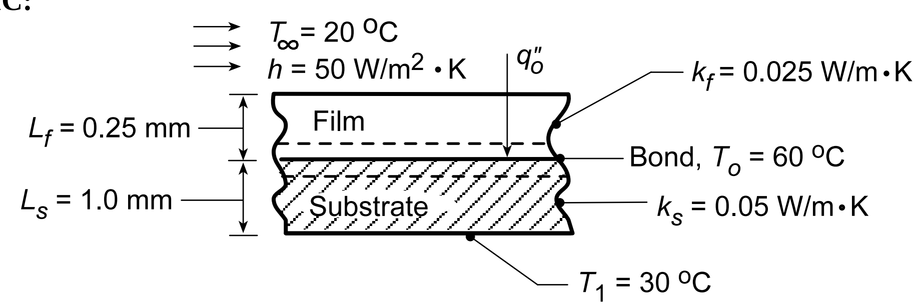
PROBLEM 3.4

KNOWN: Curing of a transparent film by radiant heating with substrate and film surface subjected to known thermal conditions.

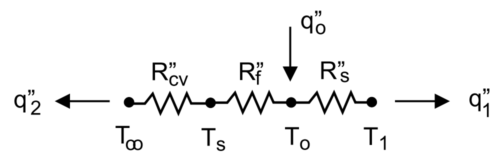
FIND: (a) Thermal circuit for this situation, (b) Radiant heat flux, q''_o (W/m^2), to maintain bond at curing temperature, T_o, (c) Compute and plot q''_o as a function of the film thickness for $0 \le L_f \le 1$ mm, and (d) If the film is not transparent, determine q''_o required to achieve bonding; plot results as a function of L_f.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat flow, (3) All the radiant heat flux q''_o is absorbed at the bond, (4) Negligible contact resistance.

ANALYSIS: (a) The thermal circuit for this situation is shown at the right. Note that terms are written on a per unit area basis.



(b) Using this circuit and performing an energy balance on the film-substrate interface,

$$q''_o = q''_1 + q''_2 \qquad\qquad q''_o = \frac{T_o - T_\infty}{R''_{cv} + R''_f} + \frac{T_o - T_1}{R''_s}$$

where the thermal resistances are

$$R''_{cv} = 1/h = 1/50\,\mathrm{W/m^2\cdot K} = 0.020\,\mathrm{m^2\cdot K/W}$$

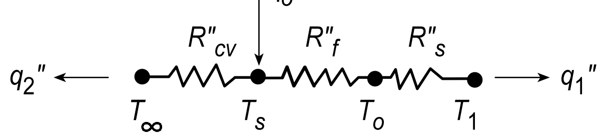
$$R''_f = L_f/k_f = 0.00025\,\mathrm{m}/0.025\,\mathrm{W/m\cdot K} = 0.010\,\mathrm{m^2\cdot K/W}$$

$$R''_s = L_s/k_s = 0.001\,\mathrm{m}/0.05\,\mathrm{W/m\cdot K} = 0.020\,\mathrm{m^2\cdot K/W}$$

$$q''_o = \frac{(60-20)^\circ\mathrm{C}}{[0.020+0.010]\mathrm{m^2\cdot K/W}} + \frac{(60-30)^\circ\mathrm{C}}{0.020\,\mathrm{m^2\cdot K/W}} = (133+1500)\,\mathrm{W/m^2} = 2833\,\mathrm{W/m^2} \qquad <$$

(c) For the transparent film, the radiant flux required to achieve bonding as a function of film thickness L_f is shown in the plot below.

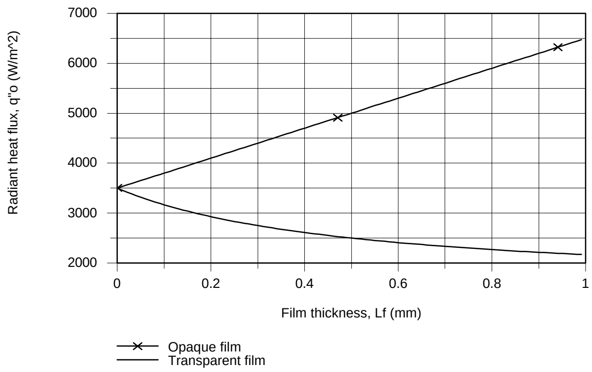
(d) If the film is opaque (not transparent), the thermal circuit is shown below. In order to find q''_o, it is necessary to write two energy balances, one around the T_s node and the second about the T_o node.



.
The results of the analyses are plotted below.

Continued...

PROBLEM 3.4 (Cont.)

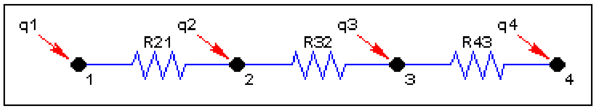


COMMENTS: (1) When the film is transparent, the radiant flux is absorbed on the bond. The flux required decreases with increasing film thickness. Physically, how do you explain this? Why is the relationship not linear?

(2) When the film is opaque, the radiant flux is absorbed on the surface, and the flux required increases with increasing thickness of the film. Physically, how do you explain this? Why is the relationship linear?

(3) The IHT Thermal Resistance Network Model was used to create a model of the film-substrate system and generate the above plot. The Workspace is shown below.

// Thermal Resistance Network Model:
// The Network:



// Heat rates into node j,qij, through thermal resistance Rij
q21 = (T2 - T1) / R21
q32 = (T3 - T2) / R32
q43 = (T4 - T3) / R43

// Nodal energy balances
q1 + q21 = 0
q2 - q21 + q32 = 0
q3 - q32 + q43 = 0
q4 - q43 = 0

/* Assigned variables list: deselect the qi, Rij and Ti which are unknowns; set qi = 0 for embedded nodal points at which there is no external source of heat. */

T1 = Tinf	// Ambient air temperature, C
//q1 =	// Heat rate, W; film side
T2 = Ts	// Film surface temperature, C
q2 = 0	// Radiant flux, W/m^2; zero for part (a)
T3 = To	// Bond temperature, C
q3 = qo	// Radiant flux, W/m^2; part (a)
T4 = Tsub	// Substrate temperature, C
//q4 =	// Heat rate, W; substrate side

// Thermal Resistances:

R21 = 1 / (h * As)	// Convection resistance, K/W
R32 = Lf / (kf * As)	// Conduction resistance, K/W; film
R43 = Ls / (ks * As)	// Conduction resistance, K/W; substrate

// Other Assigned Variables:

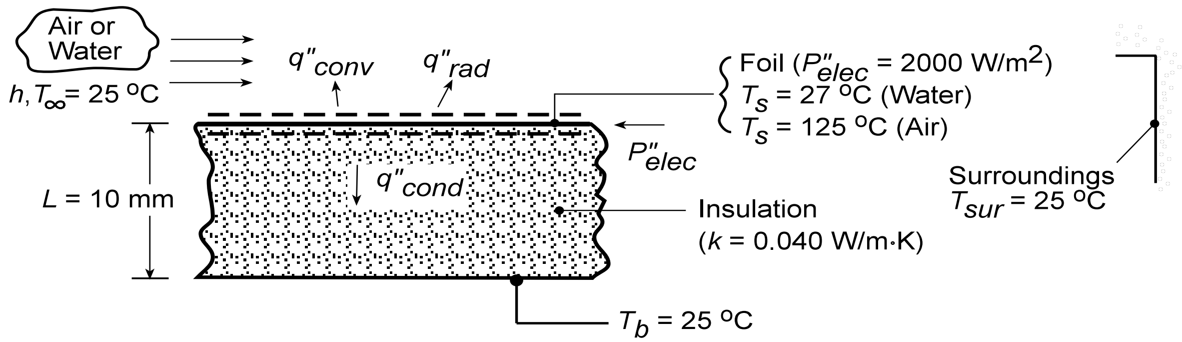
Tinf = 20	// Ambient air temperature, C
h = 50	// Convection coefficient, W/m^2.K
Lf = 0.00025	// Thickness, m; film
kf = 0.025	// Thermal conductivity, W/m.K; film
To = 60	// Cure temperature, C
Ls = 0.001	// Thickness, m; substrate
ks = 0.05	// Thermal conductivity, W/m.K; substrate
Tsub = 30	// Substrate temperature, C
As = 1	// Cross-sectional area, m^2; unit area

PROBLEM 3.6

KNOWN: Design and operating conditions of a heat flux gage.

FIND: (a) Convection coefficient for water flow (T_s = 27°C) and error associated with neglecting conduction in the insulation, (b) Convection coefficient for air flow (T_s = 125°C) and error associated with neglecting conduction and radiation, (c) Effect of convection coefficient on error associated with neglecting conduction for T_s = 27°C.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction, (3) Constant k.

ANALYSIS: (a) The electric power dissipation is balanced by convection to the water and conduction through the insulation. An energy balance applied to a control surface about the foil therefore yields

$$P''_{elec} = q''_{conv} + q''_{cond} = h\left(T_s - T_\infty\right) + k\left(T_s - T_b\right)/L$$

Hence,

$$h = \frac{P''_{elec} - k\left(T_s - T_b\right)/L}{T_s - T_\infty} = \frac{2000\,\mathrm{W/m^2} - 0.04\,\mathrm{W/m\cdot K}\left(2\,\mathrm{K}\right)/0.01\,\mathrm{m}}{2\,\mathrm{K}}$$

$$h = \frac{\left(2000-8\right)\mathrm{W/m^2}}{2\,\mathrm{K}} = 996\,\mathrm{W/m^2\cdot K} \qquad <$$

If conduction is neglected, a value of h = 1000 $W/m^2 \cdot K$ is obtained, with an attendant error of (1000 - 996)/996 = 0.40%

(b) In air, energy may also be transferred from the foil surface by radiation, and the energy balance yields

$$P''_{elec} = q''_{conv} + q''_{rad} + q''_{cond} = h\left(T_s - T_\infty\right) + \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) + k\left(T_s - T_b\right)/L$$

Hence,

$$h = \frac{P''_{elec} - \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) - k\left(T_s - T_\infty\right)/L}{T_s - T_\infty}$$

$$= \frac{2000\,\mathrm{W/m^2} - 0.15\times5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\left(398^4 - 298^4\right)\mathrm{K^4} - 0.04\,\mathrm{W/m\cdot K}\,(100\,\mathrm{K})/0.01\,\mathrm{m}}{100\,\mathrm{K}}$$

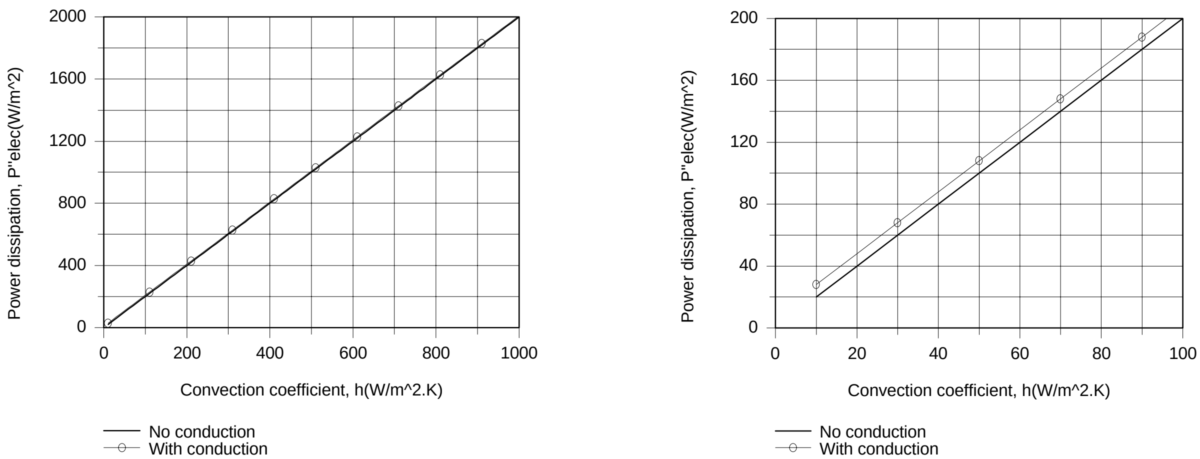
$$= \frac{\left(2000-146-400\right)\mathrm{W/m^2}}{100\,\mathrm{K}} = 14.5\,\mathrm{W/m^2\cdot K} \qquad <$$

Continued...

PROBLEM 3.6 (Cont.)

If conduction, radiation, or conduction and radiation are neglected, the corresponding values of h and the percentage errors are 18.5 $W/m^2 \cdot K$ (27.6%), 16 $W/m^2 \cdot K$ (10.3%), and 20 $W/m^2 \cdot K$ (37.9%).

(c) For a fixed value of T_s = 27°C, the conduction loss remains at q''_{cond} = 8 W/m^2, which is also the fixed difference between P''_{elec} and q''_{conv}. Although this difference is not clearly shown in the plot for $10 \le h \le 1000$ $W/m^2 \cdot K$, it is revealed in the subplot for $10 \le 100$ $W/m^2 \cdot K$.



Errors associated with neglecting conduction decrease with increasing h from values which are significant for small h (h < 100 $W/m^2 \cdot K$) to values which are negligible for large h.

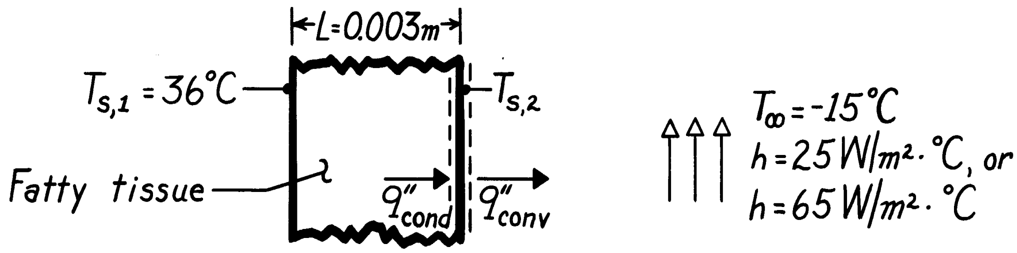
COMMENTS: In liquids (large h), it is an excellent approximation to neglect conduction and assume that all of the dissipated power is transferred to the fluid.

PROBLEM 3.7

KNOWN: A layer of fatty tissue with fixed inside temperature can experience different outside convection conditions.

FIND: (a) Ratio of heat loss for different convection conditions, (b) Outer surface temperature for different convection conditions, and (c) Temperature of still air which achieves same cooling as moving air (*wind chill* effect).

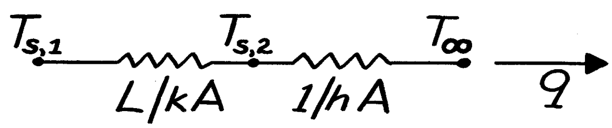
SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction through a plane wall, (2) Steady-state conditions, (3) Homogeneous medium with constant properties, (4) No internal heat generation (metabolic effects are negligible), (5) Negligible radiation effects.

PROPERTIES: *Table A-3*, Tissue, fat layer: k = 0.2 W/m·K.

ANALYSIS: The thermal circuit for this situation is



Hence, the heat rate is

$$q = \frac{T_{s,1} - T_\infty}{R_{tot}} = \frac{T_{s,1} - T_\infty}{L/kA + 1/hA}.$$

Therefore,

$$\frac{q''_{calm}}{q''_{windy}} = \frac{\left[\frac{L}{k} + \frac{1}{h}\right]_{windy}}{\left[\frac{L}{k} + \frac{1}{h}\right]_{calm}}.$$

Applying a surface energy balance to the outer surface, it also follows that

$$q''_{cond} = q''_{conv}.$$

Continued …..

PROBLEM 3.7 (Cont.)

Hence,

$$\frac{k}{L}\left(T_{s,1}-T_{s,2}\right)=h\left(T_{s,2}-T_\infty\right)$$

$$T_{s,2}=\frac{T_\infty+\dfrac{k}{hL}T_{s,1}}{1+\dfrac{k}{hL}}.$$

To determine the wind chill effect, we must determine the heat loss for the windy day and use it to evaluate the hypothetical ambient air temperature, T'_∞, which would provide the same heat loss on a calm day, Hence,

$$q''=\frac{T_{s,1}-T_\infty}{\left[\dfrac{L}{k}+\dfrac{1}{h}\right]_{windy}}=\frac{T_{s,1}-T'_\infty}{\left[\dfrac{L}{k}+\dfrac{1}{h}\right]_{calm}}$$

From these relations, we can now find the results sought:

(a) $$\frac{q''_{calm}}{q''_{windy}}=\frac{\dfrac{0.003\text{ m}}{0.2\text{ W/m}\cdot\text{K}}+\dfrac{1}{65\text{ W/m}^2\cdot\text{K}}}{\dfrac{0.003\text{ m}}{0.2\text{ W/m}\cdot\text{K}}+\dfrac{1}{25\text{ W/m}^2\cdot\text{K}}}=\frac{0.015+0.0154}{0.015+0.04}$$

$$\frac{q''_{calm}}{q''_{windy}}=0.553$$ <

(b) $$T_{s,2}\Big]_{calm}=\frac{-15^\circ\text{C}+\dfrac{0.2\text{ W/m}\cdot\text{K}}{\left(25\text{ W/m}^2\cdot\text{K}\right)\left(0.003\text{ m}\right)}36^\circ\text{C}}{1+\dfrac{0.2\text{ W/m}\cdot\text{K}}{\left(25\text{ W/m}^2\cdot\text{K}\right)\left(0.003\text{ m}\right)}}=22.1^\circ\text{C}$$ <

$$T_{s,2}\Big]_{windy}=\frac{-15^\circ\text{C}+\dfrac{0.2\text{ W/m}\cdot\text{K}}{\left(65\text{ W/m}^2\cdot\text{K}\right)\left(0.003\text{m}\right)}36^\circ\text{C}}{1+\dfrac{0.2\text{ W/m}\cdot\text{K}}{\left(65\text{ W/m}^2\cdot\text{K}\right)\left(0.003\text{m}\right)}}=10.8^\circ\text{C}$$ <

(c) $$T'_\infty=36^\circ\text{C}-\left(36+15\right)^\circ\text{C}\frac{\left(0.003/0.2+1/25\right)}{\left(0.003/0.2+1/65\right)}=-56.3^\circ\text{C}$$ <

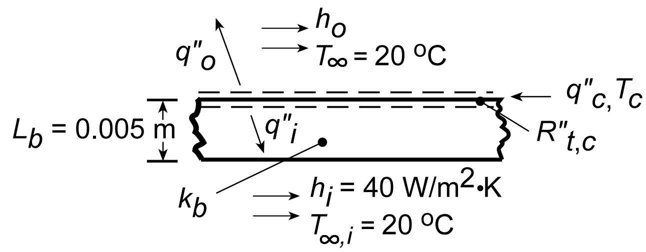
COMMENTS: The wind chill effect is equivalent to a decrease of $T_{s,2}$ by 11.3°C and increase in the heat loss by a factor of $(0.553)^{-1}$ = 1.81.

PROBLEM 3.27

KNOWN: Operating conditions for a board mounted chip.

FIND: (a) Equivalent thermal circuit, (b) Chip temperature, (c) Maximum allowable heat dissipation for dielectric liquid (h_o = 1000 W/m^2·K) and air (h_o = 100 W/m^2·K). Effect of changes in circuit board temperature and contact resistance.

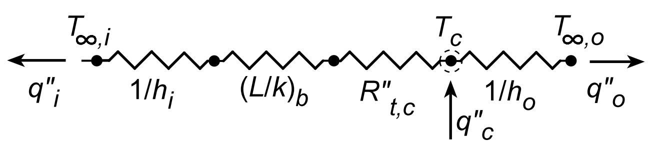
SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Negligible chip thermal resistance, (4) Negligible radiation, (5) Constant properties.

PROPERTIES: *Table A-3*, Aluminum oxide (polycrystalline, 358 K): k_b = 32.4 W/m·K.

ANALYSIS: (a)



(b) Applying conservation of energy to a control surface about the chip $\left(\dot{E}_{in} - \dot{E}_{out} = 0\right)$,

$$q''_c - q''_i - q''_o = 0$$

$$q''_c = \frac{T_c - T_{\infty,i}}{1/h_i + (L/k)_b + R''_{t,c}} + \frac{T_c - T_{\infty,o}}{1/h_o}$$

With $q''_c = 3\times10^4\ \text{W/m}^2$, h_o = 1000 W/m^2·K, k_b = 1 W/m·K and $R''_{t,c} = 10^{-4}\ \text{m}^2\cdot\text{K/W}$,

$$3\times10^4\ \text{W/m}^2 = \frac{T_c - 20^\circ\text{C}}{\left(1/40 + 0.005/1 + 10^{-4}\right)\text{m}^2\cdot\text{K/W}} + \frac{T_c - 20^\circ\text{C}}{(1/1000)\,\text{m}^2\cdot\text{K/W}}$$

$$3\times10^4\ \text{W/m}^2 = (33.2T_c - 664 + 1000T_c - 20{,}000)\ \text{W/m}^2\cdot\text{K}$$

$1003T_c = 50{,}664$

$T_c = 49°\text{C}$. <

(c) For T_c = 85°C and h_o = 1000 W/m^2·K, the foregoing energy balance yields

$$q''_c = 67{,}160\ \text{W/m}^2$$ <

with q''_o = 65,000 W/m^2 and q''_i = 2160 W/m^2. Replacing the dielectric with air (h_o = 100 W/m^2·K), the following results are obtained for different combinations of k_b and $R''_{t,c}$.

Continued...

PROBLEM 3.27 (Cont.)

k_b (W/m·K)	$R''_{t,c}$ (m^2·K/W)	q''_i (W/m^2)	q''_o (W/m^2)	q''_c (W/m^2)
1	10^{-4}	2159	6500	8659
32.4	10^{-4}	2574	6500	9074
1	10^{-5}	2166	6500	8666
32.4	10^{-5}	2583	6500	9083

<

COMMENTS: 1. For the conditions of part (b), the total internal resistance is 0.0301 m^2·K/W, while the outer resistance is 0.001 m^2·K/W. Hence

$$\frac{q''_o}{q''_i} = \frac{\left(T_c - T_{\infty,o}\right)/R''_o}{\left(T_c - T_{\infty,i}\right)/R''_i} = \frac{0.0301}{0.001} = 30\,.$$

and only approximately 3% of the heat is dissipated through the board.

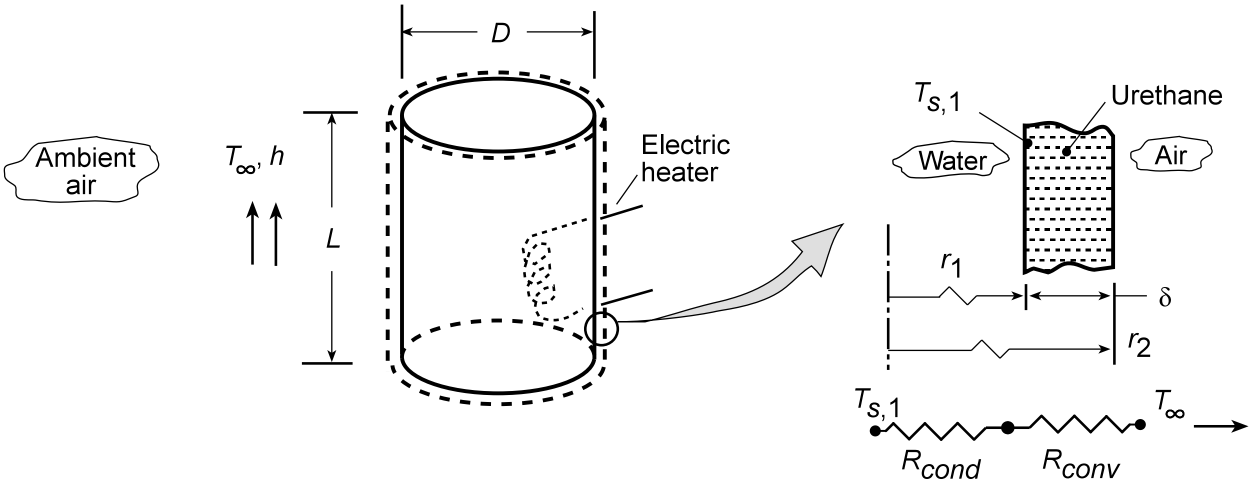
2. With h_o = 100 W/m^2·K, the outer resistance increases to 0.01 m^2·K/W, in which case $q''_o/q''_i = R''_i/R''_o$ = 0.0301/0.01 = 3.1 and now almost 25% of the heat is dissipated through the board. Hence, although measures to reduce R''_i would have a negligible effect on q''_c for the liquid coolant, some improvement may be gained for air-cooled conditions. As shown in the table of part (b), use of an aluminum oxide board increase q''_i by 19% (from 2159 to 2574 W/m^2) by reducing R''_i from 0.0301 to 0.0253 m^2·K/W. Because the initial contact resistance ($R''_{t,c} = 10^{-4}\,\mathrm{m^2 \cdot K/W}$) is already much less than R''_i, any reduction in its value would have a negligible effect on q''_i. The largest gain would be realized by increasing h_i, since the inside convection resistance makes the dominant contribution to the total internal resistance.

PROBLEM 3.36

KNOWN: Temperature and volume of hot water heater. Nature of heater insulating material. Ambient air temperature and convection coefficient. Unit cost of electric power.

FIND: Heater dimensions and insulation thickness for which annual cost of heat loss is less than $50.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction through side and end walls, (2) Conduction resistance dominated by insulation, (3) Inner surface temperature is approximately that of the water ($T_{s,1}$ = 55°C), (4) Constant properties, (5) Negligible radiation.

PROPERTIES: *Table A.3*, Urethane Foam (T = 300 K): k = 0.026 W/m·K.

ANALYSIS: To minimize heat loss, tank dimensions which minimize the total surface area, $A_{s,t}$, should be selected. With $L = 4\forall/\pi D^2$, $A_{s,t} = \pi DL + 2\left(\pi D^2/4\right) = 4\forall/D + \pi D^2/2$, and the tank diameter for which $A_{s,t}$ is an extremum is determined from the requirement

$$dA_{s,t}/dD = -4\forall/D^2 + \pi D = 0$$

It follows that

$$D = (4\forall/\pi)^{1/3} \qquad \text{and} \qquad L = (4\forall/\pi)^{1/3}$$

With $d^2A_{s,t}/dD^2 = 8\forall/D^3 + \pi > 0$, the foregoing conditions yield the desired minimum in $A_{s,t}$. Hence, for $\forall$ = 100 gal × 0.00379 m^3/gal = 0.379 m^3,

$$D_{op} = L_{op} = 0.784\,\text{m}$$ <

The total heat loss through the side and end walls is

$$q = \frac{T_{s,1} - T_\infty}{\dfrac{\ln(r_2/r_1)}{2\pi k L_{op}} + \dfrac{1}{h 2\pi r_2 L_{op}}} + \frac{2\left(T_{s,1} - T_\infty\right)}{\dfrac{\delta}{k\left(\pi D_{op}^2/4\right)} + \dfrac{1}{h\left(\pi D_{op}^2/4\right)}}$$

We begin by estimating the heat loss associated with a 25 mm thick layer of insulation. With $r_1 = D_{op}/2$ = 0.392 m and $r_2 = r_1 + \delta$ = 0.417 m, it follows that

Continued...

PROBLEM 3.36 (Cont.)

$$q = \frac{(55-20)^\circ\text{C}}{\dfrac{\ln(0.417/0.392)}{2\pi(0.026\,\text{W/m}\cdot\text{K})0.784\,\text{m}} + \dfrac{1}{(2\,\text{W/m}^2\cdot\text{K})2\pi(0.417\,\text{m})0.784\,\text{m}}}$$

$$+\frac{2(55-20)^\circ\text{C}}{\dfrac{0.025\,\text{m}}{(0.026\,\text{W/m}\cdot\text{K})\pi/4(0.784\,\text{m})^2} + \dfrac{1}{(2\,\text{W/m}^2\cdot\text{K})\pi/4(0.784\,\text{m})^2}}$$

$$q = \frac{35^\circ\text{C}}{(0.483+0.243)\,\text{K/W}} + \frac{2(35^\circ\text{C})}{(1.992+1.036)\,\text{K/W}} = (48.2+23.1)\,\text{W} = 71.3\,\text{W}$$

The annual energy loss is therefore

$$Q_{\text{annual}} = 71.3\,\text{W}(365\,\text{days})(24\ \text{h/day})\left(10^{-3}\ \text{kW/W}\right) = 625\,\text{kWh}$$

With a unit electric power cost of \$0.08/kWh, the annual cost of the heat loss is

C = (\$0.08/kWh)625 kWh = \$50.00

Hence, an insulation thickness of

δ = 25 mm $<$

will satisfy the prescribed cost requirement.

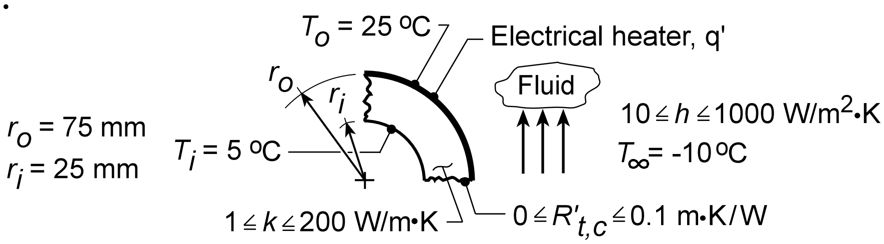
COMMENTS: Cylindrical containers of aspect ratio L/D = 1 are seldom used because of floor space constraints. Choosing L/D = 2, $\forall = \pi D^3/2$ and $D = (2\forall/\pi)^{1/3}$ = 0.623 m. Hence, L = 1.245 m, r_1 = 0.312m and r_2 = 0.337 m. It follows that q = 76.1 W and C = \$53.37. The 6.7% increase in the annual cost of the heat loss is small, providing little justification for using the optimal heater dimensions.

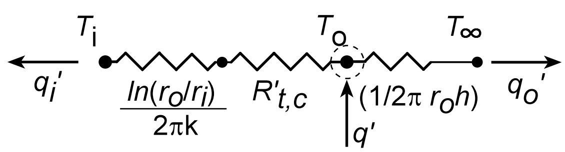
PROBLEM 3.38

KNOWN: Inner and outer radii of a tube wall which is heated electrically at its outer surface. Inner and outer wall temperatures. Temperature of fluid adjoining outer wall.

FIND: Effect of wall thermal conductivity, thermal contact resistance, and convection coefficient on total heater power and heat rates to outer fluid and inner surface.

SCHEMATIC:





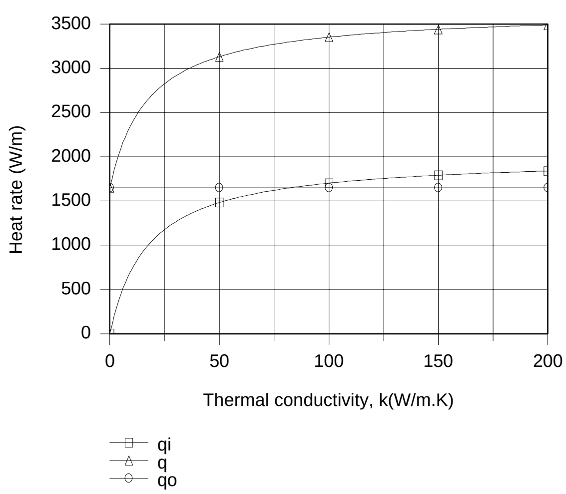
ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible temperature drop across heater, (5) Negligible radiation.

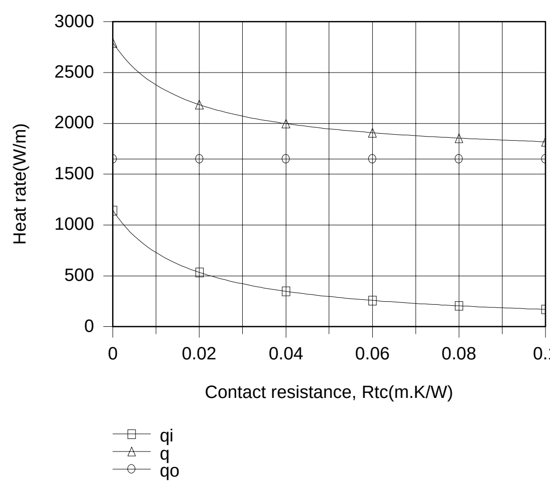
ANALYSIS: Applying an energy balance to a control surface about the heater,

$$q' = q_i' + q_o'$$

$$q' = \frac{T_o - T_i}{\dfrac{\ln\left(r_o/r_i\right)}{2\pi k} + R_{t,c}'} + \frac{T_o - T_\infty}{\left(1/2\pi r_o h\right)}$$

Selecting nominal values of k = 10 W/m·K, $R_{t,c}'$ = 0.01 m·K/W and h = 100 W/m^2·K, the following parametric variations are obtained





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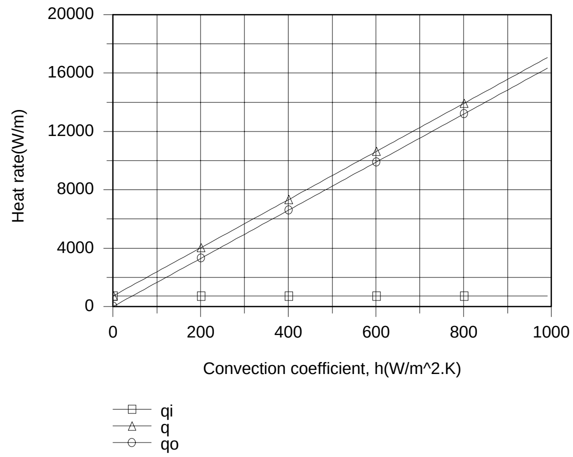
PROBLEM 3.38 (Cont.)



For a prescribed value of h, q'_o is fixed, while q'_i, and hence q', increase and decrease, respectively, with increasing k and $R'_{t,c}$. These trends are attributable to the effects of k and $R'_{t,c}$ on the total (conduction plus contact) resistance separating the heater from the inner surface. For fixed k and $R'_{t,c}$, q'_i is fixed, while q'_o, and hence q', increase with increasing h due to a reduction in the convection resistance.

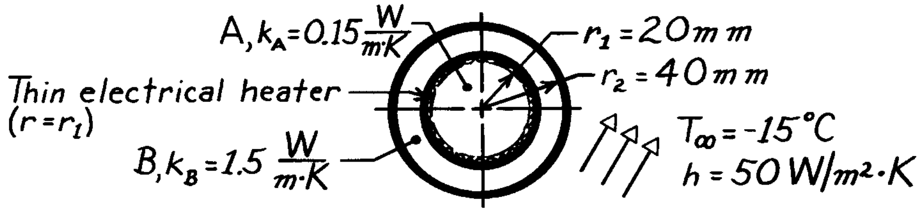
COMMENTS: For the prescribed nominal values of k, $R'_{t,c}$ and h, the electric power requirement is q' = 2377 W/m. To maintain the prescribed heater temperature, q' would increase with any changes which reduce the conduction, contact and/or convection resistances.

PROBLEM 3.41

KNOWN: Thin electrical heater fitted between two concentric cylinders, the outer surface of which experiences convection.

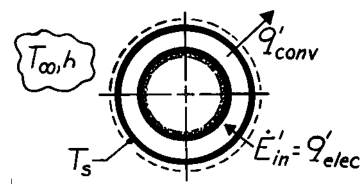
FIND: (a) Electrical power required to maintain outer surface at a specified temperature, (b) Temperature at the center

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, radial conduction, (2) Steady-state conditions, (3) Heater element has negligible thickness, (4) Negligible contact resistance between cylinders and heater, (5) Constant properties, (6) No generation.

ANALYSIS: (a) Perform an energy balance on the composite system to determine the power required to maintain $T(r_2) = T_s = 5°C$.



$$\dot{E}'_{in} - \dot{E}'_{out} + \dot{E}_{gen} = \dot{E}_{st}$$

$$+q'_{elec} - q'_{conv} = 0.$$

Using Newton's law of cooling,

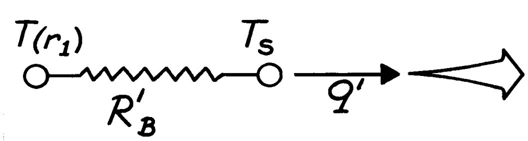
$$q'_{elec} = q'_{conv} = h \cdot 2\pi\ r_2 \left(T_s - T_\infty\right)$$

$$q'_{elec} = 50\frac{\text{W}}{\text{m}^2 \cdot \text{K}} \times 2\pi\left(0.040\text{m}\right)\ \left[5-\left(-15\right)\right]^\circ \text{C}=251\text{ W/m}.$$ <

(b) From a control volume about Cylinder A, we recognize that the cylinder must be isothermal, that is,

$$T(0) = T(r_1).$$

Represent Cylinder B by a thermal circuit:



$$q' = \frac{T\left(r_1\right) - T_s}{R'_B}$$

For the cylinder, from Eq. 3.28,

$$R'_B = \ln r_2 / r_1 / 2\pi\ k_B$$

giving

$$T\left(r_1\right) = T_s + q'R'_B = 5^\circ \text{C} + 253.1\frac{\text{W}}{\text{m}}\frac{\ln 40/20}{2\pi \times 1.5\text{ W/m} \cdot \text{K}} = 23.5^\circ \text{C}$$

Hence, $T(0) = T(r_1) = 23.5°C$. <

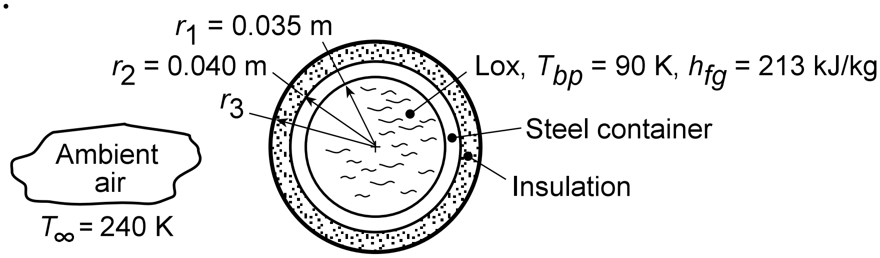
Note that k_A has no influence on the temperature T(0).

PROBLEM 3.58

KNOWN: Dimensions of spherical, stainless steel liquid oxygen (LOX) storage container. Boiling point and latent heat of fusion of LOX. Environmental temperature.

FIND: Thermal isolation system which maintains boil-off below 1 kg/day.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conditions, (2) Negligible thermal resistances associated with internal and external convection, conduction in the container wall, and contact between wall and insulation, (3) Negligible radiation at exterior surface, (4) Constant insulation thermal conductivity.

PROPERTIES: *Table A.1*, 304 Stainless steel (T = 100 K): k_s = 9.2 W/m·K; *Table A.3*, Reflective, aluminum foil-glass paper insulation (T = 150 K): k_i = 0.000017 W/m·K.

ANALYSIS: The heat gain associated with a loss of 1 kg/day is

$$q = \dot{m}h_{fg} = \frac{1\,\text{kg/day}}{86,400\,\text{s/day}}\left(2.13\times10^5\ \text{J/kg}\right) = 2.47\,\text{W}$$

With an overall temperature difference of $\left(T_\infty - T_{bp}\right)$ = 150 K, the corresponding total thermal resistance is

$$R_{tot} = \frac{\Delta T}{q} = \frac{150\,\text{K}}{2.47\,\text{W}} = 60.7\,\text{K/W}$$

Since the conduction resistance of the steel wall is

$$R_{t,cond,s} = \frac{1}{4\pi k_s}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{1}{4\pi\left(9.2\,\text{W/m}\cdot\text{K}\right)}\left(\frac{1}{0.35\,\text{m}} - \frac{1}{0.40\,\text{m}}\right) = 2.4\times10^{-3}\ \text{K/W}$$

it is clear that exclusive reliance must be placed on the insulation and that a special insulation of very low thermal conductivity should be selected. The best choice is a highly reflective foil/glass matted insulation which was developed for cryogenic applications. It follows that

$$R_{t,cond,i} = 60.7\,\text{K/W} = \frac{1}{4\pi k_i}\left(\frac{1}{r_2} - \frac{1}{r_3}\right) = \frac{1}{4\pi\left(0.000017\,\text{W/m}\cdot\text{K}\right)}\left(\frac{1}{0.40\,\text{m}} - \frac{1}{r_3}\right)$$

which yields r_3 = 0.4021 m. The minimum insulation thickness is therefore $\delta = (r_3 - r_2)$ = 2.1 mm.

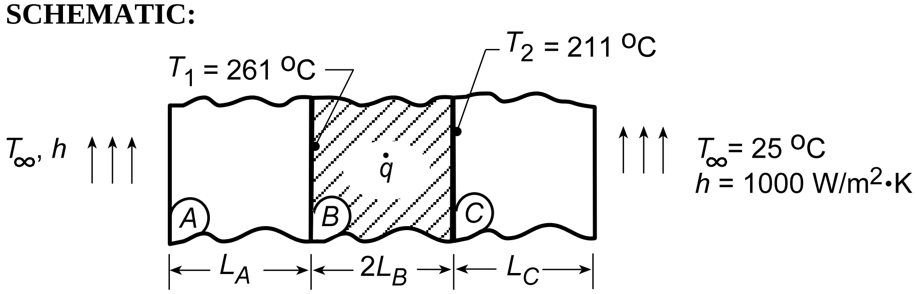
COMMENTS: The heat loss could be reduced well below the maximum allowable by adding more insulation. Also, in view of weight restrictions associated with launching space vehicles, consideration should be given to fabricating the LOX container from a lighter material.

PROBLEM 3.73

KNOWN: Composite wall with outer surfaces exposed to convection process.

FIND: (a) Volumetric heat generation and thermal conductivity for material B required for special conditions, (b) Plot of temperature distribution, (c) T_1 and T_2, as well as temperature distributions corresponding to loss of coolant condition where h = 0 on surface A.

SCHEMATIC:



L_A = 30 mm
L_B = 30 mm
L_C = 20 mm
k_A = 25 W/m·K
k_C = 50 W/m·K

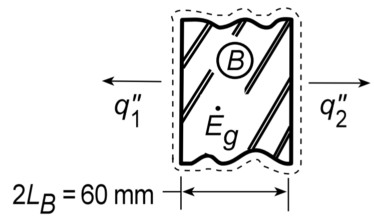
ASSUMPTIONS: (1) Steady-state, one-dimensional heat transfer, (2) Negligible contact resistance at interfaces, (3) Uniform generation in B; zero in A and C.

ANALYSIS: (a) From an energy balance on wall B,

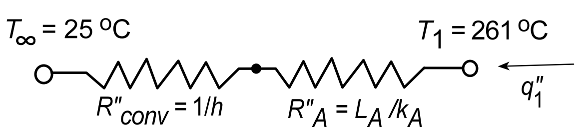
$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st}$$

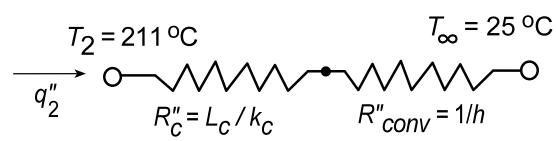
$$-q_1'' - q_2'' + 2\dot{q}L_B = 0$$

$$\dot{q}_B = \left(q_1'' + q_2''\right)/2L_B \,.$$



To determine the heat fluxes, q_1'' and q_2'', construct thermal circuits for A and C:





$$q_1'' = \left(T_1 - T_\infty\right)/\left(1/h + L_A/k_A\right)$$

$$q_1'' = \left(261 - 25\right)^\circ \mathrm{C} \Big/ \left(\frac{1}{1000\,\mathrm{W/m^2\cdot K}} + \frac{0.030\,\mathrm{m}}{25\,\mathrm{W/m\cdot K}}\right)$$

$$q_1'' = 236^\circ \mathrm{C}/\left(0.001 + 0.0012\right)\mathrm{m^2\cdot K/W}$$

$$q_1'' = 107{,}273\,\mathrm{W/m^2}$$

$$q_2'' = \left(T_2 - T_\infty\right)/\left(L_C/k_C + 1/h\right)$$

$$q_2'' = \left(211 - 25\right)^\circ \mathrm{C} \Big/ \left(\frac{0.020\,\mathrm{m}}{50\,\mathrm{W/m\cdot K}} + \frac{1}{1000\,\mathrm{W/m^2\cdot K}}\right)$$

$$q_2'' = 186^\circ \mathrm{C}/\left(0.0004 + 0.001\right)\mathrm{m^2\cdot K/W}$$

$$q_2'' = 132{,}857\,\mathrm{W/m^2}$$

Using the values for q_1'' and q_2'' in Eq. (1), find

$$\dot{q}_B = \left(106{,}818 + 132{,}143\,\mathrm{W/m^2}\right)\Big/2\times 0.030\,\mathrm{m} = 4.00\times 10^6\,\mathrm{W/m^3}\,. \quad <$$

To determine k_B, use the general form of the temperature and heat flux distributions in wall B,

$$T(x) = -\frac{\dot{q}_B}{2k_B}x^2 + C_1 x + C_2 \qquad q_x''(x) = -k_B\left[-\frac{\dot{q}}{k_B}x + C_1\right] \qquad (1,2)$$

there are 3 unknowns, C_1, C_2 and k_B, which can be evaluated using three conditions,

Continued...

PROBLEM 3.73 (Cont.)

$$T(-L_B) = T_1 = -\frac{\dot{q}_B}{2k_B}(-L_B)^2 - C_1 L_B + C_2 \qquad \text{where } T_1 = 261°C \qquad (3)$$

$$T(+L_B) = T_2 = -\frac{\dot{q}_B}{2k_B}(+L_B)^2 + C_1 L_B + C_2 \qquad \text{where } T_2 = 211°C \qquad (4)$$

$$q''_x(-L_B) = -q''_1 = -k_B\left[-\frac{\dot{q}_B}{k_B}(-L_B) + C_1\right] \qquad \text{where } q''_1 = 107{,}273\ \text{W/m}^2 \qquad (5)$$

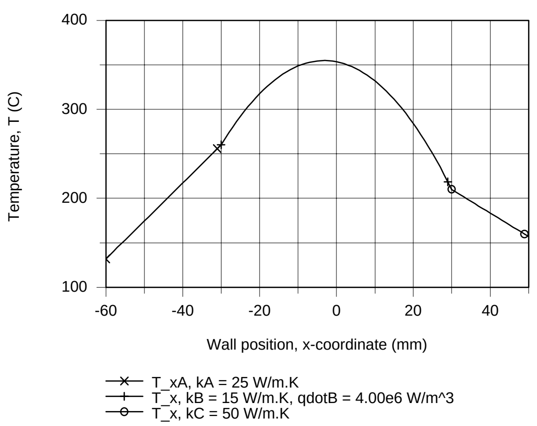
Using IHT to solve Eqs. (3), (4) and (5) simultaneously with $\dot{q}_B = 4.00 \times 10^6$ W/m^3, find

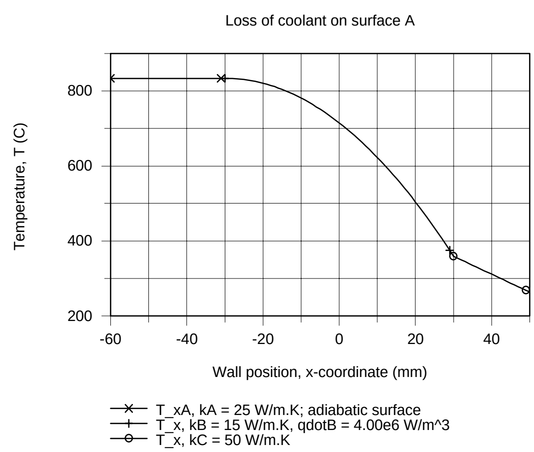
$$k_B = 15.3\ \text{W/m}\cdot\text{K}$$ <

(b) Following the method of analysis in the *IHT Example 3.6, User-Defined Functions*, the temperature distribution is shown in the plot below. The important features are (1) Distribution is quadratic in B, but non-symmetrical; linear in A and C; (2) Because thermal conductivities of the materials are different, discontinuities exist at each interface; (3) By comparison of gradients at x = -L_B and +L_B, find $q''_2 > q''_1$.

(c) Using the same method of analysis as for Part (c), the temperature distribution is shown in the plot below when h = 0 on the surface of A. Since the left boundary is adiabatic, material A will be isothermal at T_1. Find

T_1 = 835°C $\qquad$ T_2 = 360°C <



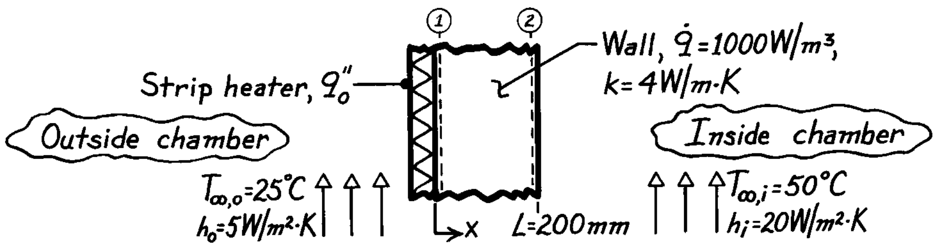


PROBLEM 3.79

KNOWN: Wall of thermal conductivity k and thickness L with uniform generation $\dot{q}$; strip heater with uniform heat flux q''_o; prescribed inside and outside air conditions (h_i, $T_{\infty,i}$, h_o, $T_{\infty,o}$).

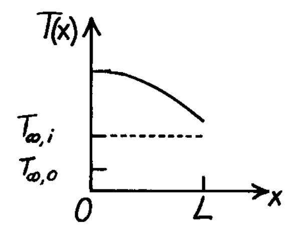
FIND: (a) Sketch temperature distribution in wall if none of the heat generated within the wall is lost to the outside air, (b) Temperatures at the wall boundaries T(0) and T(L) for the prescribed condition, (c) Value of q''_o required to maintain this condition, (d) Temperature of the outer surface, T(L), if $\dot{q}=0$ but q''_o corresponds to the value calculated in (c).

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Uniform volumetric generation, (4) Constant properties.

ANALYSIS: (a) If none of the heat generated within the wall is lost to the *outside* of the chamber, the gradient at x = 0 must be zero. Since $\dot{q}$ is uniform, the temperature distribution is parabolic, with $T(L) > T_{\infty,i}$.



(b) To find temperatures at the boundaries of wall, begin with the general solution to the appropriate form of the heat equation (Eq.3.40).

$$T(x) = -\frac{\dot{q}}{2k}x^2 + C_1 x + C_2 \qquad (1)$$

From the first boundary condition,

$$\left.\frac{dT}{dx}\right|_{x=o} = 0 \qquad \rightarrow \qquad C_1 = 0. \qquad (2)$$

Two approaches are possible using different forms for the second boundary condition.

Approach No. 1: With boundary condition $\rightarrow$ $T(0) = T_1$

$$T(x) = -\frac{\dot{q}}{2k}x^2 + T_1 \qquad (3)$$

To find T_1, perform an overall energy balance on the wall

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0$$

$$-h\left[T(L) - T_{\infty,i}\right] + \dot{q}L = 0 \qquad T(L) = T_2 = T_{\infty,i} + \frac{\dot{q}L}{h} \qquad (4)$$

Continued …..

PROBLEM 3.79 (Cont.)

and from Eq. (3) with x = L and T(L) = T_2,

$$T(L) = -\frac{\dot{q}}{2k}L^2 + T_1 \qquad \text{or} \qquad T_1 = T_2 + \frac{\dot{q}}{2k}L^2 = T_{\infty,i} + \frac{\dot{q}L}{h} + \frac{\dot{q}L^2}{2k} \tag{5,6}$$

Substituting numerical values into Eqs. (4) and (6), find

$$T_2 = 50^\circ\text{C} + 1000\ \text{W/m}^3 \times 0.200\ \text{m}/20\ \text{W/m}^2\cdot\text{K} = 50^\circ\text{C} + 10^\circ\text{C} = 60^\circ\text{C}$$ <

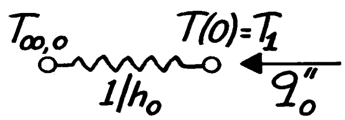
$$T_1 = 60^\circ\text{C} + 1000\ \text{W/m}^3 \times (0.200\ \text{m})^2 / 2 \times 4\ \text{W/m}\cdot\text{K} = 65^\circ\text{C}.$$ <

Approach No. 2: Using the boundary condition

$$-k\frac{dT}{dx}\bigg|_{x=L} = h\left[T(L) - T_{\infty,i}\right]$$

yields the following temperature distribution which can be evaluated at x = 0,L for the required temperatures,

$$T(x) = -\frac{\dot{q}}{2k}\left(x^2 - L^2\right) + \frac{\dot{q}L}{h} + T_{\infty,i}.$$

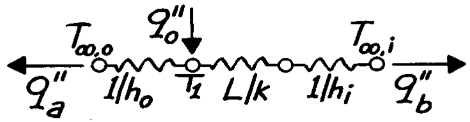


(c) The value of q''_o when T(0) = T_1 = 65°C follows from the circuit

$$q''_o = \frac{T_1 - T_{\infty,o}}{1/h_o}$$

$$q''_o = 5\ \text{W/m}^2\cdot\text{K}\,(65\text{-}25)^\circ\,\text{C} = 200\ \text{W/m}^2.$$ <

(d) With $\dot{q}=0$, the situation is represented by the thermal circuit shown. Hence,



$$q''_o = q''_a + q''_b$$

$$q''_o = \frac{T_1 - T_{\infty,o}}{1/h_o} + \frac{T_1 - T_{\infty,i}}{L/k + 1/h_i}$$

which yields

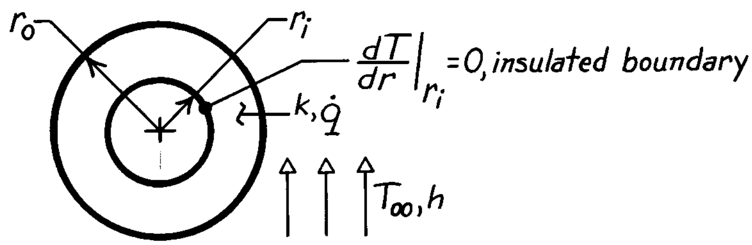
$$T_1 = 55^\circ\text{C}.$$ <

PROBLEM 3.84

KNOWN: Cylindrical shell with uniform volumetric generation is insulated at inner surface and exposed to convection on the outer surface.

FIND: (a) Temperature distribution in the shell in terms of r_i, r_o, $\dot{q}$, h, T_∞ and k, (b) Expression for the heat rate per unit length at the outer radius, $q'(r_o)$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial (cylindrical) conduction in shell, (3) Uniform generation, (4) Constant properties.

ANALYSIS: (a) The general form of the temperature distribution and boundary conditions are

$$T(r) = -\frac{\dot{q}}{4k}r^2 + C_1 \ln r + C_2$$

at $r = r_i$: $$\left.\frac{dT}{dr}\right)_{r_i} = 0 = -\frac{\dot{q}}{2k}r_i + C_1\frac{1}{r_i} + 0 \qquad C_1 = \frac{\dot{q}}{2k}r_i^2$$

at $r = r_o$: $$-k\left.\frac{dT}{dr}\right)_{r_o} = h\left[T(r_o) - T_\infty\right] \qquad \text{surface energy balance}$$

$$k\left[-\frac{\dot{q}}{2k}r_o + \left(\frac{\dot{q}}{2k}r_i^2 \cdot \frac{1}{r_o}\right)\right] = h\left[-\frac{\dot{q}}{4k}r_o^2 + \left(\frac{\dot{q}}{2k}r_i^2\right)\ln r_o + C_2 - T_\infty\right]$$

$$C_2 = -\frac{\dot{q}r_o}{2h}\left[1 + \left(\frac{r_i}{r_o}\right)^2\right] + \frac{\dot{q}r_o^2}{2k}\left[\frac{1}{2} - \left(\frac{r_i}{r_o}\right)^2 \ln r_o\right] + T_\infty$$

Hence,

$$T(r) = \frac{\dot{q}}{4k}\left(r_o^2 - r^2\right) + \frac{\dot{q}r_i^2}{2k}\ln\left(\frac{r}{r_o}\right) - \frac{\dot{q}r_o}{2h}\left[1 + \left(\frac{r_i}{r_o}\right)^2\right] + T_\infty. \qquad <$$

(b) From an overall energy balance on the shell,

$$q_r'(r_o) = \dot{E}_g' = \dot{q}\pi\left(r_o^2 - r_i^2\right). \qquad <$$

Alternatively, the heat rate may be found using Fourier's law and the temperature distribution,

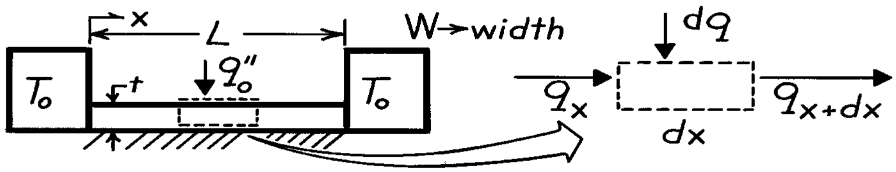
$$q_r'(r) = -k(2\pi\, r_o)\left.\frac{dT}{dr}\right)_{r_o} = -2\pi\, k r_o\left[-\frac{\dot{q}}{2k}r_o + \frac{\dot{q}r_i^2}{2k}\frac{1}{r_o} + 0 + 0\right] = \dot{q}\pi\left(r_o^2 - r_i^2\right)$$

PROBLEM 3.101

KNOWN: Dimensions of a plate insulated on its bottom and thermally joined to heat sinks at its ends. Net heat flux at top surface.

FIND: (a) Differential equation which determines temperature distribution in plate, (b) Temperature distribution and heat loss to heat sinks.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction in x (W,L>>t), (3) Constant properties, (4) Uniform surface heat flux, (5) Adiabatic bottom, (6) Negligible contact resistance.

ANALYSIS: (a) Applying conservation of energy to the differential control volume, $q_x + dq = q_{x+dx}$, where $q_{x+dx} = q_x + (dq_x/dx)\, dx$ and $dq = q_o''\,(W\cdot dx)$. Hence, $(dq_x/dx) - q_o''\,W = 0$. From Fourier's law, $q_x = -k(t\cdot W)\, dT/dx$. Hence, the differential equation for the temperature distribution is

$$-\frac{d}{dx}\left[ktW\,\frac{dT}{dx}\right] - q_o''\,W = 0 \qquad \frac{d^2T}{dx^2} + \frac{q_o''}{kt} = 0.$$ <

(b) Integrating twice, the general solution is,

$$T(x) = -\frac{q_o''}{2kt}x^2 + C_1\,x + C_2$$

and appropriate boundary conditions are $T(0) = T_o$, and $T(L) = T_o$. Hence, $T_o = C_2$, and

$$T_o = -\frac{q_o''}{2kt}L^2 + C_1L + C_2 \qquad \text{and} \qquad C_1 = \frac{q_o''L}{2kt}.$$

Hence, the temperature distribution is

$$T(x) = -\frac{q_o''L}{2kt}\left(x^2 - Lx\right) + T_o.$$ <

Applying Fourier's law at x = 0, and at x = L,

$$q(0) = -k(Wt)\; dT/dx)_{x=0} = -kWt\left[-\frac{q_o''}{kt}\right]\left[x - \frac{L}{2}\right]\Bigg|_{x=0} = -\frac{q_o''WL}{2}$$

$$q(L) = -k(Wt)dT/dx)_{x=L} = -kWt\left[-\frac{q_o''}{kt}\right]\left[x - \frac{L}{2}\right]\Bigg|_{x=L} = +\frac{q_o''WL}{2}$$

Hence the heat loss from the plates is $q = 2\left(q_o''WL/2\right) = q_o''WL$. <

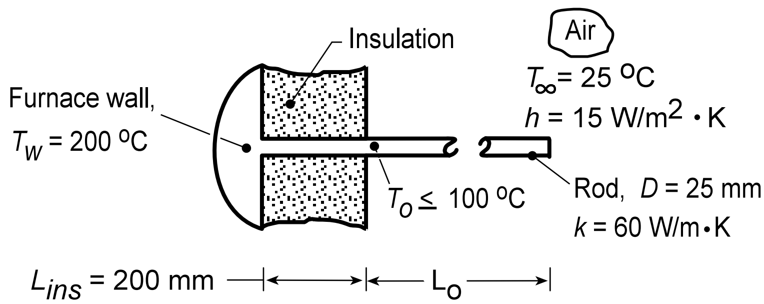
COMMENTS: (1) Note signs associated with q(0) and q(L). (2) Note symmetry about x = L/2. Alternative boundary conditions are $T(0) = T_o$ and $dT/dx)_{x=L/2} = 0$.

PROBLEM 3.111

KNOWN: Rod protruding normally from a furnace wall covered with insulation of thickness L_{ins} with the length L_o exposed to convection with ambient air.

FIND: (a) An expression for the exposed surface temperature T_o as a function of the prescribed thermal and geometrical parameters. (b) Will a rod of L_o = 100 mm meet the specified operating limit, $T_0 \le 100°C$? If not, what design parameters would you change?

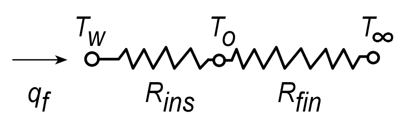
SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in rod, (3) Negligible thermal contact resistance between the rod and hot furnace wall, (4) Insulated section of rod, L_{ins}, experiences no lateral heat losses, (5) Convection coefficient uniform over the exposed portion of the rod, L_o, (6) Adiabatic tip condition for the rod and (7) Negligible radiation exchange between rod and its surroundings.

ANALYSIS: (a) The rod can be modeled as a thermal network comprised of two resistances in series: the portion of the rod, L_{ins}, covered by insulation, R_{ins}, and the portion of the rod, L_o, experiencing convection, and behaving as a fin with an adiabatic tip condition, R_{fin}. For the insulated section:

$$R_{ins} = L_{ins}/kA_c \qquad (1)$$



For the fin, Table 3.4, Case B, Eq. 3.76,

$$R_{fin} = \theta_b/q_f = \frac{1}{(hPkA_c)^{1/2}\tanh(mL_o)} \qquad (2)$$

$$m = (hP/kA_c)^{1/2} \qquad A_c = \pi D^2/4 \qquad P = \pi D \qquad (3,4,5)$$

From the thermal network, by inspection,

$$\frac{T_o - T_\infty}{R_{fin}} = \frac{T_w - T_\infty}{R_{ins} + R_{fin}} \qquad T_o = T_\infty + \frac{R_{fin}}{R_{ins} + R_{fin}}(T_w - T_\infty) \qquad (6)$$ <

(b) Substituting numerical values into Eqs. (1) - (6) with L_o = 200 mm,

$$T_o = 25°C + \frac{6.298}{6.790 + 6.298}(200 - 25)°C = 109°C$$ <

$$R_{ins} = \frac{0.200\,m}{60\,W/m\cdot K \times 4.909\times10^{-4}\,m^2} = 6.790\,K/W \qquad A_c = \pi(0.025\,m)^2/4 = 4.909\times10^{-4}\,m^2$$

$$R_{fin} = 1\Big/\left(0.0347\,W^2/K^2\right)^{1/2}\tanh(6.324\times0.200) = 6.298\,K/W$$

$$(hPkA_c) = \left(15\,W/m^2\cdot K\times\pi(0.025\,m)\times60\,W/m\cdot K\times4.909\times10^{-4}\,m^2\right) = 0.0347\,W^2/K^2$$

Continued...

PROBLEM 3.111 (Cont.)

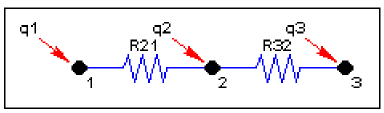
$$m = (hP/kA_c)^{1/2} = \left(15\,\text{W/m}^2\cdot\text{K}\times\pi\,(0.025\,\text{m})\Big/60\,\text{W/m}\cdot\text{K}\times 4.909\times10^{-4}\,\text{m}^2\right)^{1/2} = 6.324\,\text{m}^{-1}$$

Consider the following design changes aimed at reducing $T_o \le 100°C$. (1) Increasing length of the fin portions: with L_o = 200 mm, the fin already behaves as an infinitely long fin. Hence, increasing L_o will not result in reducing T_o. (2) Decreasing the thermal conductivity: backsolving the above equation set with T_0 = 100°C, find the required thermal conductivity is k = 14 W/m·K. Hence, we could select a stainless steel alloy; see Table A.1. (3) Increasing the insulation thickness: find that for T_o = 100°C, the required insulation thickness would be L_{ins} = 211 mm. This design solution might be physically and economically unattractive. (4) A very practical solution would be to introduce thermal contact resistance between the rod base and the furnace wall by “tack welding” (rather than a continuous bead around the rod circumference) the rod in two or three places. (5) A less practical solution would be to increase the convection coefficient, since to do so, would require an air handling unit.

COMMENTS: (1) Would replacing the rod by a thick-walled tube provide a practical solution?

(2) The *IHT Thermal Resistance Network Model* and the *Thermal Resistance Tool* for a *fin* with an *adiabatic tip* were used to create a model of the rod. The Workspace is shown below.

// Thermal Resistance Network Model:
// The Network:



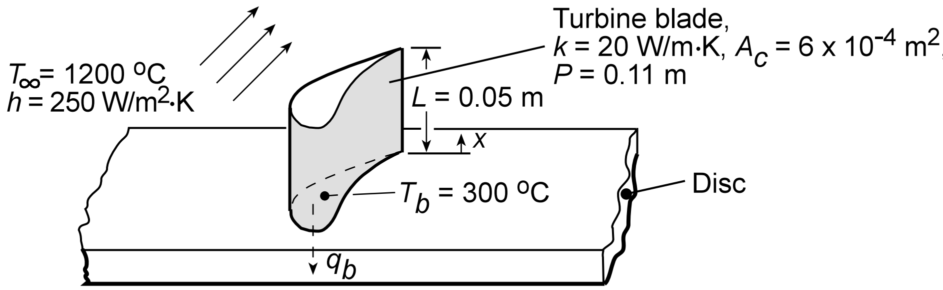

```
// Heat rates into node j,qij, through thermal resistance Rij
q21 = (T2 - T1) / R21
q32 = (T3 - T2) / R32

// Nodal energy balances
q1 + q21 = 0
q2 - q21 + q32 = 0
q3 - q32 = 0

/* Assigned variables list: deselect the qi, Rij and Ti which are unknowns; set qi = 0 for embedded nodal
points at which there is no external source of heat. */
T1 = Tw             // Furnace wall temperature, C
//q1 =              // Heat rate, W
T2 = To             // To, beginning of rod exposed length
q2 =  0             // Heat rate, W; node 2; no external heat source
T3 =  Tinf          // Ambient air temperature, C
//q3 =              // Heat rate, W

// Thermal Resistances:
// Rod - conduction resistance
R21 =  Lins / (k * Ac)        // Conduction resistance, K/W
Ac = pi * D^2 / 4             // Cross sectional area of rod, m^2

// Thermal Resistance Tools - Fin with Adiabatic Tip:
R32 =  Rfin                   //  Resistance of fin, K/W
/* Thermal resistance of a fin of uniform cross sectional area Ac, perimeter P, length L, and thermal
conductivity k with an adiabatic tip condition experiencing convection with a fluid at Tinf and coefficient h, */
Rfin = 1/ ( tanh (m*Lo) * (h * P * k * Ac ) ^ (1/2) )          // Case B, Table 3.4
m = sqrt(h*P / (k*Ac))
P = pi * D                    // Perimeter, m

// Other Assigned Variables:
Tw = 200            // Furnace wall temperature, C
k = 60              // Rod thermal conductivity, W/m.K
Lins = 0.200        // Insulated length, m
D = 0.025           // Rod diameter, m
h = 15              // Convection coefficient, W/m^2.K
Tinf = 25           // Ambient air temperature,C
Lo = 0.200          // Exposed length, m
```

PROBLEM 3.116

KNOWN: Dimensions and thermal conductivity of a gas turbine blade. Temperature and convection coefficient of gas stream. Temperature of blade base and maximum allowable blade temperature.

FIND: (a) Whether blade operating conditions are acceptable, (b) Heat transfer to blade coolant.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction in blade, (2) Constant k, (3) Adiabatic blade tip, (4) Negligible radiation.

ANALYSIS: Conditions in the blade are determined by Case B of Table 3.4.

(a) With the maximum temperature existing at x = L, Eq. 3.75 yields

$$\frac{T(L)-T_\infty}{T_b-T_\infty}=\frac{1}{\cosh mL}$$

$$m=\left(hP/kA_c\right)^{1/2}=\left(250\,W/m^2\cdot K\times 0.11\,m/20\,W/m\cdot K\times 6\times 10^{-4}\,m^2\right)^{1/2}$$

$m = 47.87\ m^{-1}$ and $mL = 47.87\ m^{-1} \times 0.05\ m = 2.39$

From Table B.1, cosh mL = 5.51. Hence,

$$T(L)=1200^\circ C+(300-1200)^\circ C/5.51=1037^\circ C$$ <

and the operating conditions are acceptable.

(b) With $M=\left(hPkA_c\right)^{1/2}\Theta_b=\left(250\,W/m^2\cdot K\times 0.11\,m\times 20\,W/m\cdot K\times 6\times 10^{-4}\,m^2\right)^{1/2}\left(-900^\circ C\right)=-517\,W$,

Eq. 3.76 and Table B.1 yield

$$q_f=M\tanh mL=-517\,W\left(0.983\right)=-508\,W$$

Hence, $q_b=-q_f=508\,W$ <

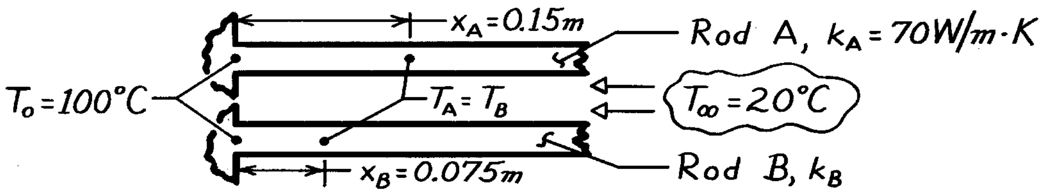
COMMENTS: Radiation losses from the blade surface and convection from the tip will contribute to reducing the blade temperatures.

PROBLEM 3.127

KNOWN: Positions of equal temperature on two long rods of the same diameter, but different thermal conductivity, which are exposed to the same base temperature and ambient air conditions.

FIND: Thermal conductivity of rod B, k_B.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Rods are infinitely long fins of uniform cross-sectional area, (3) Uniform heat transfer coefficient, (4) Constant properties.

ANALYSIS: The temperature distribution for the infinite fin has the form

$$\frac{\theta}{\theta_b} = \frac{T(x) - T_\infty}{T_o - T_\infty} = e^{-mx} \qquad m = \left[\frac{hP}{kA_c}\right]^{1/2}. \tag{1,2}$$

For the two positions prescribed, x_A and x_B, it was observed that

$$T_A(x_A) = T_B(x_B) \qquad \text{or} \qquad \theta_A(x_A) = \theta_B(x_B). \tag{3}$$

Since θ_b is identical for both rods, Eq. (1) with the equality of Eq. (3) requires that

$$m_A x_A = m_B x_B$$

Substituting for m from Eq. (2) gives

$$\left[\frac{hP}{k_A A_c}\right]^{1/2} x_A = \left[\frac{hP}{k_B A_c}\right]^{1/2} x_B.$$

Recognizing that h, P and A_c are identical for each rod and rearranging,

$$k_B = \left[\frac{x_B}{x_A}\right]^2 k_A$$

$$k_B = \left[\frac{0.075\text{m}}{0.15\text{m}}\right]^2 \times 70 \text{ W/m} \cdot \text{K} = 17.5 \text{ W/m} \cdot \text{K}.$$ <

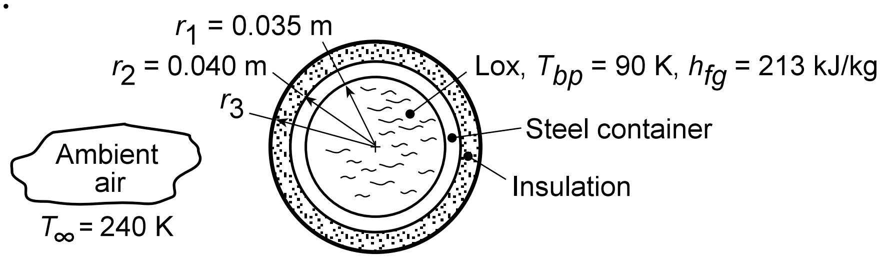
COMMENTS: This approach has been used as a method for determining the thermal conductivity. It has the attractive feature of not requiring power or temperature measurements, assuming of course, a reference material of known thermal conductivity is available.

PROBLEM 3.58

KNOWN: Dimensions of spherical, stainless steel liquid oxygen (LOX) storage container. Boiling point and latent heat of fusion of LOX. Environmental temperature.

FIND: Thermal isolation system which maintains boil-off below 1 kg/day.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conditions, (2) Negligible thermal resistances associated with internal and external convection, conduction in the container wall, and contact between wall and insulation, (3) Negligible radiation at exterior surface, (4) Constant insulation thermal conductivity.

PROPERTIES: *Table A.1*, 304 Stainless steel (T = 100 K): k_s = 9.2 W/m·K; *Table A.3*, Reflective, aluminum foil-glass paper insulation (T = 150 K): k_i = 0.000017 W/m·K.

ANALYSIS: The heat gain associated with a loss of 1 kg/day is

$$q = \dot{m}h_{fg} = \frac{1\,\text{kg/day}}{86,400\,\text{s/day}}\left(2.13\times10^5\ \text{J/kg}\right) = 2.47\,\text{W}$$

With an overall temperature difference of $\left(T_\infty - T_{bp}\right)$ = 150 K, the corresponding total thermal resistance is

$$R_{tot} = \frac{\Delta T}{q} = \frac{150\,\text{K}}{2.47\,\text{W}} = 60.7\,\text{K/W}$$

Since the conduction resistance of the steel wall is

$$R_{t,cond,s} = \frac{1}{4\pi k_s}\left(\frac{1}{r_1}-\frac{1}{r_2}\right) = \frac{1}{4\pi\left(9.2\,\text{W/m}\cdot\text{K}\right)}\left(\frac{1}{0.35\,\text{m}}-\frac{1}{0.40\,\text{m}}\right) = 2.4\times10^{-3}\,\text{K/W}$$

it is clear that exclusive reliance must be placed on the insulation and that a special insulation of very low thermal conductivity should be selected. The best choice is a highly reflective foil/glass matted insulation which was developed for cryogenic applications. It follows that

$$R_{t,cond,i} = 60.7\,\text{K/W} = \frac{1}{4\pi k_i}\left(\frac{1}{r_2}-\frac{1}{r_3}\right) = \frac{1}{4\pi\left(0.000017\,\text{W/m}\cdot\text{K}\right)}\left(\frac{1}{0.40\,\text{m}}-\frac{1}{r_3}\right)$$

which yields r_3 = 0.4021 m. The minimum insulation thickness is therefore $\delta = (r_3 - r_2)$ = 2.1 mm.

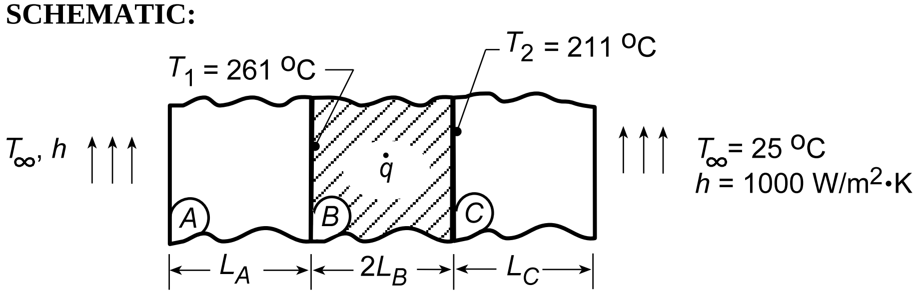
COMMENTS: The heat loss could be reduced well below the maximum allowable by adding more insulation. Also, in view of weight restrictions associated with launching space vehicles, consideration should be given to fabricating the LOX container from a lighter material.

PROBLEM 3.73

KNOWN: Composite wall with outer surfaces exposed to convection process.

FIND: (a) Volumetric heat generation and thermal conductivity for material B required for special conditions, (b) Plot of temperature distribution, (c) T_1 and T_2, as well as temperature distributions corresponding to loss of coolant condition where h = 0 on surface A.

SCHEMATIC:



$L_A = 30$ mm
$L_B = 30$ mm
$L_C = 20$ mm
$k_A = 25$ W/m·K
$k_C = 50$ W/m·K

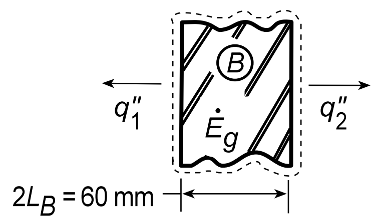
ASSUMPTIONS: (1) Steady-state, one-dimensional heat transfer, (2) Negligible contact resistance at interfaces, (3) Uniform generation in B; zero in A and C.

ANALYSIS: (a) From an energy balance on wall B,

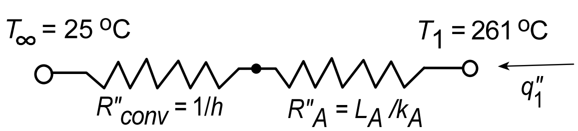
$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st}$$

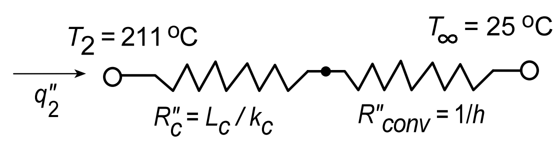
$$-q_1'' - q_2'' + 2\dot{q}L_B = 0$$

$$\dot{q}_B = \left(q_1'' + q_2''\right)/2L_B \ .$$



To determine the heat fluxes, q_1'' and q_2'', construct thermal circuits for A and C:





$$q_1'' = \left(T_1 - T_\infty\right)/\left(1/h + L_A/k_A\right) \qquad q_2'' = \left(T_2 - T_\infty\right)/\left(L_C/k_C + 1/h\right)$$

$$q_1'' = \left(261 - 25\right)^\circ C \Big/ \left(\frac{1}{1000\ W/m^2 \cdot K} + \frac{0.030\ m}{25\ W/m \cdot K}\right) \qquad q_2'' = \left(211 - 25\right)^\circ C \Big/ \left(\frac{0.020\ m}{50\ W/m \cdot K} + \frac{1}{1000\ W/m^2 \cdot K}\right)$$

$$q_1'' = 236^\circ C/\left(0.001 + 0.0012\right) m^2 \cdot K/W \qquad q_2'' = 186^\circ C/\left(0.0004 + 0.001\right) m^2 \cdot K/W$$

$$q_1'' = 107,273\ W/m^2 \qquad q_2'' = 132,857\ W/m^2$$

Using the values for q_1'' and q_2'' in Eq. (1), find

$$\dot{q}_B = \left(106,818 + 132,143\ W/m^2\right) / 2 \times 0.030\ m = 4.00 \times 10^6\ W/m^3 \ . \qquad <$$

To determine k_B, use the general form of the temperature and heat flux distributions in wall B,

$$T(x) = -\frac{\dot{q}_B}{2k_B}x^2 + C_1 x + C_2 \qquad q_x''(x) = -k_B\left[-\frac{\dot{q}}{k_B}x + C_1\right] \tag{1,2}$$

there are 3 unknowns, C_1, C_2 and k_B, which can be evaluated using three conditions,

Continued...

PROBLEM 3.73 (Cont.)

$$T(-L_B) = T_1 = -\frac{\dot{q}_B}{2k_B}(-L_B)^2 - C_1 L_B + C_2 \qquad \text{where } T_1 = 261°C \qquad (3)$$

$$T(+L_B) = T_2 = -\frac{\dot{q}_B}{2k_B}(+L_B)^2 + C_1 L_B + C_2 \qquad \text{where } T_2 = 211°C \qquad (4)$$

$$q''_x(-L_B) = -q''_1 = -k_B\left[-\frac{\dot{q}_B}{k_B}(-L_B) + C_1\right] \qquad \text{where } q''_1 = 107{,}273\ \text{W/m}^2 \qquad (5)$$

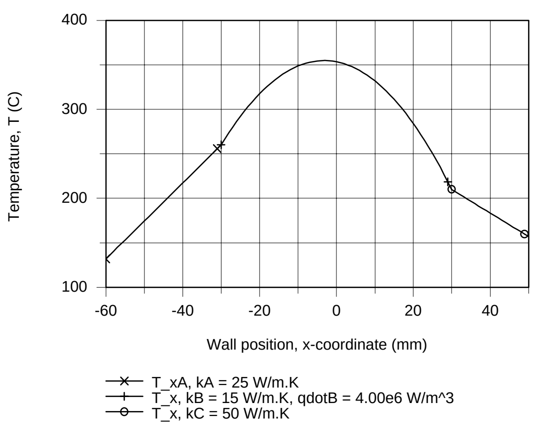
Using IHT to solve Eqs. (3), (4) and (5) simultaneously with $\dot{q}_B = 4.00 \times 10^6$ W/m^3, find

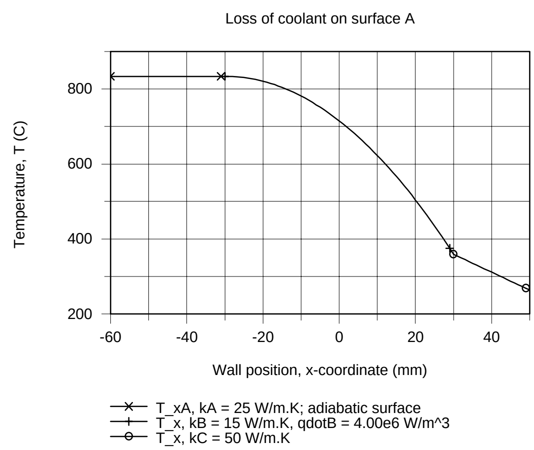
$$k_B = 15.3\ \text{W/m}\cdot\text{K}$$ <

(b) Following the method of analysis in the *IHT Example 3.6, User-Defined Functions*, the temperature distribution is shown in the plot below. The important features are (1) Distribution is quadratic in B, but non-symmetrical; linear in A and C; (2) Because thermal conductivities of the materials are different, discontinuities exist at each interface; (3) By comparison of gradients at x = -L_B and +L_B, find $q''_2 > q''_1$.

(c) Using the same method of analysis as for Part (c), the temperature distribution is shown in the plot below when h = 0 on the surface of A. Since the left boundary is adiabatic, material A will be isothermal at T_1. Find

T_1 = 835°C $\qquad$ T_2 = 360°C <



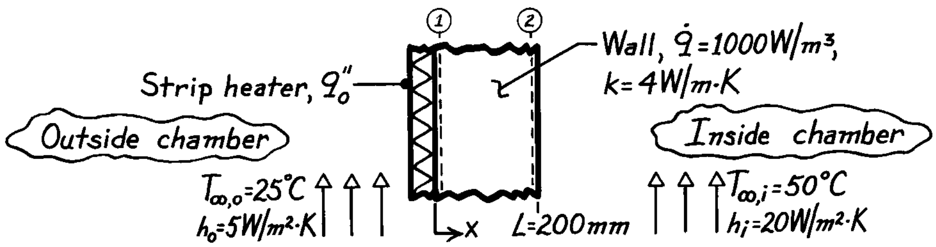


PROBLEM 3.79

KNOWN: Wall of thermal conductivity k and thickness L with uniform generation $\dot{q}$; strip heater with uniform heat flux q''_o; prescribed inside and outside air conditions (h_i, $T_{\infty,i}$, h_o, $T_{\infty,o}$).

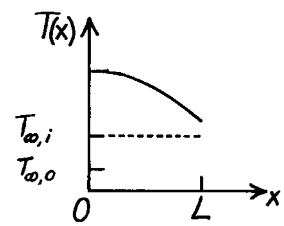
FIND: (a) Sketch temperature distribution in wall if none of the heat generated within the wall is lost to the outside air, (b) Temperatures at the wall boundaries T(0) and T(L) for the prescribed condition, (c) Value of q''_o required to maintain this condition, (d) Temperature of the outer surface, T(L), if $\dot{q}=0$ but q''_o corresponds to the value calculated in (c).

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Uniform volumetric generation, (4) Constant properties.

ANALYSIS: (a) If none of the heat generated within the wall is lost to the *outside* of the chamber, the gradient at x = 0 must be zero. Since $\dot{q}$ is uniform, the temperature distribution is parabolic, with $T(L) > T_{\infty,i}$.



(b) To find temperatures at the boundaries of wall, begin with the general solution to the appropriate form of the heat equation (Eq.3.40).

$$T(x) = -\frac{\dot{q}}{2k}x^2 + C_1 x + C_2 \tag{1}$$

From the first boundary condition,

$$\left.\frac{dT}{dx}\right|_{x=o} = 0 \quad \rightarrow \quad C_1 = 0. \tag{2}$$

Two approaches are possible using different forms for the second boundary condition.

Approach No. 1: With boundary condition $\rightarrow$ $T(0) = T_1$

$$T(x) = -\frac{\dot{q}}{2k}x^2 + T_1 \tag{3}$$

To find T_1, perform an overall energy balance on the wall

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0$$

$$-h\left[T(L) - T_{\infty,i}\right] + \dot{q}L = 0 \qquad T(L) = T_2 = T_{\infty,i} + \frac{\dot{q}L}{h} \tag{4}$$

Continued …..

and from Eq. (3) with x = L and T(L) = T_2,

$$T(L) = -\frac{\dot{q}}{2k}L^2 + T_1 \qquad \text{or} \qquad T_1 = T_2 + \frac{\dot{q}}{2k}L^2 = T_{\infty,i} + \frac{\dot{q}L}{h} + \frac{\dot{q}L^2}{2k} \tag{5,6}$$

Substituting numerical values into Eqs. (4) and (6), find

$$T_2 = 50^\circ\text{C} + 1000\ \text{W/m}^3 \times 0.200\ \text{m}/20\ \text{W/m}^2\cdot\text{K} = 50^\circ\text{C} + 10^\circ\text{C} = 60^\circ\text{C}$$ <

$$T_1 = 60^\circ\text{C} + 1000\ \text{W/m}^3 \times (0.200\ \text{m})^2 / 2\times 4\ \text{W/m}\cdot\text{K} = 65^\circ\text{C}.$$ <

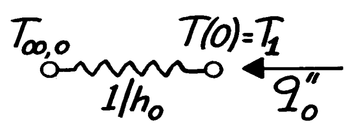
Approach No. 2: Using the boundary condition

$$-k\frac{dT}{dx}\bigg|_{x=L} = h\left[T(L) - T_{\infty,i}\right]$$

yields the following temperature distribution which can be evaluated at x = 0,L for the required temperatures,

$$T(x) = -\frac{\dot{q}}{2k}\left(x^2 - L^2\right) + \frac{\dot{q}L}{h} + T_{\infty,i}.$$

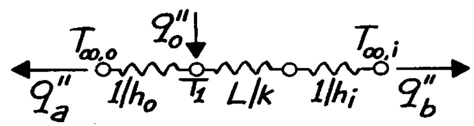
(c) The value of q''_o when T(0) = T_1 = 65°C follows from the circuit



$$q''_o = \frac{T_1 - T_{\infty,o}}{1/h_o}$$

$$q''_o = 5\ \text{W/m}^2\cdot\text{K}\,(65\text{-}25)^\circ\text{C} = 200\ \text{W/m}^2.$$ <

(d) With $\dot{q}=0$, the situation is represented by the thermal circuit shown. Hence,



$$q''_o = q''_a + q''_b$$

$$q''_o = \frac{T_1 - T_{\infty,o}}{1/h_o} + \frac{T_1 - T_{\infty,i}}{L/k + 1/h_i}$$

which yields

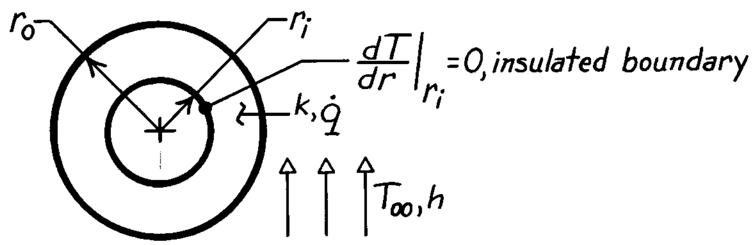
$$T_1 = 55^\circ\text{C}.$$ <

PROBLEM 3.84

KNOWN: Cylindrical shell with uniform volumetric generation is insulated at inner surface and exposed to convection on the outer surface.

FIND: (a) Temperature distribution in the shell in terms of r_i, r_o, $\dot{q}$, h, T_∞ and k, (b) Expression for the heat rate per unit length at the outer radius, $q'(r_o)$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial (cylindrical) conduction in shell, (3) Uniform generation, (4) Constant properties.

ANALYSIS: (a) The general form of the temperature distribution and boundary conditions are

$$T(r) = -\frac{\dot{q}}{4k} r^2 + C_1 \ln r + C_2$$

at $r = r_i$: $\qquad \left.\frac{dT}{dr}\right)_{r_i} = 0 = -\frac{\dot{q}}{2k} r_i + C_1 \frac{1}{r_i} + 0 \qquad C_1 = \frac{\dot{q}}{2k} r_i^2$

at $r = r_o$: $\qquad -k \left.\frac{dT}{dr}\right)_{r_o} = h\left[T(r_o) - T_\infty\right] \qquad$ surface energy balance

$$k\left[-\frac{\dot{q}}{2k} r_o + \left(\frac{\dot{q}}{2k} r_i^2 \cdot \frac{1}{r_o}\right)\right] = h\left[-\frac{\dot{q}}{4k} r_o^2 + \left(\frac{\dot{q}}{2k} r_i^2\right) \ln r_o + C_2 - T_\infty\right]$$

$$C_2 = -\frac{\dot{q} r_o}{2h}\left[1 + \left(\frac{r_i}{r_o}\right)^2\right] + \frac{\dot{q} r_o^2}{2k}\left[\frac{1}{2} - \left(\frac{r_i}{r_o}\right)^2 \ln r_o\right] + T_\infty$$

Hence,

$$T(r) = \frac{\dot{q}}{4k}\left(r_o^2 - r^2\right) + \frac{\dot{q} r_i^2}{2k} \ln\left(\frac{r}{r_o}\right) - \frac{\dot{q} r_o}{2h}\left[1 + \left(\frac{r_i}{r_o}\right)^2\right] + T_\infty. \qquad <$$

(b) From an overall energy balance on the shell,

$$q_r'(r_o) = \dot{E}_g' = \dot{q}\pi\left(r_o^2 - r_i^2\right). \qquad <$$

Alternatively, the heat rate may be found using Fourier's law and the temperature distribution,

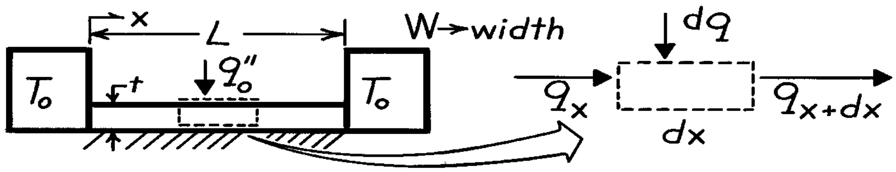
$$q_r'(r) = -k(2\pi\, r_o) \left.\frac{dT}{dr}\right)_{r_o} = -2\pi\, k r_o\left[-\frac{\dot{q}}{2k} r_o + \frac{\dot{q} r_i^2}{2k} \frac{1}{r_o} + 0 + 0\right] = \dot{q}\pi\left(r_o^2 - r_i^2\right)$$

PROBLEM 3.101

KNOWN: Dimensions of a plate insulated on its bottom and thermally joined to heat sinks at its ends. Net heat flux at top surface.

FIND: (a) Differential equation which determines temperature distribution in plate, (b) Temperature distribution and heat loss to heat sinks.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction in x (W,L>>t), (3) Constant properties, (4) Uniform surface heat flux, (5) Adiabatic bottom, (6) Negligible contact resistance.

ANALYSIS: (a) Applying conservation of energy to the differential control volume, $q_x + dq = q_{x+dx}$, where $q_{x+dx} = q_x + (dq_x/dx)\,dx$ and $dq = q_o''\,(W\cdot dx)$. Hence, $(dq_x/dx) - q_o''\,W = 0$. From Fourier's law, $q_x = -k(t\cdot W)\,dT/dx$. Hence, the differential equation for the temperature distribution is

$$-\frac{d}{dx}\left[ktW\,\frac{dT}{dx}\right] - q_o''\,W = 0 \qquad \frac{d^2T}{dx^2} + \frac{q_o''}{kt} = 0. \qquad <$$

(b) Integrating twice, the general solution is,

$$T(x) = -\frac{q_o''}{2kt}x^2 + C_1\,x + C_2$$

and appropriate boundary conditions are $T(0) = T_o$, and $T(L) = T_o$. Hence, $T_o = C_2$, and

$$T_o = -\frac{q_o''}{2kt}L^2 + C_1L + C_2 \qquad \text{and} \qquad C_1 = \frac{q_o''L}{2kt}.$$

Hence, the temperature distribution is

$$T(x) = -\frac{q_o''L}{2kt}\left(x^2 - Lx\right) + T_o. \qquad <$$

Applying Fourier's law at x = 0, and at x = L,

$$q(0) = -k(Wt)\;dT/dx)_{x=0} = -kWt\left[-\frac{q_o''}{kt}\right]\left[x - \frac{L}{2}\right]\Bigg|_{x=0} = -\frac{q_o''WL}{2}$$

$$q(L) = -k(Wt)dT/dx)_{x=L} = -kWt\left[-\frac{q_o''}{kt}\right]\left[x - \frac{L}{2}\right]\Bigg|_{x=L} = +\frac{q_o''WL}{2}$$

Hence the heat loss from the plates is $q = 2(q_o''WL/2) = q_o''WL$. $<$

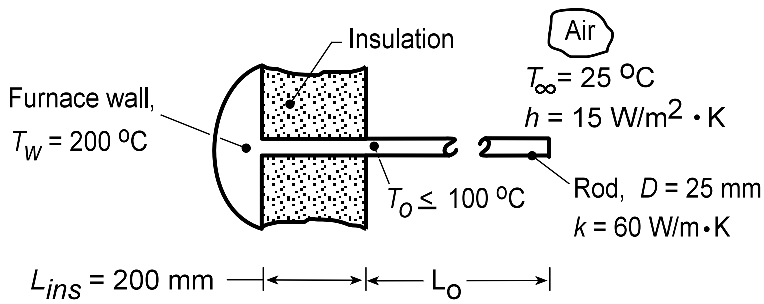
COMMENTS: (1) Note signs associated with q(0) and q(L). (2) Note symmetry about x = L/2. Alternative boundary conditions are $T(0) = T_o$ and $dT/dx)_{x=L/2} = 0$.

PROBLEM 3.111

KNOWN: Rod protruding normally from a furnace wall covered with insulation of thickness L_{ins} with the length L_o exposed to convection with ambient air.

FIND: (a) An expression for the exposed surface temperature T_o as a function of the prescribed thermal and geometrical parameters. (b) Will a rod of L_o = 100 mm meet the specified operating limit, $T_0 \le 100°C$? If not, what design parameters would you change?

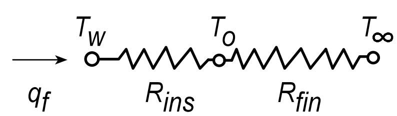
SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in rod, (3) Negligible thermal contact resistance between the rod and hot furnace wall, (4) Insulated section of rod, L_{ins}, experiences no lateral heat losses, (5) Convection coefficient uniform over the exposed portion of the rod, L_o, (6) Adiabatic tip condition for the rod and (7) Negligible radiation exchange between rod and its surroundings.

ANALYSIS: (a) The rod can be modeled as a thermal network comprised of two resistances in series: the portion of the rod, L_{ins}, covered by insulation, R_{ins}, and the portion of the rod, L_o, experiencing convection, and behaving as a fin with an adiabatic tip condition, R_{fin}. For the insulated section:

$$R_{ins} = L_{ins}/kA_c \qquad (1)$$



For the fin, Table 3.4, Case B, Eq. 3.76,

$$R_{fin} = \theta_b/q_f = \frac{1}{(hPkA_c)^{1/2}\tanh(mL_o)} \qquad (2)$$

$$m = (hP/kA_c)^{1/2} \qquad A_c = \pi D^2/4 \qquad P = \pi D \qquad (3,4,5)$$

From the thermal network, by inspection,

$$\frac{T_o - T_\infty}{R_{fin}} = \frac{T_w - T_\infty}{R_{ins} + R_{fin}} \qquad T_o = T_\infty + \frac{R_{fin}}{R_{ins} + R_{fin}}(T_w - T_\infty) \qquad (6) \quad <$$

(b) Substituting numerical values into Eqs. (1) - (6) with L_o = 200 mm,

$$T_o = 25^\circ C + \frac{6.298}{6.790 + 6.298}(200 - 25)^\circ C = 109^\circ C \quad <$$

$$R_{ins} = \frac{0.200\,m}{60\,W/m\cdot K \times 4.909\times 10^{-4}\,m^2} = 6.790\,K/W \qquad A_c = \pi(0.025\,m)^2/4 = 4.909\times 10^{-4}\,m^2$$

$$R_{fin} = 1\Big/\left(0.0347\,W^2/K^2\right)^{1/2}\tanh(6.324\times 0.200) = 6.298\,K/W$$

$$(hPkA_c) = \left(15\,W/m^2\cdot K \times \pi(0.025\,m) \times 60\,W/m\cdot K \times 4.909\times 10^{-4}\,m^2\right) = 0.0347\,W^2/K^2$$

Continued...

PROBLEM 3.111 (Cont.)

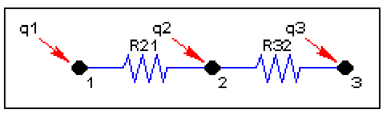
$$m = \left(hP/kA_c\right)^{1/2} = \left(15\,\mathrm{W/m^2\cdot K}\times\pi\,(0.025\,\mathrm{m})\Big/60\,\mathrm{W/m\cdot K}\times 4.909\times10^{-4}\,\mathrm{m^2}\right)^{1/2} = 6.324\,\mathrm{m^{-1}}$$

Consider the following design changes aimed at reducing $T_o \le 100°C$. (1) Increasing length of the fin portions: with L_o = 200 mm, the fin already behaves as an infinitely long fin. Hence, increasing L_o will not result in reducing T_o. (2) Decreasing the thermal conductivity: backsolving the above equation set with T_0 = 100°C, find the required thermal conductivity is k = 14 W/m·K. Hence, we could select a stainless steel alloy; see Table A.1. (3) Increasing the insulation thickness: find that for T_o = 100°C, the required insulation thickness would be L_{ins} = 211 mm. This design solution might be physically and economically unattractive. (4) A very practical solution would be to introduce thermal contact resistance between the rod base and the furnace wall by "tack welding" (rather than a continuous bead around the rod circumference) the rod in two or three places. (5) A less practical solution would be to increase the convection coefficient, since to do so, would require an air handling unit.

COMMENTS: (1) Would replacing the rod by a thick-walled tube provide a practical solution?

(2) The *IHT Thermal Resistance Network Model* and the *Thermal Resistance Tool* for a *fin* with an *adiabatic tip* were used to create a model of the rod. The Workspace is shown below.

// Thermal Resistance Network Model:
// The Network:



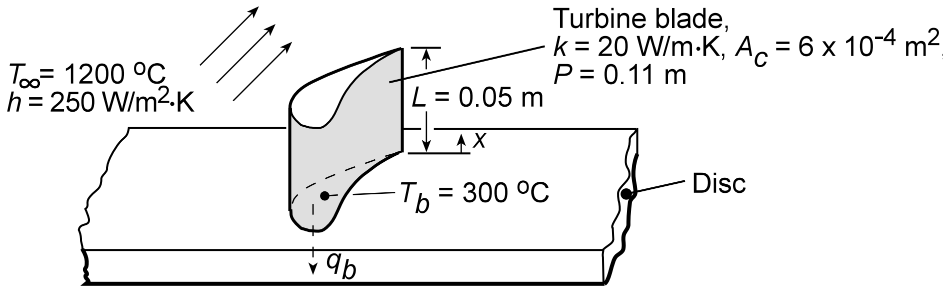

```
// Heat rates into node j,qij, through thermal resistance Rij
q21 = (T2 - T1) / R21
q32 = (T3 - T2) / R32

// Nodal energy balances
q1 + q21 = 0
q2 - q21 + q32 = 0
q3 - q32 = 0

/* Assigned variables list: deselect the qi, Rij and Ti which are unknowns; set qi = 0 for embedded nodal
points at which there is no external source of heat. */
T1 = Tw          // Furnace wall temperature, C
//q1 =           // Heat rate, W
T2 = To          // To, beginning of rod exposed length
q2 =  0          // Heat rate, W; node 2; no external heat source
T3 =  Tinf       // Ambient air temperature, C
//q3 =           // Heat rate, W

// Thermal Resistances:
// Rod - conduction resistance
R21 =  Lins / (k * Ac)       // Conduction resistance, K/W
Ac = pi * D^2 / 4            // Cross sectional area of rod, m^2

// Thermal Resistance Tools - Fin with Adiabatic Tip:
R32 =  Rfin                  //  Resistance of fin, K/W
/* Thermal resistance of a fin of uniform cross sectional area Ac, perimeter P, length L, and thermal
conductivity k with an adiabatic tip condition experiencing convection with a fluid at Tinf and coefficient h, */
Rfin = 1/ ( tanh (m*Lo) * (h * P * k * Ac ) ^ (1/2) )       // Case B, Table 3.4
m = sqrt(h*P / (k*Ac))
P = pi * D                   // Perimeter, m

// Other Assigned Variables:
Tw = 200         // Furnace wall temperature, C
k = 60           // Rod thermal conductivity, W/m.K
Lins = 0.200     // Insulated length, m
D = 0.025        // Rod diameter, m
h = 15           // Convection coefficient, W/m^2.K
Tinf = 25        // Ambient air temperature,C
Lo = 0.200       // Exposed length, m
```

PROBLEM 3.116

KNOWN: Dimensions and thermal conductivity of a gas turbine blade. Temperature and convection coefficient of gas stream. Temperature of blade base and maximum allowable blade temperature.

FIND: (a) Whether blade operating conditions are acceptable, (b) Heat transfer to blade coolant.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction in blade, (2) Constant k, (3) Adiabatic blade tip, (4) Negligible radiation.

ANALYSIS: Conditions in the blade are determined by Case B of Table 3.4.

(a) With the maximum temperature existing at x = L, Eq. 3.75 yields

$$\frac{T(L)-T_\infty}{T_b-T_\infty}=\frac{1}{\cosh mL}$$

$$m=\left(hP/kA_c\right)^{1/2}=\left(250\,W/m^2\cdot K\times 0.11\,m/20\,W/m\cdot K\times 6\times 10^{-4}\,m^2\right)^{1/2}$$

$m = 47.87\ m^{-1}$ and $mL = 47.87\ m^{-1} \times 0.05\ m = 2.39$

From Table B.1, cosh mL = 5.51. Hence,

$$T(L)=1200^\circ C+(300-1200)^\circ C/5.51=1037^\circ C$$ <

and the operating conditions are acceptable.

(b) With $M=\left(hPkA_c\right)^{1/2}\Theta_b=\left(250\,W/m^2\cdot K\times 0.11\,m\times 20\,W/m\cdot K\times 6\times 10^{-4}\,m^2\right)^{1/2}\left(-900^\circ C\right)=-517\,W$,
Eq. 3.76 and Table B.1 yield

$$q_f=M\tanh mL=-517\,W\,(0.983)=-508\,W$$

Hence, $q_b=-q_f=508\,W$ <

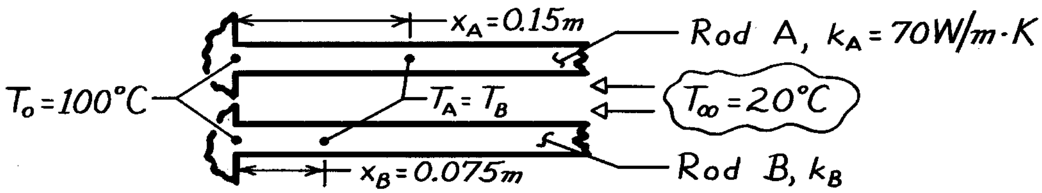
COMMENTS: Radiation losses from the blade surface and convection from the tip will contribute to reducing the blade temperatures.

PROBLEM 3.127

KNOWN: Positions of equal temperature on two long rods of the same diameter, but different thermal conductivity, which are exposed to the same base temperature and ambient air conditions.

FIND: Thermal conductivity of rod B, k_B.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Rods are infinitely long fins of uniform cross-sectional area, (3) Uniform heat transfer coefficient, (4) Constant properties.

ANALYSIS: The temperature distribution for the infinite fin has the form

$$\frac{\theta}{\theta_b} = \frac{T(x) - T_\infty}{T_o - T_\infty} = e^{-mx} \qquad m = \left[\frac{hP}{kA_c}\right]^{1/2}. \tag{1,2}$$

For the two positions prescribed, x_A and x_B, it was observed that

$$T_A(x_A) = T_B(x_B) \qquad \text{or} \qquad \theta_A(x_A) = \theta_B(x_B). \tag{3}$$

Since θ_b is identical for both rods, Eq. (1) with the equality of Eq. (3) requires that

$$m_A x_A = m_B x_B$$

Substituting for m from Eq. (2) gives

$$\left[\frac{hP}{k_A A_c}\right]^{1/2} x_A = \left[\frac{hP}{k_B A_c}\right]^{1/2} x_B.$$

Recognizing that h, P and A_c are identical for each rod and rearranging,

$$k_B = \left[\frac{x_B}{x_A}\right]^2 k_A$$

$$k_B = \left[\frac{0.075\text{m}}{0.15\text{m}}\right]^2 \times 70 \text{ W/m} \cdot \text{K} = 17.5 \text{ W/m} \cdot \text{K}.$$ <

COMMENTS: This approach has been used as a method for determining the thermal conductivity. It has the attractive feature of not requiring power or temperature measurements, assuming of course, a reference material of known thermal conductivity is available.