBLEM 2.33

KNOWN: Heat diffusion with internal heat generation for one-dimensional cylindrical, radial coordinate system.

FIND: Heat diffusion equation.

SCHEMATIC:



ASSUMPTIONS: (1) Homogeneous medium.

ANALYSIS: Control volume has volume, $V = A_r \cdot dr = 2\pi r \cdot dr \cdot 1$, with unit thickness normal to page. Using the conservation of energy requirement, Eq. 1.11a,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st}$$

$$q_r - q_{r+dr} + \dot{q}V = \rho V c_p \frac{\partial T}{\partial t}.$$

Fourier's law, Eq. 2.1, for this one-dimensional coordinate system is

$$q_r = -kA_r \frac{\partial T}{\partial r} = -k \times 2\pi r \cdot 1 \times \frac{\partial T}{\partial r}.$$

At the outer surface, r+dr, the conduction rate is

$$q_{r+dr} = q_r + \frac{\partial}{\partial r}(q_r)dr = q_r + \frac{\partial}{\partial r}\left[-k \cdot 2\pi r \cdot \frac{\partial T}{\partial r}\right]dr.$$

Hence, the energy balance becomes

$$q_r - \left[q_r + \frac{\partial}{\partial r}\left[-k2\pi r \frac{\partial T}{\partial r}\right]dr\right] + \dot{q} \cdot 2\pi r dr = \rho \cdot 2\pi r dr \cdot c_p \frac{\partial T}{\partial t}$$

Dividing by the factor $2\pi r$ dr, we obtain

$$\frac{1}{r}\frac{\partial}{\partial r}\left[kr\frac{\partial T}{\partial r}\right] + \dot{q} = \rho c_p \frac{\partial T}{\partial t}.$$ <

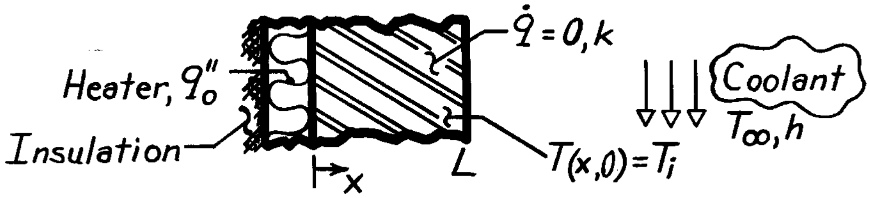
COMMENTS: (1) Note how the result compares with Eq. 2.20 when the terms for the ϕ,z coordinates are eliminated. (2) Recognize that we did not require $\dot{q}$ and k to be independent of r.

PROBLEM 2.47

KNOWN: Plane wall, initially at a uniform temperature T_i, is suddenly exposed to convection with a fluid at T_∞ at one surface, while the other surface is exposed to a constant heat flux q''_o.

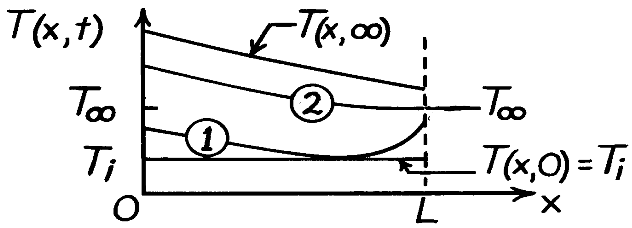
FIND: (a) Temperature distributions, T(x,t), for initial, steady-state and two intermediate times, (b) Corresponding heat fluxes on $q''_x - x$ coordinates, (c) Heat flux at locations x = 0 and x = L as a function of time, (d) Expression for the steady-state temperature of the heater, T(0,∞), in terms of q''_o, T_∞, k, h and L.

SCHEMATIC:



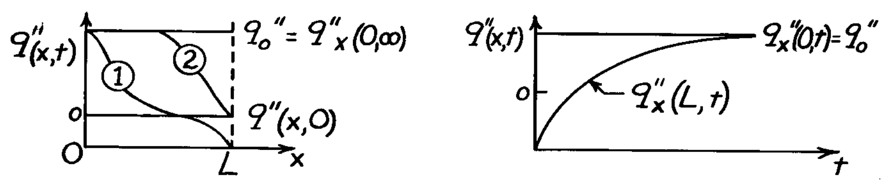
ASSUMPTIONS: (1) One-dimensional conduction, (2) No heat generation, (3) Constant properties.

ANALYSIS: (a) For $T_i < T_\infty$, the temperature distributions are



Note the constant gradient at x = 0 since $q''_x(0) = q''_o$.

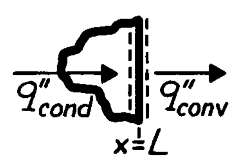
(b) The heat flux distribution, $q''_x(x,t)$, is determined from knowledge of the temperature gradients, evident from Part (a), and Fourier's law.



(c) On $q''_x(x,t) - t$ coordinates, the heat fluxes at the boundaries are shown above.

(d) Perform a surface energy balance at x = L and an energy balance on the wall:

$$q''_{cond} = q''_{conv} = h\left[T(L,\infty) - T_\infty\right] \quad (1), \qquad q''_{cond} = q''_o. \quad (2)$$



For the wall, under steady-state conditions, Fourier's law gives

$$q''_o = -k\frac{dT}{dx} = k\frac{T(0,\infty) - T(L,\infty)}{L}. \qquad (3)$$

Combine Eqs. (1), (2), (3) to find:

$$T(0,\infty) = T_\infty + \frac{q''_o}{1/h + L/k}.$$